NE 180
Midterm II
Fall Semester 2000
Solutions

November 15, 2000

1. a.

$$q_\perp^i = -\kappa_\perp^i \nabla T_i$$

$$= \frac{2.0 n_i k T_i}{m_i \omega_{ci}^2 \tau_i} \nabla T_i$$

$$T_i(r) = T_i(0)(1 - 0.9^2) = 2.85\,\text{keV}$$

$$n\tau_i = \frac{2.09 \times 10^7 (m_i/m_p)^{1/2} T_i(eV)^{3/2}}{\ln \Lambda}\ \text{(cgs units)}$$

$$= 2.87 \times 10^{11}\text{cm}^{-3}\text{s}^{-1}$$

$$= 2.87 \times 10^{17}\text{m}^{-3}\text{s}^{-1}$$

$$n_i = 2 \times 10^{20}(1 - 0.9^2) = 3.8 \times 10^{18}\text{m}^{-3}$$

$$\tau_i = n\tau_i / n_i = 7.5607 \times 10^{-3}\text{s}$$

$$\omega_{ci} = qB/m_i = (1.67{\times}10^{-19})(4)/(2.5{\times}1.67{\times}10^{-27}) = 1.53293{\times}10^8\ \text{s}^{-1}$$

$$\kappa_\perp^i = \frac{(2.0)(3.8 \times 10^{18})(2.85 \times 10^3)(1.6 \times 10^{-19})}{(2.5 \times 1.67 \times 10^{-27})(1.53293 \times 10^8)^2 (7.5607 \times 10^{-3})}$$

$$= 4.67 \times 10^{16}\text{m}^{-1}\,\text{s}^{-1}$$

$$\nabla T_i = -2(0.9)/2T_i(0) = -1.35 \times 10^4 \,\mathrm{eV\,m^{-1}}$$
$$= 2.16 \times 10^{-15} \,\mathrm{J\,m^{-1}}$$
$$q_i = (2.16 \times 10^{-15})(4.67 \times 10^{16}) = 100.918 \mathrm{W\,m^{-2}}$$
$$\mathrm{Area} = 4\pi^2 Rr = 426.377 \mathrm{m^2}$$
$$\boxed{P^i_{cond} = q_i \cdot A = 43.0 \text{ kW}}$$

1. b.

$$\eta_\parallel = 2.8 \times 10^{-8} T_e (keV)^{-3/2} \Omega\,\mathrm{m}$$
$$\boxed{= 8.85 \times 10^{-10} \,\Omega\,\mathrm{m}}$$

1. c.

$$\text{Plasma resistance} = \mathcal{R} = 2\eta_\parallel R/a^2 = 2.66 \times 10^{-9} \Omega$$
$$\boxed{P_\Omega = I^2 \mathcal{R} = 265.6 \,\mathrm{kW}}$$

1. d.

$$q^e_\parallel = -\kappa^e_\parallel \nabla_\parallel T_e$$
$$= \frac{3.2 n_e k T_e \tau_e}{m_e} \nabla_\parallel T_e$$
$$n\tau_e = \frac{3.44 \times 10^5 T_i (eV)^{3/2}}{\ln \Lambda} \text{ (cgs units)}$$
$$= 1.965 \times 10^{10} \mathrm{cm^{-3}s}$$
$$= 1.965 \times 10^{16} \mathrm{m^{-3}s}$$
$$\kappa^e_\parallel = \frac{(3.2)(10^4)(1.6 \times 10^{-19})(1.965 \times 10^{16})}{9.11 \times 10^{-31}}$$

$$= 1.1 \times 10^{32} \mathrm{m}^{-1}\,\mathrm{s}^{-1}$$

$$q_{\parallel}^{e} = (1.1 \times 10^{32})(10^{4})(1.6 \times 10^{-19})/1000$$

$$= 1.768 \times 10^{14}\,\mathrm{W\,m}^{-2}$$

$$\boxed{f = \frac{P_{cond}^{e}}{A q_{\parallel}^{e}} = \frac{5 \times 10^{7}}{(1.768 \times 10^{14})(426.367)} = 6.63 \times 10^{-10}}$$

2. a.

$$f_B = \frac{\rho R}{\rho R + 6} = 0.625$$

$$Y = 10^{8} \rightarrow f_B M = \frac{10^{8}(5)}{(17.6 \times 10^{6})(1.6 \times 10^{-19})(6.02 \times 10^{23})}$$

$$M = 471.9\mu\mathrm{g} = (4\pi/3)\rho R^{3}$$

$$R^{2} = \frac{\rho R^{3}}{\rho R}$$

$$\boxed{R = 33.57\mu, \qquad \rho = 2979.3\,\mathrm{g\,cm}^{-2}}$$

2. b.

$$\lambda_{crit} = \frac{c}{f_{pe}}$$

$$n_e = \frac{\rho}{M} N_A = \frac{2979.3}{(2.5)(1.67 \times 10^{-24})(6.02 \times 10^{23})} = 7.174 \times 10^{26}\,\mathrm{cm}^{-3}$$

$$f_{pe} = 89 \times 10^{9}(n_e/10^{14})^{1/2} = 2.3838 \times 10^{17}\,\mathrm{Hz}$$

$$\lambda_{crit}(\mathring{A}) = \frac{3 \times 10^{10}(10^{8})}{2.3838 \times 10^{17}}$$

$$\boxed{\lambda_{crit} = 12.58\,\mathring{A}}$$

2. c.

$$c_s = \left(\frac{\gamma p}{\rho}\right)^{1/2} = \left(\frac{\gamma(T_e + T_i)}{m_i}\right)^{1/2} = \left(\frac{5/3(2 \times 10^4)(1.6 \times 10^{-19})}{(2.5)(1.67 \times 10^{-27})}\right)^{1/2}$$

$$= 7.99 \times 10^5 \,\mathrm{m\,s^{-1}}$$

$$= 7.99 \times 10^7 \,\mathrm{cm\,s^{-1}}$$

$$\boxed{\tau_{hydro} = \frac{R}{4c_s} = \frac{0.00336}{(4)(7.99 \times 10^7)} = 1.04 \times 10^{-11}\,\mathrm{s}}$$

2. d.

$$T(^\circ\mathrm{K}) = 11608\,T(\mathrm{eV}) = 1.1608 \times 10^8\,^\circ\mathrm{K}$$

$$q_{rad} = \sigma T^4 = (5.6699 \times 10^{-8})(1.1608 \times 10^8)^4 = 1.02 \times 10^{25}\,\mathrm{W\,m^{-2}}$$

$$\boxed{P_{rad} = 4\pi R^2 q_{rad} = 4\pi(33.6 \times 10^{-6})^2(1.02 \times 10^{25}) = 1.45 \times 10^{17}\,\mathrm{W}}$$

$$(P_{rad}\tau_{hydro} = 1.53\,\mathrm{MJ})$$