

UNIVERSITY OF CALIFORNIA, BERKELEY
 College of Engineering
 Department of Electrical Engineering and Computer Sciences

EE 105: Microelectronic Devices and Circuits

Spring 2019

Prof. Ming Wu

MIDTERM EXAMINATION #1

Time allotted: 80 minutes

NAME: Solution
 (print) Last First Signature

STUDENT ID#: _____

INSTRUCTIONS:

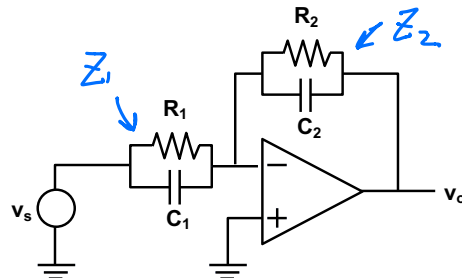
1. **SHOW YOUR WORK.** (Make your methods clear to the grader!)
2. Clearly mark (underline or box) your answers.
3. Specify the units of your answer to receive full credit.
4. Unless stated in the problem, use the values of physical constants provided below.
5. You can use approximations within 20% accuracy any time.
6. Calculator is allowed. (Cell phone is not allowed).

**** If you need more space for your answer, use the blank pages in the back. Clearly label which problem is your answer for ****

Commonly used constants and physical parameters:		
Electronic charge	q	1.6×10^{-19} C
Boltzmann's constant	k	8.62×10^{-5} eV/K
Thermal voltage at 300K	$V_T = kT/q$	0.026 V
Relative permittivity of Si	$\epsilon_{r,Si}$	12
Relative permittivity of SiO ₂	$\epsilon_{r,ox}$	4
Vacuum permittivity	ϵ_0	8.854×10^{-14} F/cm

Points	Problem 1	25	
	Problem 2	25	
	Problem 3	25	
	Problem 4	25	
	Total	100	

1) A simple filter is shown below. Assume the Op Amp is ideal.



- Derive the transfer function $H(j\omega) = v_o/v_s$.
- Assume $C_1 = 100 \text{ pF}$, $C_2 = 10 \text{ pF}$, $R_1 = 10 \text{ k}\Omega$, $R_2 = 1 \text{ k}\Omega$, draw the magnitude Bode plot. Clearly label the graph, including all the breakpoints (in magnitude and frequency).
- Draw the phase Bode plot using the same numerical values. Clearly label the graph.

$$(a) \quad v_o = 0 - \frac{v_s}{Z_1} \cdot Z_2 \Rightarrow \frac{v_o}{v_s} = - \frac{Z_2}{Z_1}$$

$$Z_1 = \frac{R_1 \cdot \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{R_1}{1 + j\omega R_1 C_1}, \quad Z_2 = \frac{R_2}{1 + j\omega R_2 C_2}$$

$$H(j\omega) = - \frac{R_2}{R_1} \frac{1 + j\omega R_1 C_1}{1 + j\omega R_2 C_2} \quad (6)$$

$$(b) \quad \text{zero} = \omega_z = \frac{1}{R_1 C_1} = \frac{1}{10 \text{ k}\Omega \cdot 100 \text{ pF}} = 10^6 \text{ rad/s} \quad (2)$$

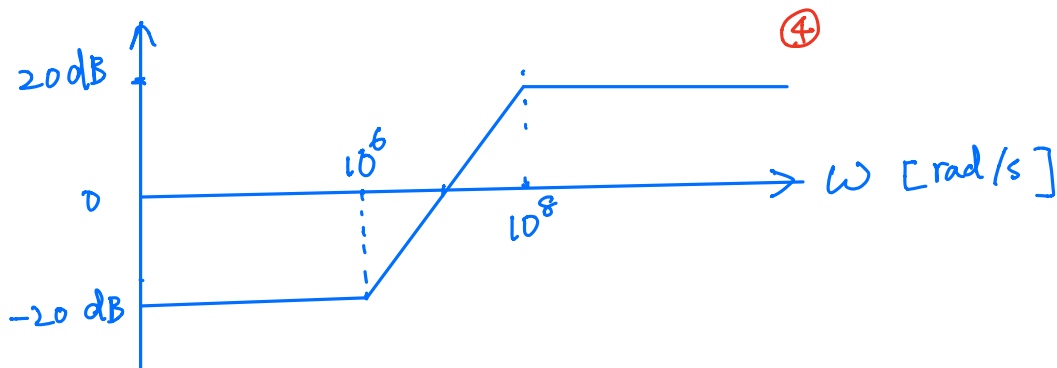
$$\text{pole} = \omega_p = \frac{1}{R_2 C_2} = \frac{1}{1 \text{ k}\Omega \cdot 10 \text{ pF}} = 10^8 \text{ rad/s} \quad (2)$$

