Name: ______________________________

SID: ______________________________

Section: ______________________

Physics 7B Midterm 2 - Fall 2018
Professor A. Lanzara

Total Points: 100 (5 Problems)

This exam is out of 100 points. Show all your work and take particular care to explain your steps. Partial credit will be given. Use symbols defined in problems and define any new symbols you introduce. If a problem requires that you obtain a numerical result, first write a symbolic answer and then plug in numbers. Label any drawings you make. **Good luck!**

Problem 1 (20 pts.)

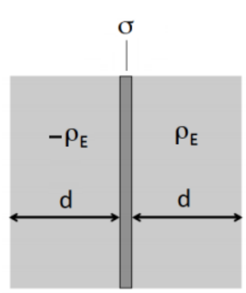
A very large thin plate has uniform surface charge density σ. Touching it on the right is a long wide slab of insulator with thickness d with uniform charge density ρ_E. On the left side there is a similar slab with uniform charge density $-\rho_E$. Determine the electric field in the regions below. Neglect all edge effects.

(a) (5 pts.) To the left of the leftmost slab.

(b) (5 pts.) To the right of the rightmost slab.

(c) (5 pts.) Inside the leftmost slab.

(d) (5 pts.) Inside the rightmost slab.



Problem 2 (20 pts.)

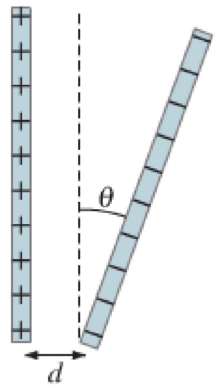
A total amount of positive charge Q is spread onto a non-conducting flat circular annulus of inner radius a and outer radius b. The charge is distributed so that the charge surface density is given by $\sigma = k/r$, where r is the distance from the center of the annulus to any point on it.

(a) (10 pts.) Take the x-axis to have $x = 0$ at the center of the annulus. Point P is an arbitrary point on the x-axis with position $(x, 0, 0)$. Determine the potential at P.

(b) (5 pts.) Starting with the potential from (a), determine the x, y, and z components of the electric field at the same point.

(c) (5 pts.) Determine the acceleration (magnitude and direction) of a charge q placed at point x. Assume the charge has mss m.

Problem 3 (20 pts.) Suppose one plate of a parallel-plate capacitor is tilted so it makes a small angle θ with the other plate, as shown below. Determine a formula for the capacitance C in terms of A, d, and θ, where A is the area of each plate. Assume the plates are square. Hint: Consider the capacitor as many capacitors in parallel.



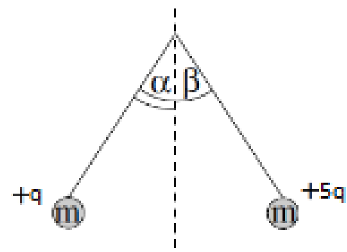
Problem 4 (20 pts.) A thin wire of length L and cross-sectional area A oriented in the x-direction is made of an ohmic material whose resistivity varies along the length of the wire according to the empirical law $\rho = \rho_0 e^{-x/L}$.

(a) (10 pts.) What is the total resistance of the wire?

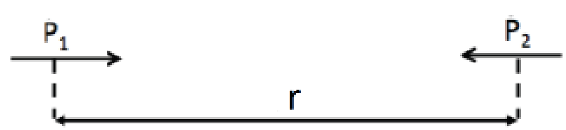
(b) (10 pts.) Describe how the field within the wire varies with position if the end at $x = 0$ is at potential V_0 greater than at the end at $x = L$.

Problem 5 (20 pts.)

(a) (10 pts.) Two balls of equal mass m are hung from silk threads of equal length, as shown below. If the sphere on the right has a charge 5q and the one on the left has charge q, at equilibrium the two spheres form angles α and β with the vertical. How are the angles α and β related? Explain your reasoning.



(b) (10 pts.) Two electric dipoles with dipole moment $\mathbf{P}_1$ and $\mathbf{P}_2$ are anti-aligned with one another as shown below. Assume the distance r between the dipoles is much greater than the length d of either dipole. If we turn on an electric field E_0 parallel to $\mathbf{P}_1$, what happens to each dipole? What is the maximum torque that the field can exert on each dipole?



For a point charge Q:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \qquad V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

$$C = \frac{\epsilon_0 A}{d} \qquad R = \frac{\rho l}{A}$$

$$K = \frac{1}{2}mv^2$$

Volume element in spherical coordinates: $dV = r^2 \sin(\theta) dr d\theta d\phi$

Volume element in cylindrical coordinates: $dV = r dr dz d\phi$

Area element in polar coordinates: $dA = r dr d\theta$

$$\int \frac{1}{(a^2+x^2)^{1/2}}dx = \ln\left(x + \sqrt{a^2+x^2}\right) + C \qquad \int \frac{1}{(a^2+x^2)}dx = \left(\frac{1}{a}\right)\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{(a^2+x^2)^{3/2}}dx = \frac{x}{a^2\sqrt{a^2+x^2}} + C \qquad \int \frac{1}{(a^2+x^2)^2}dx = \frac{1}{2a^3}\left(\frac{ax}{a^2+x^2} + \tan^{-1}\left(\frac{x}{a}\right)\right) + C$$

$$\int \frac{x}{(a^2+x^2)^{1/2}}dx = \sqrt{a^2+x^2} + C \qquad \int \frac{x}{(a^2+x^2)^{3/2}}dx = -\frac{1}{\sqrt{a^2+x^2}} + C$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}\cdots \qquad (1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots$$

$$\sin(x) = x - \frac{x^3}{6} + \cdots \qquad e^x = 1 + x + \frac{x^2}{2} + \cdots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots$$