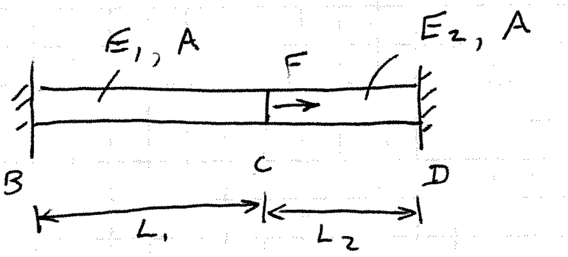
# MIDTERM 2 - SOLUTION | MEC 85 | SPRING 2018

1.

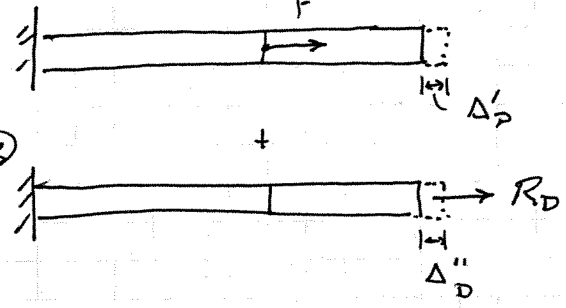



$E_1 = E$ , $E_2 = \beta E$

$L_1 = \alpha L$ , $L_2 = (1-\alpha)L$

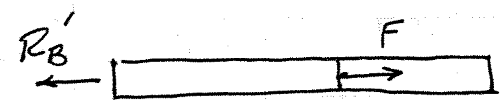
$k_1 = \dfrac{AE}{\alpha L}$ , $k_2 = \left(\dfrac{\beta}{1-\alpha}\right)\dfrac{AE}{L}$

SOLVE BY SUPERPOSITION:
- REMOVE CONSTRAINT AT D
- DETERMINE DEFLECTION AT D DUE TO F
- REMOVE F + IMPOSE REACTION AT D
- DETERMINE DEFLECTION AT D
- DEFLECTIONS MUST ADD TO ZERO AT D




$$\Delta'_D + \Delta''_D = 0$$

FBD FOR A :




$$\Sigma F_x = 0 \Rightarrow R'_B = F$$

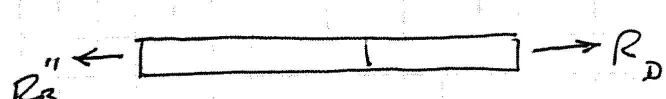
FORCE - DEFLECTION RELATION: FOR ROD 1, THE INTERNAL AXIAL FORCE IS CONSTANT $P'_{BC} = F$ AND THE DEFLECTION OF C IS

$$\Delta'_C = \frac{P'_{BC} L_1}{A E_1} = \frac{F(\alpha L)}{AE}$$

THERE IS NO FORCE ACTING BETWEEN C + D, SO

$$\Delta'_D = \Delta'_C$$

FBD FOR B :




$$\Sigma F_x = 0 \Rightarrow R''_B = R_D$$

FORCE - DEFLECTION: INTERNAL AXIAL FORCE IS UNIFORM BETWEEN B + D $\Rightarrow$

DEFLECTION OF C DUE TO $R_D$: $\Delta_C'' = \dfrac{R_D L_1}{A E_1} = \dfrac{R_D (\alpha L)}{AE}$

~~DEFLECTION~~ EXTENSION OF ROD CD DUE TO $R_D$:

$$\Delta_{CD}'' = \frac{R_D L_2}{A E_2} = \frac{R_D (1-\alpha) L}{\beta A E}$$

$$\Delta_D'' = \Delta_C'' + \Delta_{CD}'' = \frac{R_D L}{AE}\left(\alpha + \frac{1-\alpha}{\beta}\right)$$

RETURNING TO COMPATIBILITY:

$$\Delta_D' + \Delta_D'' = 0 = \frac{FL}{AE}(\alpha) + \frac{R_D L}{AE}\left(\alpha + \frac{1-\alpha}{\beta}\right)$$

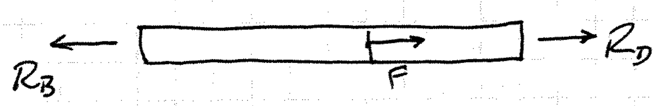
$$\Rightarrow R_D\left(\frac{1-\alpha+\alpha\beta}{\beta}\right) = -\alpha F$$

(a)

$$\Rightarrow \boxed{R_D = -\frac{\alpha\beta}{1-\alpha+\alpha\beta} F}$$

(FORCE ACTS TO THE LEFT)

FBD OF ENTIRE SYSTEM (FROM ORIGINAL SKETCH)




$$\Sigma F_x = 0 = F + R_D - R_B \Rightarrow R_B = F + R_D$$

$$\boxed{R_B = F\left[1 - \frac{\alpha\beta}{1-\alpha+\alpha\beta}\right] = \frac{(1-\alpha)}{1-\alpha+\alpha\beta} F}$$

(b) DISPLACEMENT OF C: IN THE FULL STATICALLY INDETERMINATE PROBLEM, WE NOTE THAT THE AXIAL FORCES IN BC + CD ARE $R_B$ + $R_D$, RESPECTIVELY.