Bode Plot:

$$\omega \rightarrow 0, \quad H(j\omega) \rightarrow - \frac{R_2}{R_1} = - \frac{1}{10}, \quad 20 \log |H(j\omega)| \rightarrow -20 \text{ dB} \quad (2)$$

$$\omega \rightarrow \infty \quad H(j\omega) \rightarrow - \frac{R_2}{R_1} \cdot \frac{R_1 C_1}{R_2 C_2} = - \frac{C_1}{C_2} = - \frac{100 \text{ pF}}{10 \text{ pF}} = -10$$

$$20 \log |H(j\omega)| \rightarrow 20 \text{ dB} \quad (2)$$



$$(c) \quad \omega \rightarrow 0, \quad \angle H(j\omega) = 180^\circ \quad \textcircled{2}$$

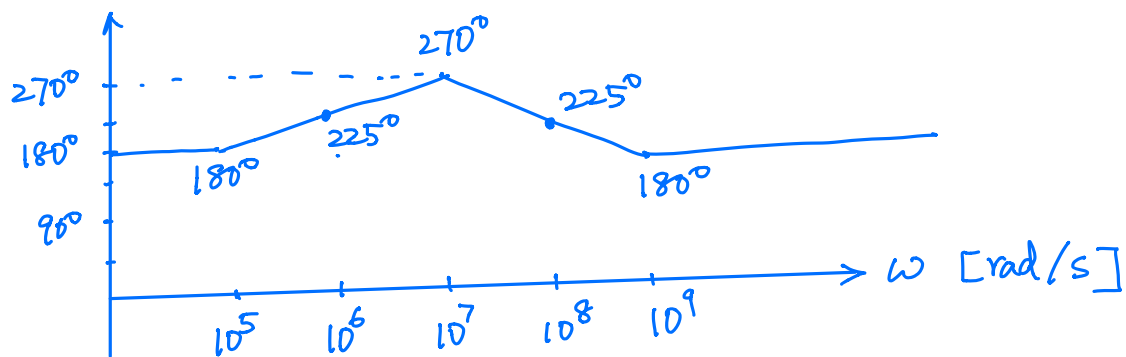
$$\omega \rightarrow \infty, \quad \angle H(j\omega) = 180^\circ \quad \textcircled{2}$$

$$\omega = \omega_z = \frac{1}{R_1 C_1}, \quad H(j\omega) \approx -\frac{R_2}{R_1} \cdot \frac{1+j}{1}, \quad \angle H(j\omega) = 180^\circ + 45^\circ \quad \textcircled{1}$$

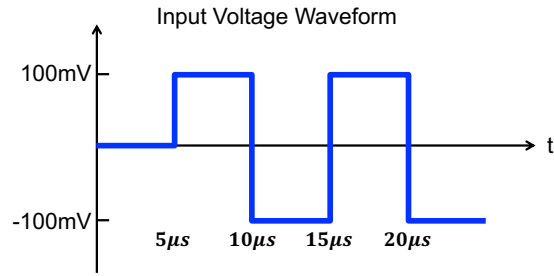
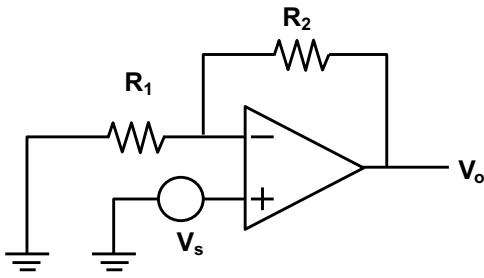
$$\omega = 10\omega_z, \quad H(j\omega) \approx -\frac{R_2}{R_1} \cdot \frac{j}{1}, \quad \angle H(j\omega) = 180^\circ + 90^\circ \quad \textcircled{1}$$

$$\omega = \omega_p, \quad H(j\omega) = -\frac{R_2}{R_1} \cdot \frac{j}{1+j}, \quad \angle H(j\omega) = 180^\circ + 90^\circ - 45^\circ \quad \textcircled{1}$$

