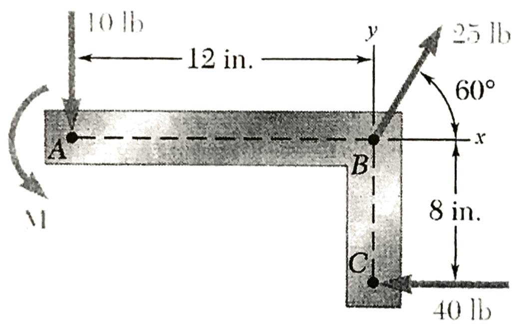
Thursday, March 9, 2017, 3:30–5 PM.

- Write your name at the top of each page as indicated.
- Write all answers in the space provided; continue on the back sides if necessary.
- Do not remove or add any pages.
- For all answers, where appropriate, provide units.
- For time management, plan to spend about 15 minutes on each question.

Good luck!

PROBLEM 1: 20 pts total

The three forces as shown and a counter-clockwise couple of magnitude M = 80 lb-in. are applied to an angle bracket. Find the direction (*i.e.*, the angle with respect to the *x*-axis) and location (*i.e.*, the *x*-coordinate) of the pure force resultant, *i.e.*, the resultant force such that the statically equivalent force-couple combination has a zero couple.



Sum of forces acting on the system:

$$\vec{\Sigma F} = (-40 + 25.\cos 60^{\circ})\underline{i} + (-10 + 25\sin 60^{\circ})\underline{j}$$

or

$$\vec{\Sigma F} = -27.5\underline{i} + 11.65\underline{j} \ (lb)$$

Sum of moments about point B:

$$+\circlearrowleft \vec{\Sigma M}_B = (80 - 40\times 8 + 10\times 12)\underline{k} = -120\,\underline{k} \ (lb\text{-}in)$$

Equivalent system: i.e, the sum of forces and the sum of moments should be equivalent to the original problem

Therefore,

$$\underline{F}^{eq} = -27.5\underline{i} + 11.65\underline{j} \ (lb) \text{ and}$$

$\longrightarrow$ turn page

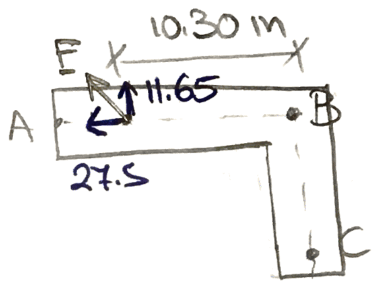
Equivalent force applied somewhere along AB:

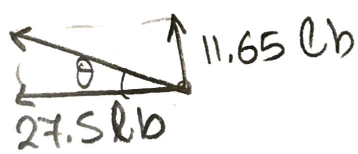
$$\overset{+}{\circlearrowleft} \sum \vec{M}_O = (x\underline{i}) \times (-27.5\underline{i} + 11.65\underline{j}) = 11.65x\,\underline{k}$$

and that should be equal to what we had before, i.e. $-120\,\underline{k}$

Thus, $11.65 \cdot x = -120 \Rightarrow \boxed{x = -10.30 \text{ in}}$

Therefore we have equivalent force $\underline{F} = -27.5\underline{i} + 11.65\underline{j}$ (lb) at $x = -10.30$ in (from origin B).





$\tan\theta = \dfrac{11.65}{27.5} \Rightarrow$

$\theta = 22.96^\circ$ or $\theta \simeq 23^\circ$ (angle with respect to the x-axis).

PROBLEM 2: 20 pts total

A 200-mm lever AB and a 240-mm-diameter pulley are welded to the axle BE, which is supported by frictionless journal bearings at C and D. A horizontal cord is attached to the pulley, and provides a tension T that resists rotation of the pulley. With the lever AB in a horizontal orientation, a vertical force of 720 N is applied at A.