SO $\Delta_C = \dfrac{R_B L_1}{A E_1}$ OR $\Delta_C = -\dfrac{R_D L_2}{A E_2}$

$$\boxed{\Delta_c = \frac{R_B L_1}{A E_1} = \frac{\alpha(1-\alpha)}{1-\alpha+\alpha\beta}\frac{FL}{AE}}$$

OR

$$\Delta_c = \frac{-R_D L_2}{A E_2} = \left(\frac{\alpha\beta}{1-\alpha+\alpha\beta}\right)\cancel{\mathcal{F}}\,\frac{F(1-\alpha)L}{\beta AE} = \frac{\alpha(1-\alpha)}{1-\alpha+\alpha\beta}\frac{FL}{AE} \checkmark$$

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"SANITY CHECKS"

- IF $\alpha \to 0$, $L_1 \to 0$ + $k_1 \to \infty$

  WE WOULD EXPECT $\Delta_c \to 0$, $R_B \to F$, $R_D \to 0$

  AND THIS IS WHAT THE SOLUTION GIVES US ✓

- IF $\alpha \to 1$, $L_1 \to L$, $L_2 \to 0$, $k_2 \to \infty$

  WE EXPECT $\Delta_c \to 0$, $R_B \to 0$, $R_D \to -F$

  + AGAIN OUR RESULTS ARE CONSISTENT ✓

- IF $\beta \to 0$, $E_2 \to 0$ + $k_2 \to 0$

  WE EXPECT $\Delta_c = \frac{FL_1}{AE} \to \alpha\frac{FL}{AE}$, $R_B \to F$, $R_D \to 0$

  ✓

- IF $\beta \to \infty$, $E_2 \to \infty$ + $k_2 \to \infty$

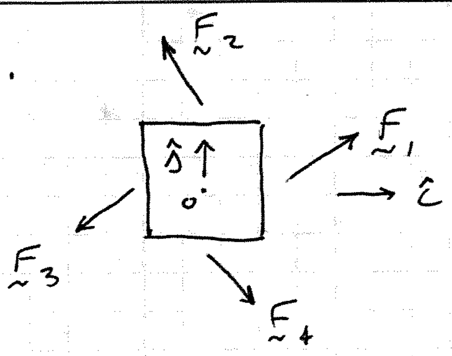
  WE EXPECT $\Delta_c \to 0$, $R_B \to 0$, $R_D \to -F$ ✓

- IF $\alpha = 1/2$, $\beta = 1$, $L_1 = L_2$, $E_1 = E_2$ +

  SYMMETRY $\Rightarrow R_B = \frac{1}{2}F$, $R_D = -\frac{1}{2}F$

  SO EACH END SUPPORTS HALF OF THE CENTRALLY APPLIED LOAD ... ✓

2.




ASSUME THAT

$$\underset{\sim}{F}_2 = F_2\, \underset{\sim}{e}_2 = F_2(-\sin\phi\,\hat{i} + \cos\phi\,\hat{j})$$

$$\underset{\sim}{F}_3 = F_3\, \underline{e}_3 = F_3(-\cos\phi\,\hat{i} - \sin\phi\,\hat{j})$$

$$\underset{\sim}{F}_4 = F_4\, \underline{e}_4 = F_4(\sin\phi\,\hat{i} - \cos\phi\,\hat{j})$$

(a)

EQUILIBRIUM EQUATIONS:

$$\Sigma F_x = 0 = (F - F_3)\cos\phi + (-F_2 + F_4)\sin\phi \quad (1)$$

$$\Sigma F_y = 0 = (F - F_3)\sin\phi + (F_2 - F_4)\cos\phi \quad (2)$$

$$\Sigma M_o = 0 = \frac{a}{2}(F + F_3)\sin\phi + \frac{a}{2}(F_2 + F_4)\sin\phi \quad (3)$$

SOLVE FOR $F_2$, $F_3$, $F_4$:

MULTIPLY (1) BY $\cos\phi$, (2) BY $\sin\phi$ AND ADD $\Rightarrow$

$$0 = (F - F_3)(\cos^2\phi + \sin^2\phi) = F - F_3 \Rightarrow \underline{\underline{F_3 = F}}$$