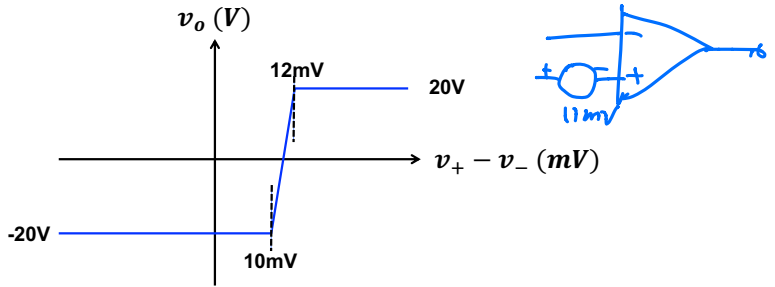
$$\omega = 10\omega_p, \quad H(j\omega) = -\frac{R_2}{R_1} \cdot \frac{j}{j}, \quad \angle H(j\omega) = 180^\circ$$



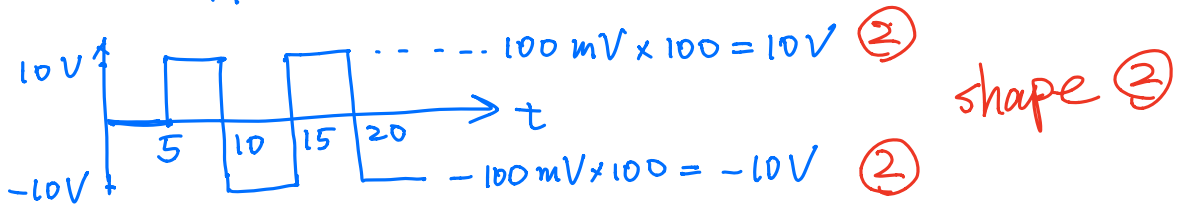
2) Consider the amplifier below, where $R_1 = 1\text{ k}\Omega$ and $R_2 = 99\text{ k}\Omega$. The input waveform is shown on the right.



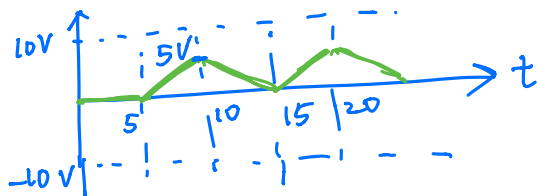
- If the Op amp is ideal, draw the output waveform.
- If the Op amp has a finite slew rate of $1\text{ V}/\mu\text{s}$, draw the output waveform. Clearly label the key features of the waveform. Justify the values with calculation.
- Now consider the Op amp with a non-zero offset voltage. The *open-loop* transfer curve is shown on the right. Draw the output waveform with both non-zero offset voltage and finite slew rate of $1\text{ V}/\mu\text{s}$.
- With the Op amp characteristics shown in c), if the *open loop* bandwidth of the Op amp is 1 kHz , what is the bandwidth of the amplifier?



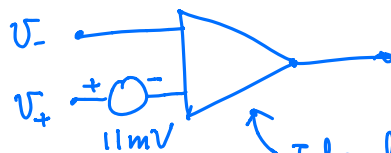
$$(a) \quad V_o = V_s + \frac{V_s}{R_1} \cdot R_2 = V_s \left(1 + \frac{R_2}{R_1}\right) = 100 V_s$$



$$(b) \quad SR = 1\text{ V}/\mu\text{s} = \text{max slope}$$



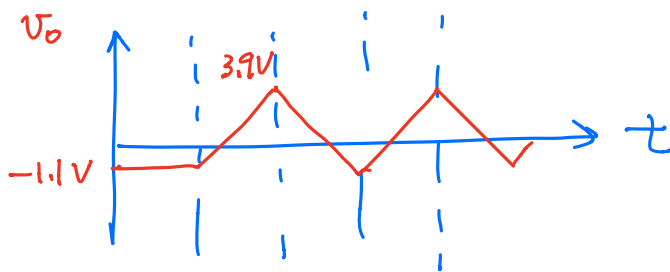
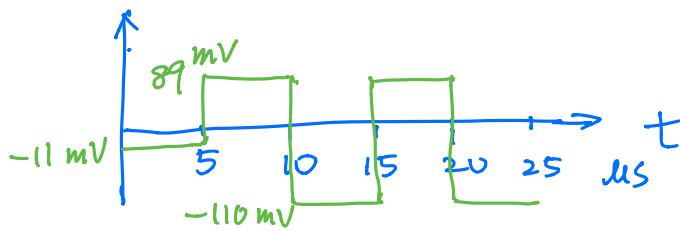
(c) Offset voltage = 11 mV . Equivalent circuit is



Polarity ③

Ideal Op Amp without offset.

