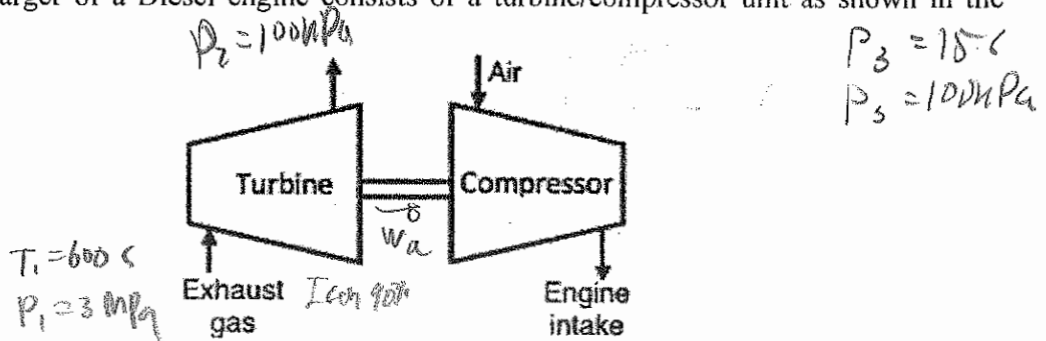


Name _____

MIDTERM EXAMINATION #2

April 7th, 2017

1. The turbocharger of a Diesel engine consists of a turbine/compressor unit as shown in the figure.



The turbine extracts work from the engine exhaust and transfer it to the compressor, which in turn uses it to increase the pressure of the air entering the engine. The exhaust gas entering the turbine can be consider to be air, it enter the turbine at 600 C, and 3 MPa and exits the turbine at 100 KPa. The air enters the compressor at 15 C and 100 KPa. The kinetic and potential energy through the compressor and turbine can be neglected and take air to have $c_p = 1.013 \text{ kJ/kg}\cdot\text{K}$ and $R = 0.287 \text{ kJ/kg}\cdot\text{K}$. Considering that the turbine has an isentropic efficiency of 90% and the compressor of 80%, calculate:

- (central w/)
- A) The work that the turbine transfers to the compressor
 - B) The pressure at the compressor exit (engine intake)

① $P_1 = 3 \text{ MPa}$ $T_1 = 600^\circ\text{C} = 873 \text{ K} \rightarrow h_1 = 900 \text{ kJ/kg}$

② $P_2 = 100 \text{ kPa}$ $T_2 = 330 \text{ K} \rightarrow h_{2s} = 330.34 \text{ kJ/kg}$

③ Isentropic $s_2 = s_1$

④ $h_2 - h_1 = -\dot{w}$

$\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \frac{T_2}{T_1}$ $T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 873 \text{ K} \left(\frac{100}{3000}\right)^{\frac{1.4-1}{1.4}} = 330 \text{ K}$

$-\dot{w} = h_{2s} - h_1 = c_p (T_{2s} - T_1) = 1.013 \text{ kJ/kg}\cdot\text{K} (330 \text{ K} - 873 \text{ K}) = -550 \text{ kJ/kg}$

$\dot{w}_s = 550 \text{ kJ/kg}$

$\eta_T = \frac{w_a}{\dot{w}_s} = 0.90$ $w_a = \eta_T \dot{w}_s = (0.90)(550 \text{ kJ/kg})$

$= 495 \text{ kJ/kg}$

30

$$\textcircled{2} \quad T_3 = 15^\circ\text{C} = \cancel{288\text{K}} = 288\text{K} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} h_{290} = 290.16 \text{ kJ/kg} \\ = h_3 \end{array}$$

$$P_3 = 100 \text{ kPa}$$

$\textcircled{1}$

$$\textcircled{3} \quad \beta_c = \frac{w_s}{w_a} = \frac{h_{4s} - h_3}{h_{24} - h_3}$$

answer) $w_a = -495 \text{ kJ/kg}$ from part a)