MULTIPLY (1) BY $\sin\phi$ & (2) BY $\cos\phi$ AND SUBTRACT $\Rightarrow$

$$0 = (F_4 - F_2)\sin\phi\cos\phi \Rightarrow \underline{\underline{F_4 = F_2}}$$

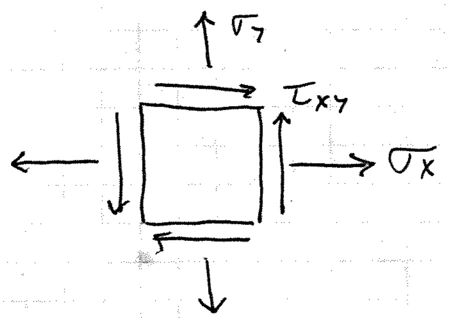
MOMENT EQUATION: ALL TERMS INVOLVE $a\sin\phi/2 \Rightarrow$

$$0 = F + F_3 + F_2 + F_4$$

$$= 2F + 2F_2 \Rightarrow \underline{\underline{F_2 = F_4 = -F}}$$

$$\boxed{\begin{aligned} \underset{\sim}{F}_2 &= F(\sin\phi\,\hat{i} - \cos\phi\,\hat{j}) \\ \underset{\sim}{F}_3 &= -F(\cos\phi\,\hat{i} + \sin\phi\,\hat{j}) \\ \underset{\sim}{F}_4 &= F(-\sin\phi\,\hat{i} + \cos\phi\,\hat{j}) \end{aligned}}$$

(b) THE STRESS IS UNIFORM SO ANY ELEMENT WILL HAVE THE STRESS STATE




$$\sigma_x = S\cos\phi$$

$$\sigma_y = -S\overset{\cos}{\cancel{\sin}}\phi$$

$$\tau_{xy} = S\sin\phi$$

(c) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 0 \pm \sqrt{S^2\cos^2\phi + S^2\sin^2\phi} = \pm S$$

$$\boxed{\sigma_1 = S, \quad \sigma_2 = -S}$$

PRINCIPAL ANGLE

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2S\sin\phi}{2S\cos\phi} = \tan\phi$$

$$\boxed{\theta_p = \phi/2}$$

(d) TRICK QUESTION: THE SHEAR STRESS IN THE PRINCIPAL DIRECTIONS IS ALWAYS ZERO.

(e) $$\varepsilon_z = \frac{1}{E}\left(\overset{0\text{ - PLANE STRESS}}{\sigma_z} - \nu\sigma_x - \nu\sigma_y\right)$$

$$\boxed{\varepsilon_z = \frac{-\nu}{E}(\sigma_x + \sigma_y) = 0}$$

3. $u = a_1 x + a_2 y + a_3 xy$

$v = b_1 x + b_2 y + b_3 xy$

(a)

LET'S LOOK AT POINTS B, D & C:

(B) $x = 1,\ y = 0 \Rightarrow u = a_1,\ v = b_1$

FROM THE FIGURE, $u = 0.02,\ v = 0$

$$\boxed{a_1 = 0.02,\quad b_1 = 0}$$

(D) $x = 0,\ y = 1 \Rightarrow u = a_2,\ v = b_2$

FIGURE $\Rightarrow u = 0,\ v = -0.02$

$$\boxed{a_2 = 0,\quad b_2 = -0.02}$$

(C) $x = 1,\ y = 1 \Rightarrow u = a_1 + \cancel{a_2}^{\,0} + a_3 = 0.04$

$v = \cancel{b_1}^{\,0} + b_2 + b_3 = 0.02$

$$\boxed{\begin{aligned} a_3 &= 0.04 - a_1 = 0.02 \\ b_3 &= 0.02 - b_2 = 0.04 \end{aligned}}$$

$$\boxed{\begin{aligned} u &= 0.02x + 0.02xy \\ v &= -0.02y + 0.04xy \end{aligned}}$$

(b)

$$\varepsilon_x = \frac{\partial u}{\partial x} = a_1 + a_3 y = 0.02(1 + y)$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = b_2 + b_3 x = -0.02(1 - 2x)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = a_3 x + b_3 y$$

$$\gamma_{xy} = 0.02(x + 2y)$$

(c) NOT UNIFORM. THE STRAINS DEPEND UPON $x$ & $y$.

(d) $\gamma_{xy}(x=1, y=1) = 0.06$

$$\boxed{\text{INTERIOR ANGLE} = \frac{\pi}{2} - \gamma_{xy} = \frac{\pi}{2} - 0.06}$$