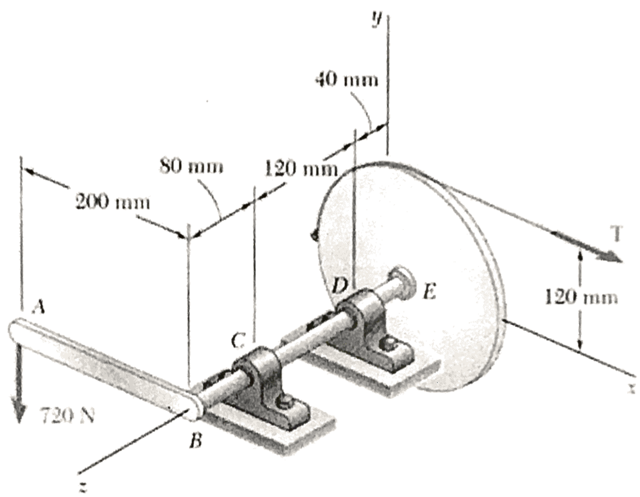
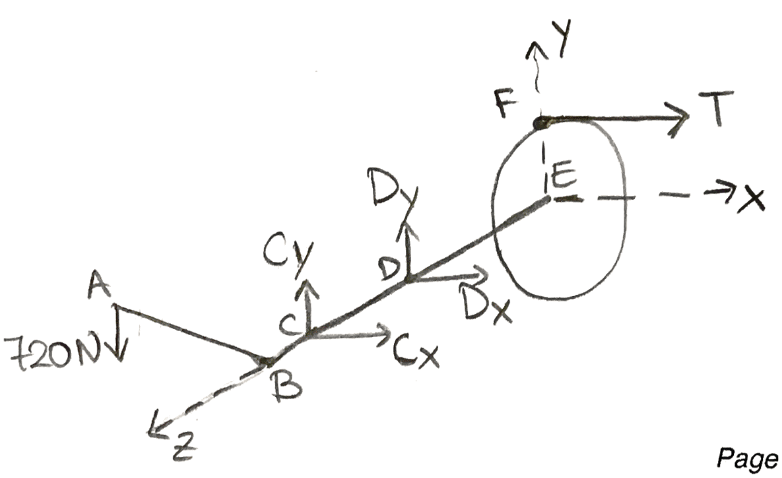


Fig. P4.95

a- For this situation, what is the tension T? **(7 points)**
b- Determine the magnitude of the reaction forces at each of C and D. **(10 points)**
c- Is there anything unusual or of concern about this configuration? Explain. **(3 points)**

Support your answers with clearly labeled free-body diagrams.

a- We always start with a nice and neat FBD...



and then we count the unknowns:
5 unknowns since the journal bearings are frictionless
C_x, C_y, D_x, D_y and T
5 unknowns and 6 equil. eqns for the 3D problem ⇒ ☺

a- Moments about z-axis:

$$\circlearrowleft^{+} \Sigma M_z = 0 \Rightarrow 720 \times 200 - 120T = 0 \Rightarrow \boxed{T = 1200\,N}$$

b- Moments about x-axis at point C:

$$\circlearrowleft^{+} \Sigma M_C^{x\text{-axis}} = 0 \Rightarrow 120D_y + 80 \times 720 = 0 \Rightarrow \boxed{D_y = -480\,N}$$

Moments about y-axis at point C:

$$\circlearrowleft^{+} \Sigma M_C^{y\text{-axis}} = 0 \Rightarrow -120D_x - 160 \times 1200 = 0 \Rightarrow \boxed{D_x = -1600\,N}$$

Equilibrium of forces

$$\Sigma F_x = 0 \Rightarrow C_x + D_x + T = 0 \Rightarrow \boxed{C_x = 400\,N}$$

$$\Sigma F_y = 0 \Rightarrow C_y + D_y - 720 = 0 \Rightarrow \boxed{C_y = 1200\,N}$$