$\textcircled{3} \quad (P_4 = ?) \quad T_4 = ?$

$\textcircled{4} \quad h_4 - h_3 = Q - W$

Isentropic

$$s_4 = s_3 \quad \Delta s = 0 \quad \Delta Q = 0$$

$$h_4 - h_3 = Q - W$$

$$\beta_c = \frac{w_s}{w_a}$$

$$w_s = \beta_c w_a = (0.80)(-495 \text{ kJ/kg})$$

$$w_s = -396 \text{ kJ/kg}$$

$$s_2 - s_1 = C_p \ln \frac{T_2}{T_1}$$

$$h_{4s} - h_3 = -w_s$$

$$h_{4s} = -w_s + h_3 = +396 \text{ kJ/kg} + 290.16 \text{ kJ/kg} = \cancel{686} \text{ kJ/kg}$$

$$h_{675\text{K}} \approx 686 \text{ kJ/kg}$$

$$T_{4s} = 675 \text{ K}$$

$$\left(\frac{T_4}{T_3}\right) = \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}}$$

$$T_3 = \frac{T_4}{\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}}} = \frac{675\text{K}}{\left(\frac{P_4}{100\text{kPa}}\right)^{\frac{1.4-1}{1.4}}}$$

$$P_4 = P_3 \left(\frac{T_4}{T_3}\right)^{\frac{k}{k-1}} = (100 \text{ kPa}) \left(\frac{675\text{K}}{288\text{K}}\right)^{\frac{1.4}{0.4}}$$

$$P_{4s} = 2085 \text{ kPa}$$

Incompressible.

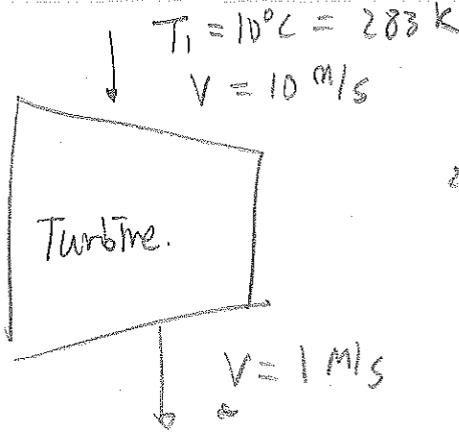
steady

$$\dot{V} = \frac{m^3}{kg} \rightarrow \frac{m}{s} \left(m^2 \frac{s}{m} \right)$$
 (Area s^2)

2. Consider the turbine in a hydroelectric power plant, that operates with water from a dam. The liquid water enter the turbine at 10°C , and with a velocity of 10 m/s , and exits the turbine with a velocity of 1 m/s . The potential energy through the turbine can be neglected. Considering that the turbine has an isentropic efficiency of 95% , calculate the power per unit mass flow rate (kg/s) extracted from the turbine.

Water \rightarrow H₂O

$$\eta_T = 0.95 = \frac{\dot{W}_a}{\dot{W}_s} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$



$\Delta PE = 0$

calc power/mass $\frac{\text{kJ/s}}{\text{kg}}$

① control volume

② $T_1 = 283\text{ K}$, $V = 10\text{ m/s}$

③ $V = 1\text{ m/s}$

④ Isentropic $\Delta S = 0$, $S_2 = S_1$

⑤ Tools $h_2 - h_1 + KE = \dot{Q} - \dot{W}$

$\dot{W}_a = 0.95 \dot{W}_s$

$T_2 = T_1$

$$S_2 - S_1 = \int \frac{dQ}{T}$$

$C_p = C_v = C$

$$-\dot{W}_s = h_2 - h_1 + KE$$

$$= h_2 - h_1 + \left(\frac{1}{2} (1\text{ m/s})^2 - \frac{1}{2} (10\text{ m/s})^2 \right)$$

~~$T_2 = 10^\circ\text{C}$~~ $T_2 = 10^\circ\text{C}$

$$S_2 - S_1 = C_{avg} \ln \frac{T_2}{T_1}$$

$-49.5 \frac{\text{kJ}}{\text{kg}}$

$T_2 = 10^\circ\text{C}$ why $\rightarrow h_2 - h_1 = 0$ or $S_2 - S_1 = 0$

$$-\dot{W}_s = \dot{m} \left(\frac{1}{2} (1\text{ m/s})^2 - \frac{1}{2} (10\text{ m/s})^2 \right) = -49.5 \dot{m}$$

