Chemical Engineering 150A
Midterm Exam
Monday, February 25, 2019
9:10 am – 10:00 am

The exam is 100 points total.

Name: ________________________________ (in Uppercase)

Student ID: ____________________________

You are allowed one 8.5”×11” sheet of paper with your notes on both sides and a calculator for this exam.

The exam should have 15 pages (front and back) including the cover page.

Instructions:

1) Please write your answers in the box if provided.
2) Do your calculations in the space provided for the corresponding part. Any work done outside of specified area will not be graded.
3) Please sign below saying that you agree to the UC Berkeley honor code.
4) The exam contains one problem with sub-parts.
5) Use the blank white full pages behind the question pages as scratch sheets (your work here will not be graded).

Honor Code:
As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.

Signature: _

1.a	1.b	1.c	1.d	1.e	1.f	1.g	1.h	1.i	Total

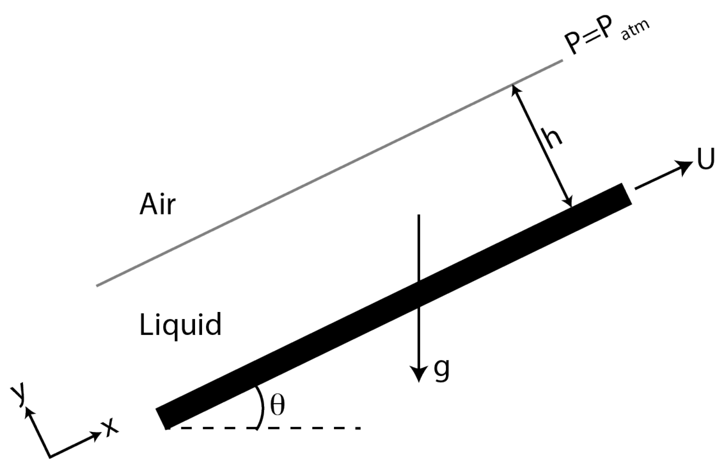
Problem 1. (100 points)

A film of incompressible, Newtonian liquid of density $\boldsymbol{\rho}$ sits on a plate. This plate is at an angle $\boldsymbol{\theta}$ from the horizontal and is moving at a constant speed **U**. The top of the liquid film is exposed to air at atmospheric pressure, and the film thickness **h** is constant along the length of the plate. The system is in a steady-state.

Assume that the pressure and velocity profiles within the liquid are of the following forms:

1. $\underline{v} = v_x(y)\underline{e}_x$
2. $P = P(y)$

A schematic of this setup is given below along with the coordinate system.



(a) Is the flow profile compatible with incompressibility? Prove it. (5 points)

Condition for incompressibility: $\underline{\nabla} \cdot \underline{v} = 0$

Substitute the velocity profile $\boldsymbol{v} = v_x(y)\boldsymbol{e}_x$

$$\frac{\partial v_x(y)}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

Flow is incompressible

(b) The fluid flowing between the plates can be described by the following constitutive relationships between shear stress and velocity gradients, where μ is the coefficient of viscosity.

Please **circle** the components that are non-zero. (10 points)

$$\tau_{xx} = 2\mu \frac{\partial v_x}{\partial x} \qquad \tau_{xy} = \mu \left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right] \qquad \tau_{xz} = \mu \left[\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right]$$

$$\tau_{yx} = \mu \left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right] \qquad \tau_{yy} = 2\mu \frac{\partial v_y}{\partial y} \qquad \tau_{yz} = \mu \left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right]$$

$$\tau_{zx} = \mu \left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right] \qquad \tau_{zy} = \mu \left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right] \qquad \tau_{zz} = 2\mu \frac{\partial v_z}{\partial z}$$

(c) Write the Cauchy microscopic momentum balance *only* in the y-direction. Simplify the momentum balance. Solve the ordinary differential equation (ODE) and use appropriate boundary conditions to obtain an expression for P(y).
Write the answer in the box. (20 points)

$$\frac{\partial(\rho v_y)}{\partial t} + \frac{\partial}{\partial x}(\rho v_y v_x) + \frac{\partial}{\partial y}(\rho v_y v_y) + \frac{\partial}{\partial z}(\rho v_y v_z) = -\frac{\partial P}{\partial y} + \frac{\partial}{\partial x}(\tau_{xy}) + \frac{\partial}{\partial y}(\tau_{yy}) + \frac{\partial}{\partial z}(\tau_{zy}) + \rho g_y$$

$$\boldsymbol{g} = (-g \sin\theta)\, \boldsymbol{e}_x + (-g\cos\theta)\, \boldsymbol{e}_y$$

$$\frac{\partial(\rho v_y)}{\partial t} + \frac{\partial}{\partial x}(\rho v_y v_x) + \frac{\partial}{\partial y}(\rho v_y v_y) + \frac{\partial}{\partial z}(\rho v_y v_z) = -\frac{\partial P}{\partial y} + \frac{\partial}{\partial x}(\tau_{xy}) + \frac{\partial}{\partial y}(\tau_{yy}) + \frac{\partial}{\partial z}(\tau_{zy}) + \rho g_y$$

