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Introduction to Solid Mechanics
ME C85/CE C30

Final Exam

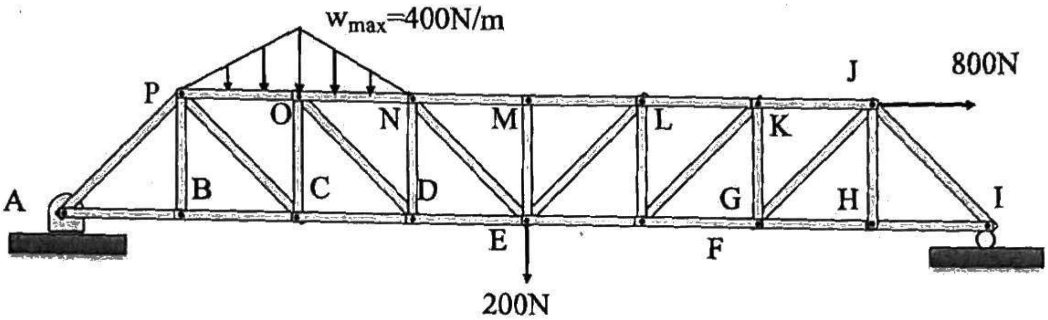
Fall, 2018

1. Do not open the exam until you are told to begin.
2. Put your name, SID and problem number on each of your answer sheets.
3. You may use both sides of each sheet, but be sure to start each problem on the front of a new sheet. We will be separating the exam problems for efficient grading and if you do not start a problem on a new sheet, it is possible that we will miss some of your work.
4. You may not use a calculator, but you may use a straightedge to help you draw figures.
5. You may use two 8-1/2 x 11 sheets of notes, but not your book or any other notes.
6. Store everything else out of sight.
7. Turn off cell phones.
8. Please read the entire exam before starting work. There is a great deal of information in the text of each problem, as well as in the figures.
9. You may solve the problems in whichever order you choose, of course, but pay attention to the clock so that you have sufficient time to work on all of the problems.
10. We will not answer questions during the exam. Write your concerns or interpretations of the problem(s) on your answer sheets.
11. Be concise and write clearly. Identify your answers by putting boxes around them.
12. Only one person at a time may leave the room to go to the restroom. Leave your cell phone with whoever is proctoring the exam.
13. You may leave when you have finished the exam - no need to wait until 6:00 if you finish before that time.
14. Time will be strictly enforced. At the end of the exam time, you must put down your pencil or pen and immediately turn in your exam. Failure to do so may result in loss of points.

You may solve **one** of the following 3 problems similar to those from Midterm 1

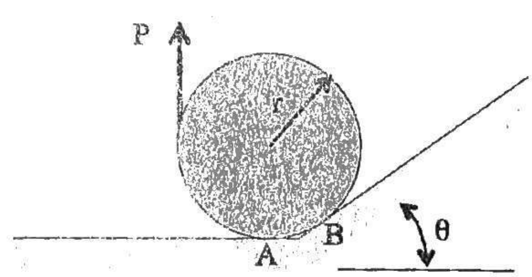
Problem MT1-1 (30 Points) The structure shown below consists of 29 rigid and (assumed) massless members that are connected to one another by ideal pins at the 16 labeled joints. It is loaded by point forces applied at joints E and J, and a distributed force that ramps linearly from 0 at P to w_{max}=400 N/m at O and back down to 0 at N. All horizontal and vertical members are $1m$ in length.

a) Determine the reaction forces at A and I.
b) Determine the forces in members LM, EM, EN, and DE. Be sure to identify whether a member is in tension or compression.



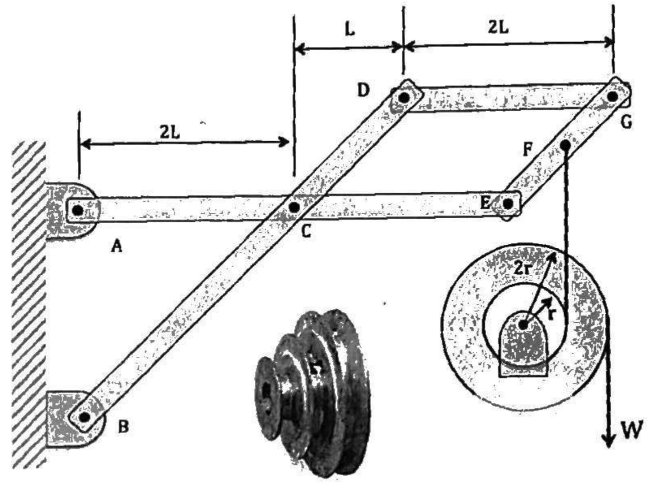
Problem MT1-2 (35 Points) A cylinder of radius r and weight W is at the intersection between a horizontal plane (the "floor") and an inclined plane that makes an angle θ with the horizontal. The cylinder is in contact with the floor at point A and with the inclined surface at point B. A vertical force P is applied at the left side of the cylinder. Let μ be the coefficient of static friction at both A and B, and let μ be sufficiently large that the cylinder does not slip with respect to the contacting surfaces, but instead begins to roll up the incline when P is large enough.