$\dot{W}_s = 49.5 \dot{m}$

28

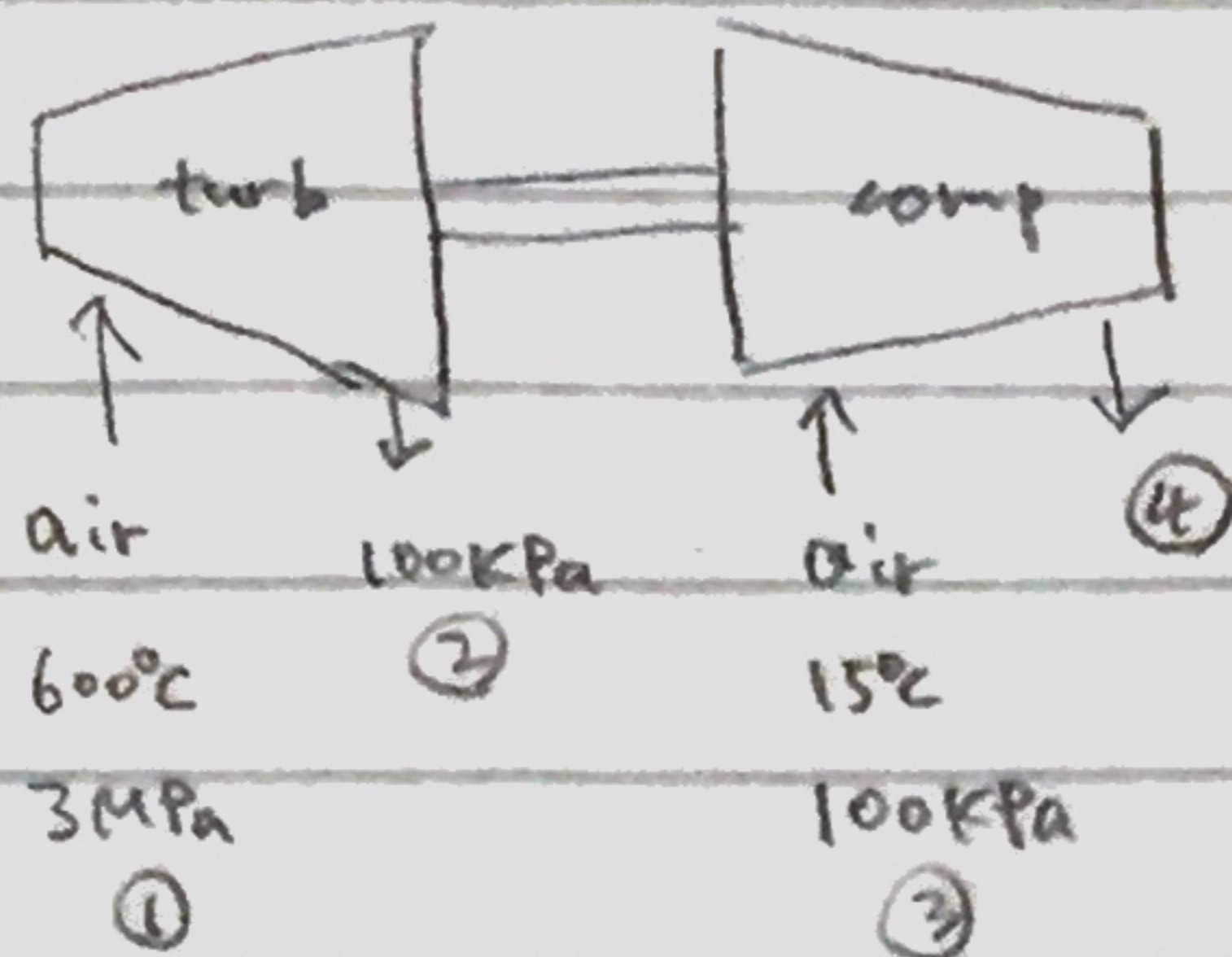
$$\dot{W}_a = 0.95 \dot{W}_s = 0.95 (49.5 \dot{m}) = 47.0 \dot{m} = 47.0 \frac{\text{kJ}}{\text{kg}} \dot{m} \frac{\text{kg}}{\text{s}}$$

$\frac{\text{kW}}{\text{kg/s}}$

$47.0 \dot{m} \text{ kW}$

$47.0 \dot{m} \text{ kJ/s}$

①



$$\Delta KE, \Delta PE \approx 0$$

$$\eta_{\text{isen, turb}} = 0.9$$

$$\eta_{\text{isen, comp}} = 0.8$$

$$\left(\begin{aligned} k &= 1.395 = \frac{c_p}{c_p - R} \\ c_p &= 1.013 \text{ kJ/kg}\cdot\text{K} \end{aligned} \right)$$

