EE 120: Signals and Systems
Department of Electrical Engineering and Computer Sciences — MIDTERM 1
UC BERKELEY — 20 February 2019

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Discussion Section Time: ______________ SID (All Digits): ______________

- **(20 Points)** Print your *official* name (not your e-mail address) and *all* digits of your student ID number legibly, and indicate your lab time, on *every* page.
- This exam should take up to 100 minutes to complete. However, you may use up to a maximum of 110 minutes *in one sitting*, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided $8.5" \times 11"$ sheet of handwritten, original notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- We will provide you with scratch paper. Do not use your own.
- **The exam printout consists of pages numbered 1 through 10.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the ten numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't evaluate it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

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MT1.1 (50 Points) Consider a DT-LTI filter F whose impulse response is given by

$$\forall n \in \mathbb{Z}, \quad f(n) = \delta(n) + \delta(n-1) + \delta(n-2).$$

(a) (20 Points) The frequency response of the filter can be expressed as follows:

$$\forall \omega \in \mathbb{R}, \quad F(\omega) = A(\omega) e^{i\alpha\omega},$$

where $A(\omega) = 1 + 2\cos(\omega)$ and α is a real-valued quantity.

(i) (5 Points) Determine α explicitly.

(ii) (10 Points) Provide a well-labeled plot of the magnitude response $|F(\omega)|$.

(iii) (5 Points) Let the input to the filter be

$$\forall n \in \mathbb{Z}, \quad x(n) = 1 + (-1)^n + \cos\left(\frac{2\pi n}{3}\right).$$

Determine a reasonably simple expression for, and provide a well-labeled plot of, the corresponding output $y(n)$.

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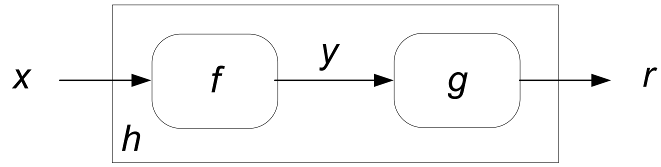
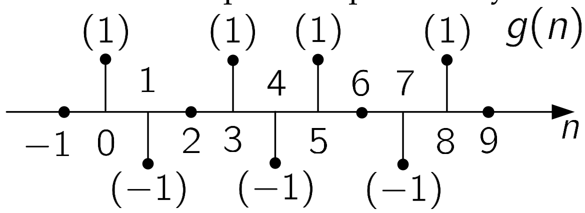
(b) (30 Points) We place the filter F in cascade (series) with another DT-LTI filter G:



All the nonzero values of the impulse response of system G appear below:



(i) (10 Points) Show that the impulse response of the overall system H is

$$\forall n \in \mathbb{Z}, \quad h(n) = \delta(n) + \delta(n-5) + \delta(n-10).$$

(ii) (5 Points) Determine a reasonably expression for the output $r(n)$ if the input is

$$\forall n \in \mathbb{Z}, \quad x(n) = 1 + \cos\left(\frac{\pi n}{5}\right) + \sin\left(\frac{2\pi n}{15}\right).$$

(iii) (15 Points) We can factor the polynomial $p(z) = z^{10} + z^5 + 1$ as $p(z) = a(z)\, b(z)$, where

$$a(z) = a_8 z^8 + a_7 z^7 + \cdots + a_1 z + a_0 \quad \text{and} \quad b(z) = b_2 z^2 + b_1 z + b_0.$$

Determine the factorization polynomials $a(z)$ and $b(z)$ explicitly (by determining their respective coefficients numerically).

All the coefficients are integers. Don't do anything wild. Think first before you tackle this part. It should not involve much work (despite the large space we've given you below). The result should be stunningly beautiful, as should the process of getting there.

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MT1.2 (40 Points) Consider the DT-LTI filter F whose input-output behavior is described by

$$\forall n \in \mathbb{Z}, \quad y(n) = x(n) - 0.9\, x(n-1).$$

(a) (10 Points) Determine and provide a well-labeled plot of, $f(n)$, the impulse response of the filter.

(b) (5 Points) Is the system BIBO stable? Provide a succinct, yet clear and convincing explanation.

(c) (10 Points) Determine the output of the system in response to the input

$$\forall n \in \mathbb{Z}, \quad x(n) = 0.9^n\, u(n) + 0.9^n\, u(-n).$$

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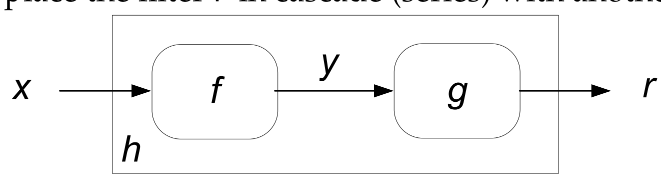
(d) (10 Points) Determine a reasonably simple expression for the frequency response $F(\omega)$, and provide well-labeled plots of the magnitude response $|F(\omega)|$ and phase response $\angle F(\omega)$.

(e) (5 Points) We place the filter F in cascade (series) with another DT-LTI filter G:



Determine the impulse response $g(n)$ and frequency response $G(\omega)$ of the system G, such that $r(n) = x(n)$. In other words, we want the system G to be the *inverse* of the system F.

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MT1.3 (40 Points) The input-output behavior of a BIBO stable, discrete-time LTI filter F is described by the linear, constant-coefficient difference equation

$$y(n) = \gamma\, y(n-1) - \gamma^*\, x(n) + x(n-1), \quad \text{where} \quad |\gamma| < 1.$$

(a) (10 Points) Provide a delay-adder-gain block diagram implementation of the filter. Your implementation must use the minimal number delay blocks needed.

(b) (20 Points) Show that the frequency response of the filter is given by

$$\forall \omega \in \mathbb{R}, \quad F(\omega) = \frac{e^{-i\omega} - \gamma^*}{1 - \gamma\, e^{-i\omega}},$$

and provide a well-labeled plot of the magnitude response $|F(\omega)|$.

(c) (10 Points) Determine a reasonably simple expression for $f(n)$, the impulse response of the filter.

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MT1.4 (50 Points) The impulse response of a CT-LTI filter H is given by

$$h(t) = \delta(t) - 2\alpha e^{-\alpha t} u(t),$$

where $\alpha > 0$. In one or more parts of this problem, you may or may not find it useful to know that if the impulse response of a BIBO stable continuous-time LTI system is

$$\beta\, e^{-\alpha t} u(t), \text{then its frequency response is } \frac{\beta}{i\omega + \alpha},$$

where α and β are, in general, complex scalars, with the value of α consistent with BIBO stability.

(a) (30 Points) Determine a reasonably simple expression for $H(\omega)$, the frequency response of the filter, and provide well-labeled plots for the magnitude response $|H(\omega)|$ and phase response $\angle H(\omega)$.

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(b) (10 Points) Determine a reasonably simple expression for, and provide a well-labeled plot of, the filter's unit-step response $s(t)$. Recall that if the input is $x(t) = u(t)$, the corresponding output is $y(t) = s(t)$ and is called the *unit-step response*.

(c) (10 Points) Show that the input-output behavior of the filter is described by the linear, constant-coefficient differential equation

$$\dot{y}(t) + \alpha\, y(t) = \dot{x}(t) - \alpha\, x(t).$$

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You may use this page for scratch work only.
With the exception of your identifying information above,
no other writing on this page will be graded.