Magnitude of forces:

$$|D| = \sqrt{D_x^2 + D_y^2} = 1670{,}4\,N$$

$$|C| = \sqrt{C_x^2 + C_y^2} = 1264{,}9\,N$$

Alternative way (longer)

$$\sum \vec{F} = \vec{0} \Rightarrow (C_x + D_x + T)\underline{i} + (C_y + D_y - 720)\underline{j} = \underline{0} \quad (N)$$

Thus, $C_x + D_x + T = 0 \quad (1)$

$C_y + D_y - 720 = 0 \quad (2)$

$$+\circlearrowleft \sum \vec{M}_C = 0 \Rightarrow (\vec{ED} - \vec{EC}) \times (D_x \underline{i} + D_y \underline{j}) + (\vec{EA} - \vec{EC}) \times (-720\underline{j}) + (\vec{EF} - \vec{EC}) \times (T\underline{i}) = \underline{0}$$

or

$$(40\underline{k} - 160\underline{k}) \times (D_x\underline{i} + D_y\underline{j}) + (-200\underline{i} + 240\underline{k} - 160\underline{k}) \times (-720\underline{j}) + (120\underline{j} - 160\underline{k}) \times (T\underline{i}) = \underline{0}$$

or

$$(-120\underline{k}) \times (D_x\underline{i} + D_y\underline{j}) + (-200\underline{i} + 80\underline{k}) \times (-720\underline{j}) + (120\underline{j} - 160\underline{k}) \times T\underline{i} = \underline{0}$$

or

$$(120D_y + 80 \times 720)\underline{i} + (-120D_x - 160T)\underline{j} + (200 \times 720 - 120T)\underline{k} = \underline{0}$$

or

$120D_y + 80 \times 720 = 0 \quad (3)$

$-120D_x - 160T = 0 \quad (4)$

$200 \times 720 - 120T = 0 \quad (5)$

$\Rightarrow$ From (3), (4) and (5)

$$\boxed{T = 1200\ N}$$

$$D_x = -\frac{160}{120}(1200) = -1600\ N$$

$$D_y = -\frac{80 \times 720}{120} = -480\ N$$

From (1) and (2) we get:

$C_x = -D_x - T = 400\ N$ and $C_y = 1200\ N$

b- Magnitude of $\underline{C} = 400\underline{i} + 1200\underline{j}$ (N)

$$|\underline{C}| = \sqrt{C_x^2 + C_y^2} = 1264.9 \text{ N}$$

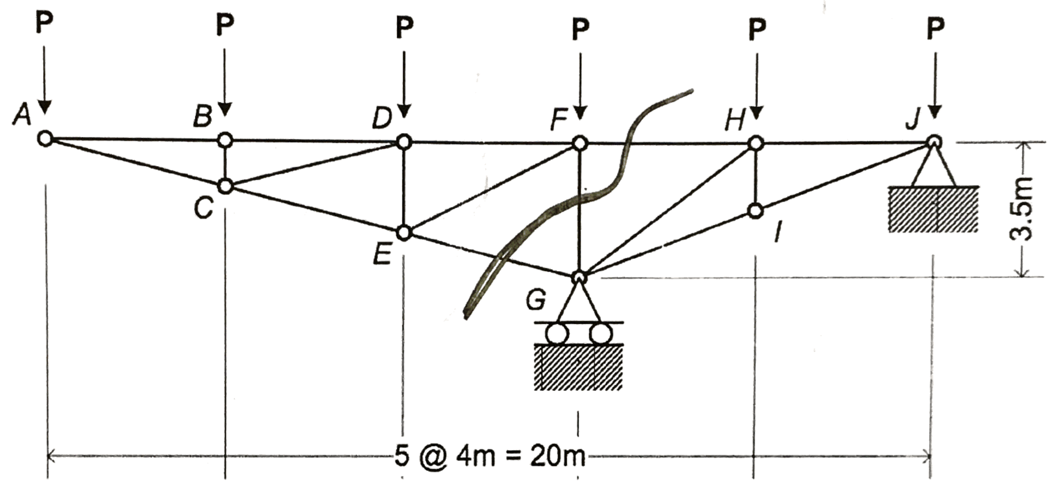
Magnitude of $\underline{D} = -1600\underline{i} - 480\underline{j}$ (N)