a) Determine the minimum value of μ that will ensure that the impending motion is rolling about B, not slipping at A and B. Express your answer in terms of W, r, and θ.
b) Determine P when this rolling motion is impending. Express your answer in terms of W, r, and θ.
c) Determine the normal and frictional forces at both A and B when motion is impending. Again, express your answers in terms of W, r, and θ.



Problem MT1-3 (35 Points) The frame shown consists of four massless rigid members. Members AE and BD are connected with a pin at C. A force is applied at point F, which is at the midpoint of member EG, from the double pulley below. This pulley has a smaller radius of r and a larger radius of $2r$. The two portions of the pulley are fixed to one another so that it acts as a single unit rotating freely about its axis. (For reference, a picture of such a pulley with three radii is shown.) Weight W hangs from the cable wrapped around the larger radius, while the cable wrapped around the smaller radius extends vertically to F. The two diagonal members, BD and EG, are at 45° to the horizontal.

a) Determine the reaction forces at supports A and B. Express your answers in terms of W, r, and L. (Note: there is no relation between r and L.)
b) Determine the force exerted on pin C by member AE. Again, express your answer in terms of W, r, and L.



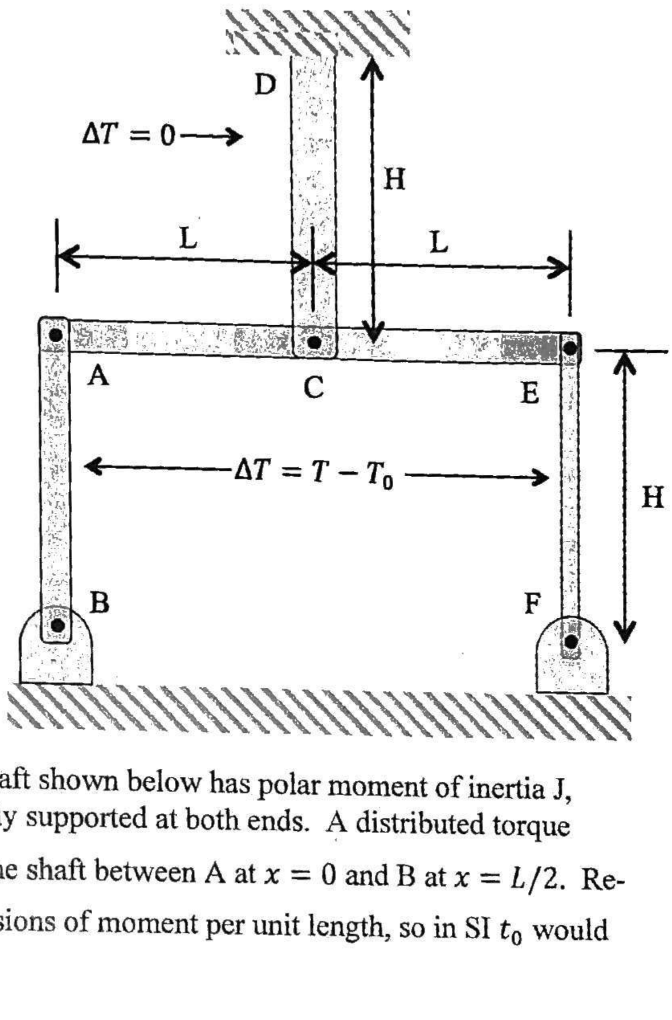
You may solve **one** of the following 2 problems similar to those from Midterm 2

Problem MT2-1 (50 points) Rigid bar ACE, which has total length 2L, is supported by three vertical elastic bars at points A, C and E. Bar ACE is horizontal at temperature T_0 and in the absence of any applied forces. The three vertical members are all of length H, and are made of the same material with Young's modulus E and linear coefficient of thermal expansion α. Member AB has cross sectional area A, member CD has area $2A$, and member EF has area $A/2$.

Let the temperature of members AB and EF increase from T_0 to T, while the temperature of bar CD does not change. Also assume that member ACE does not change length during this thermal loading.

a. Determine the stresses in the three vertical members.
b. Determine the rotation of bar ACE.

