Introduction to Solid Mechanics
ME C85/CE C30

Midterm Exam 2

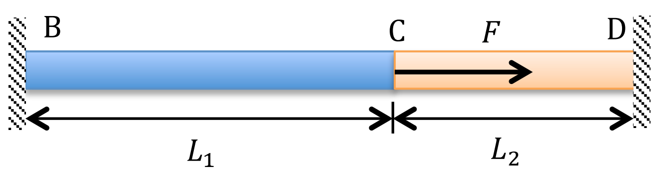
Spring 2018

1. Do not open the exam until you are told to begin.
2. There are three problems worth 80 points total. There are separate sets of answer sheets for each problem. Be sure that you have them before you start. We will collect the answer sheets separately at the end of the exam.
3. Please put your name and SID on your answer sheets.
4. We will not answer questions during the exam. Write your concerns or interpretations of the problem(s) on your answer sheets.
5. You may not use a calculator.
6. You may use one 8-1/2 x 11 sheet of notes, but not your book or any other notes.
7. Store everything else out of sight.
8. Turn off cell phones.
9. Please read the entire exam before starting work. There is important information in the text of each problem, as well as in the figures.
10. You may solve the problems in whichever order you choose, of course, but pay attention to the clock so that you have sufficient time to work on all three problems.
11. Be concise and write clearly. Identify your answers by putting boxes around them.
12. You may leave the exam room when you are finished, but you may not leave and return during the exam. Please plan accordingly.
13. Time will be strictly enforced. At 11:00, you must put down your pencil or pen and immediately turn in your exam. Failure to do so may result in loss of points.

1. **(35 points total)** A rod of total length L and uniform cross sectional area A is made of two segments and is built into rigid supports at either end. While the two segments have the same cross sectional area, they are made of different materials and have different lengths. The left segment between B and C has Young's modulus $E_1 = E$ and length $L_1 = \alpha L$ $(0 < \alpha < 1)$, while the right segment between C and D has Young's modulus $E_2 = \beta E$ $(\beta > 0)$ and length $L_2 = (1 - \alpha)L$. The rod is loaded by axial force F at C, the intersection of the two segments.

   (a) (20 points) Determine the reaction forces acting on the rod at ends B and D. Express your results in terms of F, L, E, A, α and β.
   (b) (15 points) Determine the displacement of point C, again in terms of F, L, E, A, α and β.

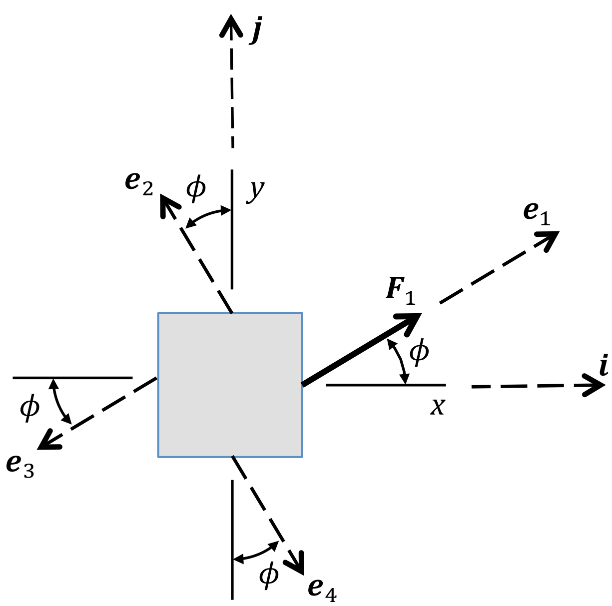
   If you have time to check your answers, presumably after you have finished all of the problems on this exam, it may be useful to consider the "reasonableness" of your results when α and/or β are near their extremes – i.e., when α is near zero or one, and/or when β is near zero or approaches infinity.



2. **(25 points total)** Consider a cube of material which is in equilibrium and is loaded in plane stress. Let the force $\boldsymbol{F}_1$ be known as $\boldsymbol{F}_1 = F\boldsymbol{e}_1$, with $F > 0$ and $\boldsymbol{e}_1$ being a unit vector making an angle ϕ with the x axis, $\boldsymbol{e}_1 = \cos\theta\boldsymbol{i} + \sin\theta\boldsymbol{j}$. The magnitudes of the other three forces $\boldsymbol{F}_i$, $i = 2, 3, 4$ (i.e., $\boldsymbol{F}_2$, $\boldsymbol{F}_3$ and $\boldsymbol{F}_4$) are not known a priori, but it is known that each acts along $\pm\boldsymbol{e}_i$, each of which makes an angle ϕ with the normal to its respective face.

Note: All forces may be considered to be uniformly distributed across the faces on which they act.

(a) (3 points) Determine $\boldsymbol{F}_2$, $\boldsymbol{F}_3$ and $\boldsymbol{F}_4$ in terms of the known quantities F and ϕ.
(b) (6 points) Let A be the area of each face and let a quantity S be defined as $S = F/A$. Determine the components of stress relative to the xy axes, σ_x, σ_y and τ_{xy}, in terms of S and ϕ.
(c) (10 points) Determine the principal stresses, along with the principal direction θ_p, all in terms of S and ϕ.
(d) (3 points) Determine the shear stress acting on an element aligned with the principal directions.
(e) (3 points) Let the material have Young's modulus E and Poisson's ratio ν. Determine the normal strain in the z-direction, ε_z.



3. **(20 points total)** A unit cube is deformed in plane strain in such a way that the displacement fields in the x and y directions are

$$u(x,y) = a_1 x + a_2 y + a_3 xy$$
$$v(x,y) = b_1 x + b_2 y + b_3 xy$$

The schematics below show the original shape (dashed lines with corners at A, B, C and D) and deformed shape (solid lines with corners at A', B', C' and D') of the xy face of this body. The figure at the left expands the vertical scale around $y = 1$, while the figure at the bottom expands the horizontal scale around $x = 1$.

(a) (6 points) Determine the six coefficients defining this deformation (i.e., the three a_i's and the three b_i's).
(b) (8 points) Determine the in-plane strains as functions of position.
(c) (3 points) Is this strain field uniform? Explain.
(d) (3 points) Determine the interior angle at point C' of the deformed material.