$$|\underline{D}| = \sqrt{D_x^2 + D_y^2} = 1670.4 \text{ N}$$

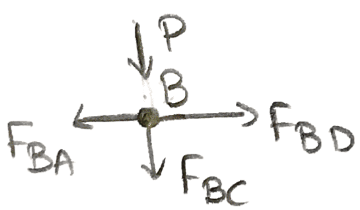
c- We can observe that the system is not restrained in the z-direction (frictionless journal bearings) (unless there is something behind the disk at E that we are not able to see given the available drawing). However, there are not any forces applied in the z-direction. Thus, the system is in equilibrium.

PROBLEM 3: 20 pts total

For the truss structure shown here, determine the force (magnitude, and whether tensile or compressive) in members FH and BC of the truss shown when P=35 kN. Support your answers with clearly labeled free-body diagrams.



From equilibrium of joint B we get:



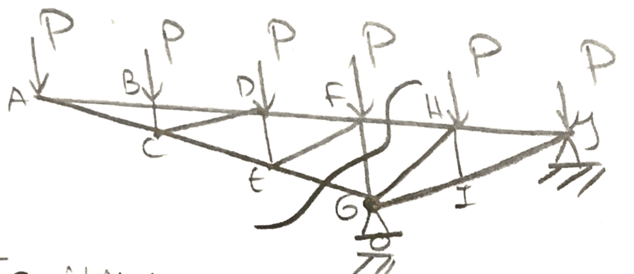
$\Sigma F_y = 0 \Rightarrow F_{BC} + P = 0 \Rightarrow F_{BC} = -P = -35\ N$

Thus, we have compressive force of magnitude equal to 35 N in member BC.

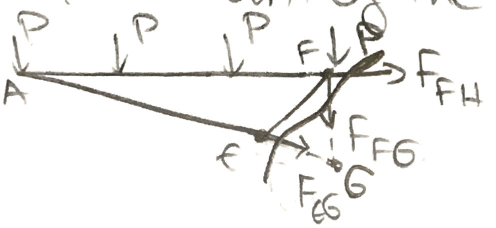
In order to calculate force in member FH we use the method of sections. We cut (this way the internal forces turn into external and using equil. eqns we are able to calculate them.) Taking the FBD of the left part of the structure we have:

(Remember: when a system is in equilibrium, every single part of it it's in equilibrium.)

There are many ways to calculate the force in member FH, but the fastest one is the following. Cutting through members FH, FG and EG we don't actually need to calculate any support reactions if we consider the FBD of the left part of the structure!



Equilibrium of the left part:

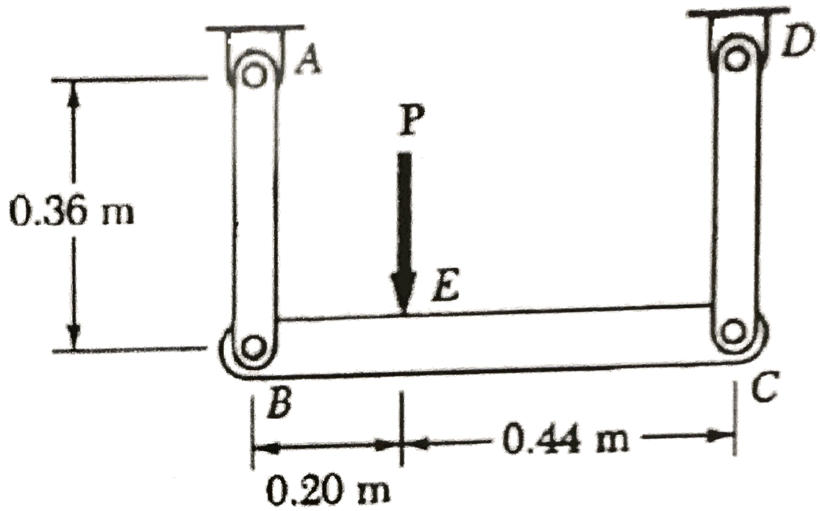


$$\overset{+}{\circlearrowleft}\Sigma M_G = 0 \Rightarrow -F_{FH}\times 3.5 + 35\times 12 + 35\times 8 + 35\times 4 = 0$$