$$\frac{\partial P}{\partial y} = -\rho g \cos\theta$$

$$P = -\rho g \cos\theta\, y + C_1(x)$$

Boundary condition: $P(y = h, x) = P_{atm}$

$$C_1(x) = P_{atm} + \rho g h \cos\theta$$

$$P = -\rho g \cos\theta \, (y - h) + P_{atm}$$

(d) Write the Cauchy microscopic momentum balance *only* in the x-direction. Simplify using the constitutive relationships from part (b). Write the final ODE for $v_x(y)$ in the box. (15 points)

$$\frac{\partial(\rho v_x)}{\partial t} + \frac{\partial}{\partial x}(\rho v_x v_x) + \frac{\partial}{\partial y}(\rho v_x v_y) + \frac{\partial}{\partial z}(\rho v_x v_z) = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}(\tau_{xx}) + \frac{\partial}{\partial y}(\tau_{yx}) + \frac{\partial}{\partial z}(\tau_{zx}) + \rho g_x$$

$$\frac{\partial(\rho v_x)}{\partial t} + \frac{\partial}{\partial x}(\rho v_x v_x) + \frac{\partial}{\partial y}(\rho v_x v_y) + \frac{\partial}{\partial z}(\rho v_x v_z) = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}(\tau_{xx}) + \frac{\partial}{\partial y}(\tau_{yx}) + \frac{\partial}{\partial z}(\tau_{zx}) + \rho g_x$$

$$\rho g \sin\theta = \frac{\partial}{\partial y}(\tau_{yx}) = \mu \frac{\partial^2 v_x}{\partial y^2}$$

$$\rho g \sin\theta = \mu \frac{\partial^2 v_x}{\partial y^2}$$

(e) Solve the ordinary differential equation derived in part (d) for the velocity profile in the fluid, $v_x(y)$. Do not solve for the constants of integration yet, which means you can leave the constants of integration as they are. If you weren't able to obtain the ODE in part (d), use $A = \frac{\partial^2 v_x}{\partial y^2}$, where A is some constant. Write the answer in the box. (5 points)

$$\rho g \sin\theta = \mu \frac{\partial^2 v_x}{\partial y^2}$$

$$\frac{\partial v_x}{\partial y} = \frac{\rho g \sin\theta}{\mu} y + C_1$$

$$v_x = \frac{\rho g \sin\theta}{2\mu} y^2 + C_1 y + C_2$$

(f) Write the boundary conditions for the flow. (10 points)

No-slip at plate:

$$v_x(y = 0) = U$$

No shear stress at gas-liquid interface:

$$\tau_{yx}(y = h) = \mu \frac{\partial v_x}{\partial y}(y = h) = 0$$

$$\frac{\partial v_x}{\partial y}(y = h) = 0$$

(g) Use your boundary conditions to solve for the constants of integration and write an expression for $v_x(y)$. Write the final answer in the box. (10 points)

$$\frac{\partial v_x}{\partial y} = \frac{\rho g \sin\theta}{\mu} y + C_1$$

$$\frac{\partial v_x}{\partial y}(y = h) = \frac{\rho g \sin\theta}{\mu} h + C_1 = 0$$

$$C_1 = -\frac{\rho g \sin\theta}{\mu} h$$

$$v_x = \frac{\rho g \sin\theta}{2\mu} y^2 - \frac{\rho g \sin\theta}{\mu} hy + C_2$$

$$v_x(y = 0) = C_2 = U$$

$$v_x = \frac{\rho g \sin\theta}{2\mu} y^2 - \frac{\rho g \sin\theta}{\mu} hy + U$$

$$v_x = \frac{\rho g \sin\theta}{\mu}\left(\frac{y^2}{2} + -hy\right) + U$$

(h) Consider the case where there is no net flow of fluid (i.e. the amount of fluid pulled up by the plate is equal to the amount pulled down by gravity). Find an expression for the plate speed, **U**, in terms of $\boldsymbol{\mu}$, **h, g,** $\boldsymbol{\rho}$, and $\boldsymbol{\theta}$. Consider the width of the plate in the z-direction to be **W.**
Write the final answer in the box. (15 points)

$$Q = \int_0^h v_x Z dy = 0$$

$$\int_0^h \left[\frac{\rho g \sin\theta}{\mu}\left(\frac{y^2}{2} + -hy\right) + U\right] dy = 0$$

$$= \frac{\rho g \sin\theta}{\mu}\left(\frac{h^3}{6} + -\frac{h^3}{2}\right) + Uh = Uh - \frac{1}{3}\frac{\rho g \sin\theta}{\mu} h^3 = 0$$

$$U = \frac{1}{3}\frac{\rho g \sin\theta}{\mu} h^2$$

$$U = \frac{1}{3}\frac{\rho g \sin\theta}{\mu} h^2$$

(i) Sketch the velocity profile for the scenario described in part (h). If you are unsure, draw based on your intuition and provide explanation for what you drew. Ensure that your sketch is consistent with your boundary conditions from part (f). (10 points)