30 A) For ideal case for turb

$$\dot{Q} = \dot{Q} + (\dot{KE} + \dot{PE} + \dot{h})_{\text{in}} - W - (\dot{KE} + \dot{PE} + \dot{h})_{\text{out}}$$

$$\begin{aligned} W_{\text{turb, ideal}} &= h_1 - h_{2s} \\ &= c_p (T_1 - T_{2s}) \end{aligned}$$

$$\left(\begin{aligned} \frac{T_{2s}}{T_1} &= \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \quad (\text{isentropic \& ideal gas}) \\ \Rightarrow T_{2s} &= 333.2 \text{ K} \end{aligned} \right)$$

$$\begin{aligned} &= 1.013 (873 - 333.2) \text{ kJ/kg} \\ &= 546.8 \text{ kJ/kg} \end{aligned}$$

$$W_{\text{turb, actual}} = W_{\text{turb, ideal}} \cdot \eta_{\text{isen, turb}} = \boxed{492.14 \text{ kJ/kg}}$$

40 B) $W_{\text{in, comp}} = -W_{\text{turb, actual}} = -492.14 \text{ kJ/kg}$

For ideal comp

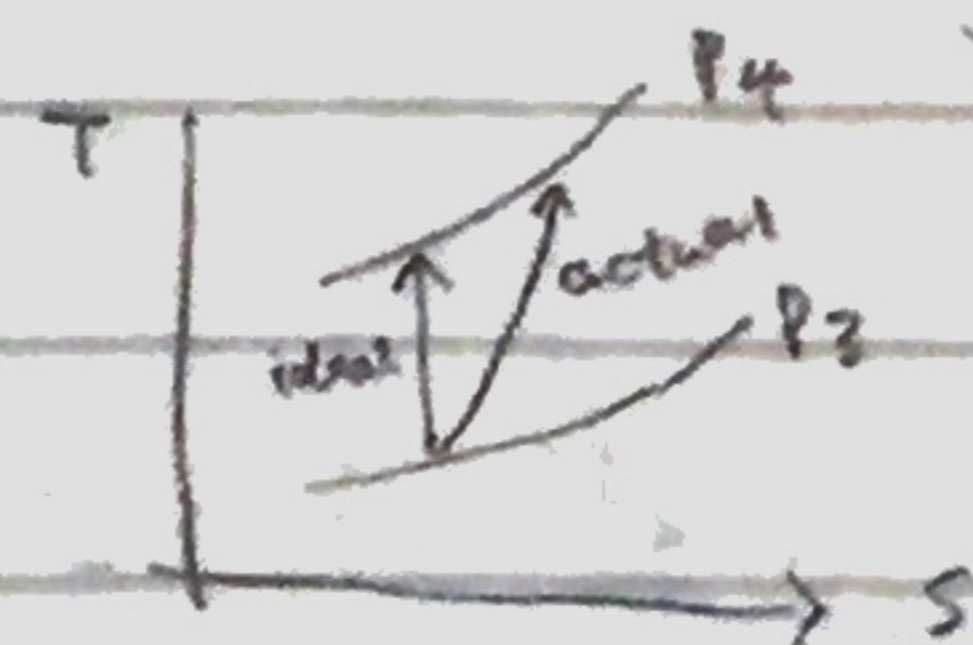
$$W_{\text{in, comp}} = h_3 - h_{4s}$$

$$0.8(-492.14 \text{ kJ/kg}) = 1.013 \text{ kJ/kg}\cdot\text{K} (288 \text{ K} - T_{4s})$$