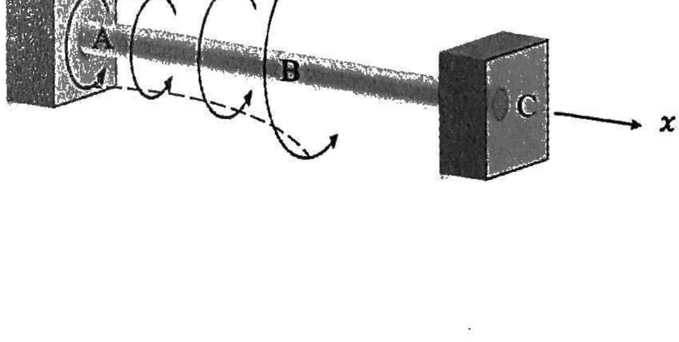
Express your answers in terms of the parameters given:
A, E, α, L, H, T_0 and T.



Problem MT2-2 (50 Points) The circular shaft shown below has polar moment of inertia J, shear modulus G and length L. It is rigidly supported at both ends. A distributed torque $t(x) = t_0\left(1 + \left(\frac{2x}{L}\right)^2\right)$ is applied along the shaft between A at $x = 0$ and B at $x = L/2$. Recall that the distributed torque has dimensions of moment per unit length, so in SI t_0 would have units of N•m/m.

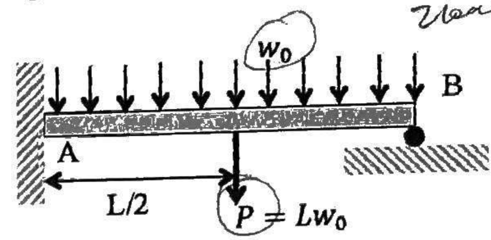
a. Determine the reaction torques at A and C.
b. Derive an equation for the twist angle for $x \le L/2$.

Express your answers in terms of the parameters given: J, L, G, and t_0.



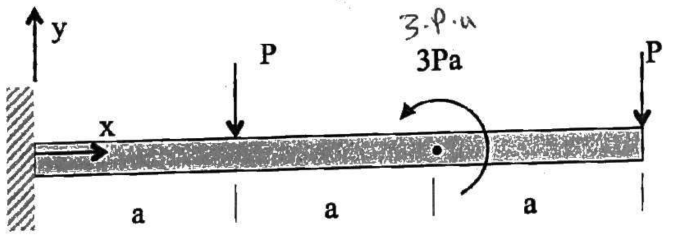
Everyone must solve the following 3 problems

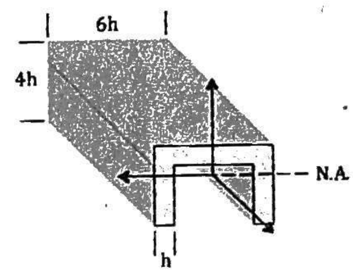
Problem F.1 (30 Points) The uniform beam in the figure below is fixed at $x = 0$, is supported on a roller at $x = L$, and is loaded as shown. It has length L, is made of a material with Young's modulus E, and has a cross section with moment of inertia I.



(a) Determine the reactions at A and B.
(b) Determine the slope of the beam at B.

Problem F.2 (50 Points) An inverted "U" beam is fixed at $x = 0$ and free at $x = 3a$. It is made of a material with Young's modulus E, and has a cross section that is 6h wide, 4h high and the horizontal and vertical segments are all h thick. The beam is loaded by point forces of magnitude P acting down at $x = a$ and $x = 3a$, and by a counterclockwise point moment of magnitude 3Pa at $x = 2a$.





(a) Determine the height of the neutral axis above the bottom (open side) of the beam. This is the axis from which y is measured.
(b) Determine the moment of inertia about the neutral axis, $I = I_z$.
(c) Draw the shear and moment diagrams. Be sure to include the magnitude and sign of these quantities at each end and at each transition in your diagrams.
(d) Determine the displacement $v(x)$ of the beam at $x = a$. Please use the symbol I for the moment of inertia until the last step when you can substitute your result from part (b).
(e) Identify the location of the point in the beam at which the magnitude of the axial stress $|\sigma_x|$ is greatest, and determine the value of σ_x at that point. Again, please use the symbol I for the moment of inertia until the last step when you can substitute your result from part (b).
(f) Determine the value of the maximum in-plane shear $\tau_{x'y'}$ at that point of maximum $|\sigma_x|$.
(g) Identify the location of the point in the beam at which the magnitude of the transverse shear stress $|\tau_{xy}|$ is greatest, and determine the value of $|\tau_{xy}|$ at that point.
(h) Determine the principal in-plane stresses σ_1 and σ_2 at that point.

Problem F.3 (20 Points) A rectangular column with thickness b, width 3b, and length L is loaded in compression. The material has Young's modulus E, Poisson's ratio ν and shear modulus G. The column can be considered pinned at both ends and is also supported against deflection in the weak direction at L/2. The end supports are not shown in the diagram below.

Determine the minimum force P that will cause this column to buckle and sketch the deflected shape after buckling.