Input waveform "seen" by Ideal Op Amp.



$$-1.1V \text{ (3)}$$

$$3.9V \text{ (3)}$$

(d) open-loop gain from the transfer curve in Part (c)

$$A_0 = \frac{(20 - (-20))V}{12mV - 10mV} = \frac{40V}{2mV} = 20 \times 10^3 = 2 \times 10^4 \text{ (3)}$$

$$\omega_b = 2\pi \times 1 \text{ KHz}$$

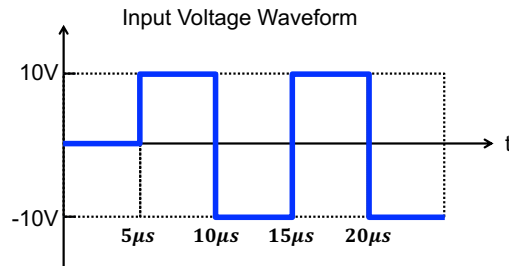
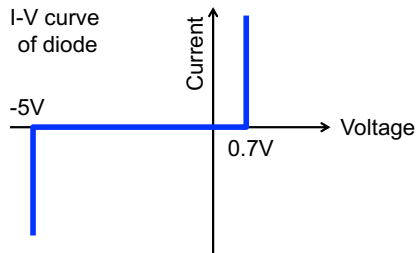
For the closed-loop amplifier

$$A_V = 1 + \frac{R_2}{R_1} = 100$$

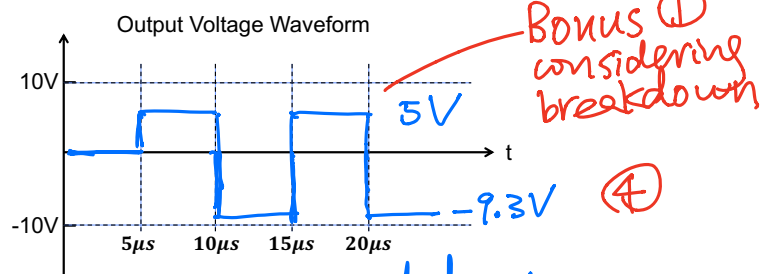
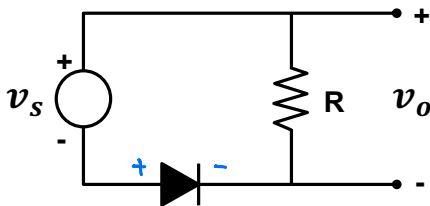
$$\omega_{3dB} = \frac{2 \times 10^4 \times 2\pi \times 1 \text{ KHz}}{10^2} = 2\pi \times 2 \times 10^2 \times 10^3 = 2\pi \times 200 \text{ KHz}$$

$$f_{3dB} = 200 \text{ KHz} \text{ (3)}$$

3) Consider an ideal diode with the I-V curve shown below. It has a turn on voltage of 0.7V and a reverse breakdown voltage of -5V. For all circuits, $R = 10\text{ k}\Omega$ and $C = 1\text{ nF}$. Draw the output waveforms corresponding to the input waveform shown below. Clearly label your waveform with all critical features. (This question does not require extensive calculation. Just label your curve clearly. Mark critical voltages and times on the graph. You can use the space below for any calculation/justification needed.)



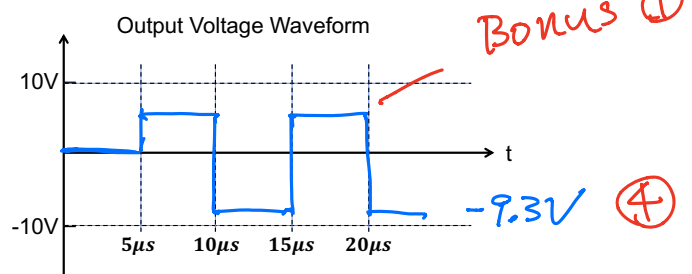
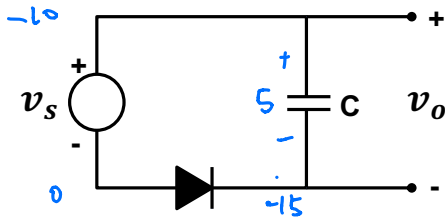
a)



Justifications/comments:

Half-wave rectifier but with diode breakdown
 $\begin{cases} V_s > 5V \Rightarrow \text{breakdown, } V_o = V_s - 5V \\ 5V > V_s > -0.7V \Rightarrow \text{OFF, } V_o = 0 \\ -0.7V > V_s \Rightarrow \text{ON, } V_o = V_s + 0.7V \end{cases}$

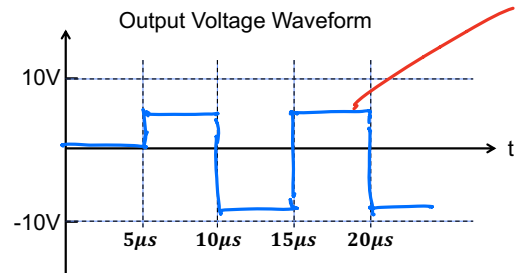
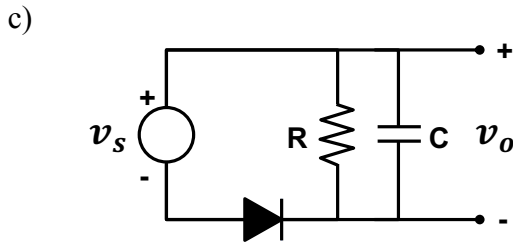
b)



Justifications/comments:

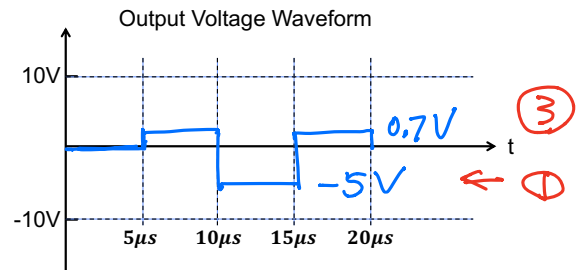
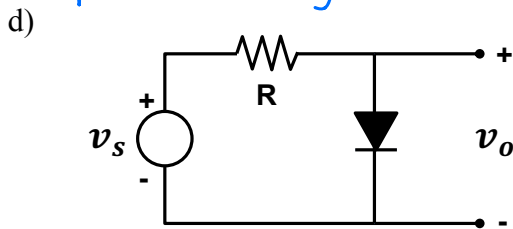
Negative peak detector
 Assume initially C is not charged.
 Diode turned on when $V_s < -0.7V$. $V_o = V_s + 0.7V$
 $V_o = -10 + 0.7 = -9.3V$
 When $V_s > 5V$, \Rightarrow Breakdown. $V_{BR} = 5V$.
 $V_o = V_s - 5V$

Bonus ④



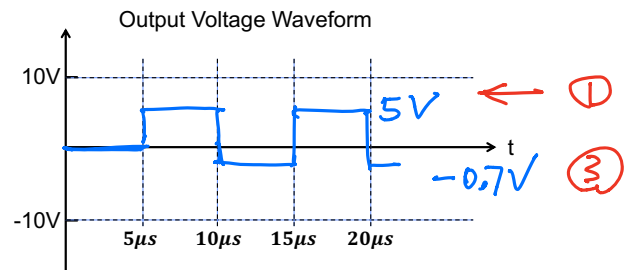
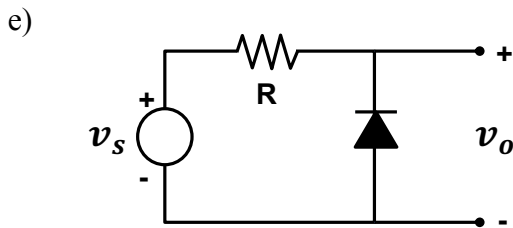
Justifications/comments:

Same as in (b). Though there is a leakage path through R, voltage is determined by V_D , V_{BR}



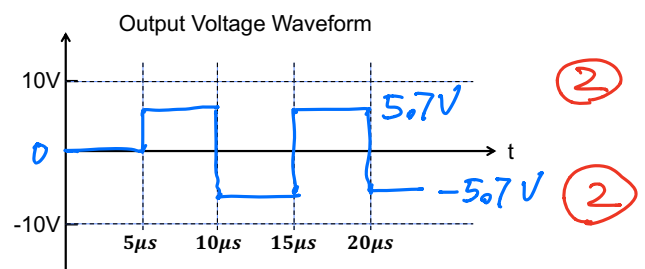
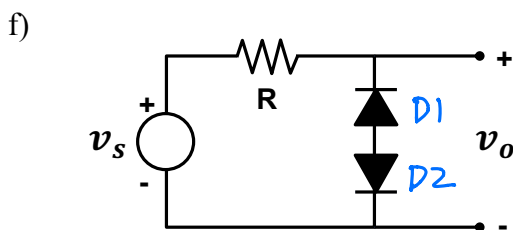
Justifications/comments:

$V_s > 0.7V$, diode is ON and $V_o = 0.7V$
 $V_s < 0.7V$, diode is OFF, $V_o = V_s$



Justifications/comments:

$V_s < -0.7V$, diode is ON, $V_o = -0.7V$
 $V_s > -0.7V$, diode is OFF $V_o = V_s$



Justifications/comments:

$V_s > |V_{BR}| + V_D = 5 + 0.7 = 5.7V$,
D1 is in breakdown, D2 is ON. $V_o = 5.7V$
 $V_s < -(|V_{BR}| + V_D) = -5.7V$
D1 is ON, D2 in breakdown. $V_o = -5.7V$
 $-5.7 < V_s < 5.7$, $V_o = V_s$

- 4) An asymmetric p-n junction has an n-doping of $N_D = 10^{16} \text{ cm}^{-3}$ and a p-doping of $N_A = 2.25 \times 10^{19} \text{ cm}^{-3}$. The intrinsic carrier concentration is $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$.
- What is the built-in voltage of the p-n junction?
 - What is the total depletion width?
 - Is the depletion region mostly in the p-side or the n-side?
 - What is the peak electric field at zero bias?
 - When the diode is forward biased, what is the dominant current in the diode? Choose among electron diffusion, electron drift, hole diffusion, and hole drift currents. Consider their peak values if they are not uniform across the diode. Justify your answers.
 - If you use the diode as a variable capacitance, what is the capacitance tuning ratio (maximum capacitance / minimum capacitance) if the bias voltage is vary between 0 to 10 V?

$$(a) \quad V_{bi} = 60 \text{ mV} \cdot \log \left(\frac{N_D \cdot N_A}{n_i^2} \right) = 60 \text{ mV} \cdot \log \left(\frac{10^{16} \cdot 2.25 \times 10^{19}}{2.25 \times 10^{20}} \right)$$

$$= 60 \text{ mV} \cdot \log(15) = 900 \text{ mV} \quad (4)$$

$$(b) \quad W = \sqrt{\frac{2\epsilon_s}{q} \cdot \left(\frac{1}{N_A} + \frac{1}{N_D} \right) \cdot V_{bi}}$$

$$\epsilon_s = 12 \times 8.854 \times 10^{-14} \text{ F/cm}$$

$$q = 1.6 \times 10^{-19} \text{ C}$$

$$W = 346 \text{ nm} \quad (4)$$

(c) x_n = depletion width on n-side
 x_p = " " " p-side

$$x_n \cdot N_D = x_p \cdot N_A$$

$$\text{Since } N_A \gg N_D \Rightarrow x_n \gg x_p$$

Depletion region mainly on n-side (4)

$$(d) \quad E_{max} = \frac{q N_D \cdot x_n}{\epsilon_s}$$

$$x_n = W \cdot \frac{N_A}{N_A + N_D} \approx W$$

$$E_{max} = 5.2 \times 10^4 \frac{\text{V}}{\text{cm}} \quad (4)$$

(e) At forward bias, diffusion currents dominate. Since p-doping is higher, hole diffusion current is the dominant term.

$$J_{p,diff} = qD_p \frac{dp}{dx} = qD_p \frac{\Delta p}{L_p} = qD_p \frac{n_i^2}{N_D} e^{\frac{V}{V_T}} \cdot \frac{1}{L_p}$$

$$J_{n,diff} = qD_n \frac{n_i^2}{N_A} e^{\frac{V}{V_T}} \cdot \frac{1}{L_n}$$

Since $N_A \gg N_D$, $J_{p,diff} \gg J_{n,diff}$ (4)

(f) Max capacitance at $V=0$

$$C_{max} = \frac{\epsilon A}{W} = C_0$$

$$C_{min} = \frac{\epsilon A}{W(V=10V)} = \frac{C_0}{\sqrt{1 + \frac{10}{V_{bc}}}}$$

$$\frac{C_{max}}{C_{min}} = \sqrt{1 + \frac{10}{V_{bc}}} = \sqrt{1 + \frac{10}{0.9}} = 3.5 \quad (4)$$