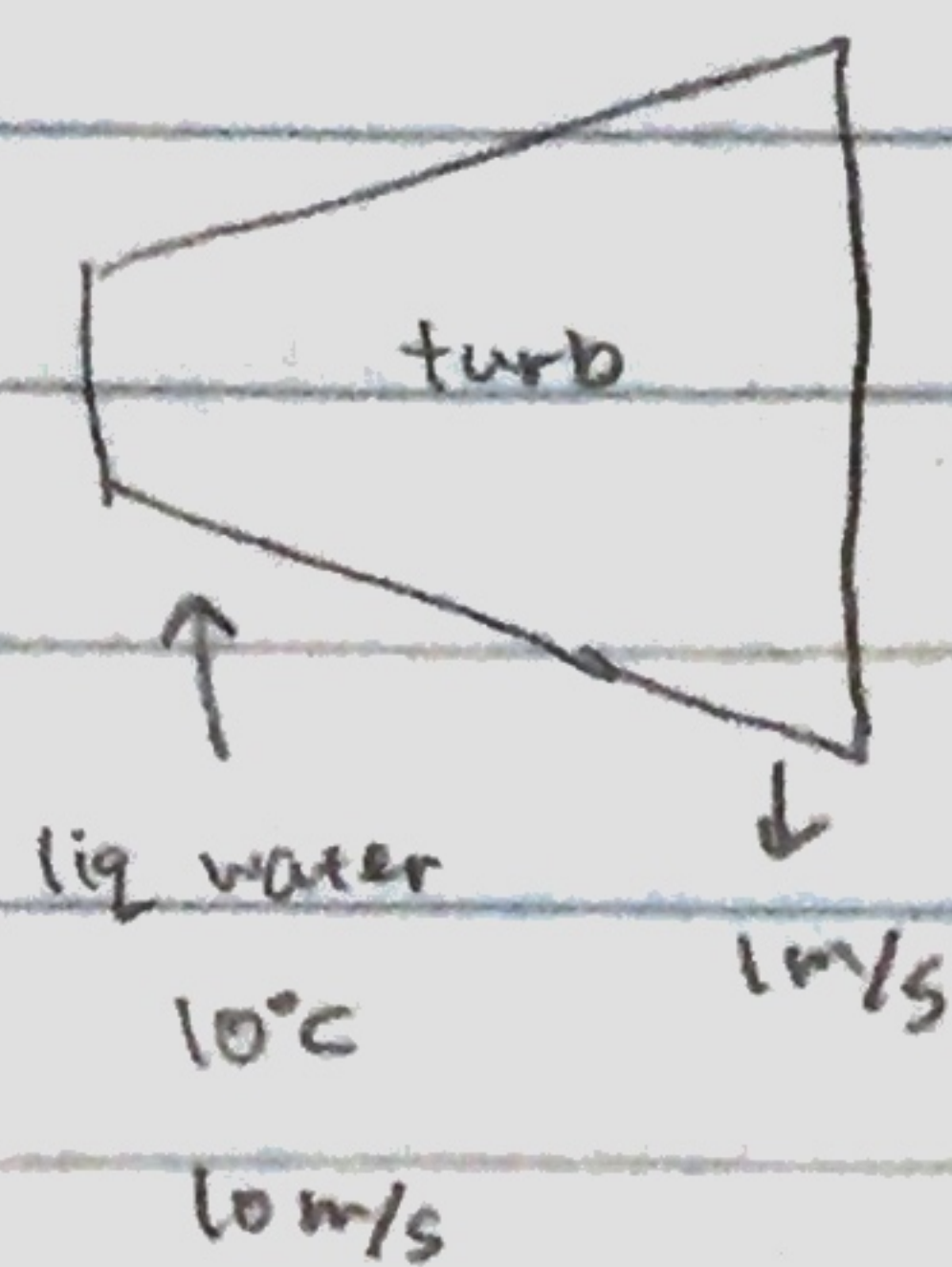
$$\Rightarrow T_{4s} = 676.6 \text{ K}$$

$$\frac{P_{4s}}{P_3} = \left(\frac{T_{4s}}{T_3} \right)^{\frac{k}{k-1}}$$

$$\Rightarrow P_{4s} = \boxed{2042 \text{ kPa}} = P_4$$



2 30



$$\Delta PE \approx 0$$

$$\eta_{isen} = 0.95$$

For ideal turbine,

$$\Delta u = \cancel{Q} + (KE + PE + h)_{in} - W - (KE + PE + h)_{out}$$

$$W_{ideal} = h_{in} - h_{out} + \frac{1}{2}(v_{in}^2 - v_{out}^2)$$

$$\left(\begin{array}{l} \text{isotropic} \\ T_{ds} = du + p dV \\ \Rightarrow du = 0 \Rightarrow cdT = 0 \\ \Rightarrow T_{in} \approx T_{out} \Rightarrow h_{in} \approx h_{out} \end{array} \right. \text{reversible}$$

$$= \frac{1}{2}(100 - 1) = 49.5 \left(\frac{m^2}{s^2} \right)$$

$$W_{actual} = 0.95 \cdot 49.5 = \boxed{47.03 \left(\frac{m^2}{s^2} \right)} = \boxed{47.03 \frac{J}{kg}}$$