$$\Rightarrow \boxed{F_{FH} = 240\ \text{kN}}$$

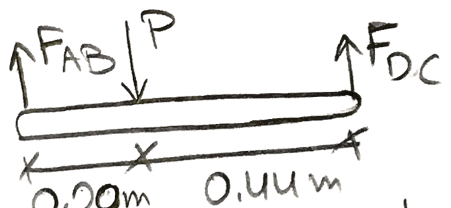
PROBLEM 4: 20 pts total

1. Elastic members AB and CD are pinned (frictionless) at each end, and have the same dimensions. The Young's modulus for member AB is 75 x 10^3 MPa and for CD is 200 x 10^3 MPa. These elastic members support a rigid member BC, which is loaded at point E by a vertical force P. For this situation, calculate the value of the ratio of the deflection of point B to the deflection of point C. **(10 points)**



2. If P = 2000 N, what is the maximum tensile stress in member AB? Assume AB is flat with a rectangular cross-section, 20 thick (into page) and 40 mm wide at mid-section, and that the pin at A fits through a 20-mm (outer diameter) hole in AB. **(10 points)**

1. FBD



$\Sigma F_y = 0 \Rightarrow F_{AB} + F_{DC} = P$ and $\overset{+}{\circlearrowleft} \Sigma M_B = 0 \Rightarrow -P \times 0.20 + F_{DC} \times 0.64 = 0$

Thus, $F_{DC} = 0.3125P$

$F_{AB} = 0.6875P$

$\delta_{AB} = \dfrac{F_{AB} \cdot l_{AB}}{E_{AB} \cdot A_{AB}}$ and $\delta_{DC} = \dfrac{F_{DC} \cdot l_{DC}}{E_{DC} A_{DC}}$

Deflection at point B: $\delta_B = \delta_{AB}$ and

Deflection at point C: $\delta_C = \delta_{DC}$

same dimensions for AB, DC members

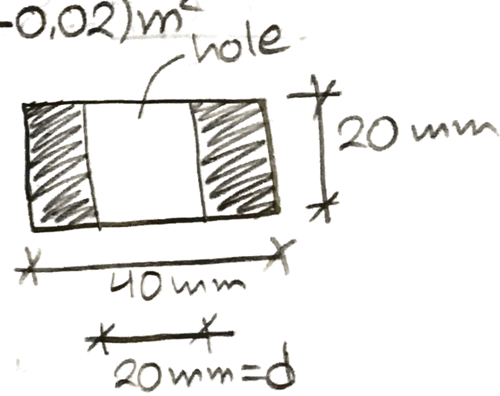
$$\text{ratio} = \frac{\delta_B}{\delta_C} = \frac{\delta_{AB}}{\delta_{DC}} = \frac{\dfrac{F_{AB}\cancel{l_{AB}}}{E_{AB}\cdot\cancel{A_{AB}}}}{\dfrac{F_{DC}\cdot\cancel{l_{DC}}}{E_{DC}\cancel{A_{DC}}}} = \frac{\dfrac{0.6875\cancel{P}}{75\times\cancel{10^3}}}{\dfrac{0.3125\cancel{P}}{200\times\cancel{10^3}}} = 5.87$$

2. Max tensile stress in AB:

$$\sigma^{max} = \frac{F_{AB}}{A_{net}} \quad \text{where } A_{net} = 0.02\times(0.04-0.02)\,m^2$$

or

$$A_{net} = 4\times10^{-4}\,m^2$$



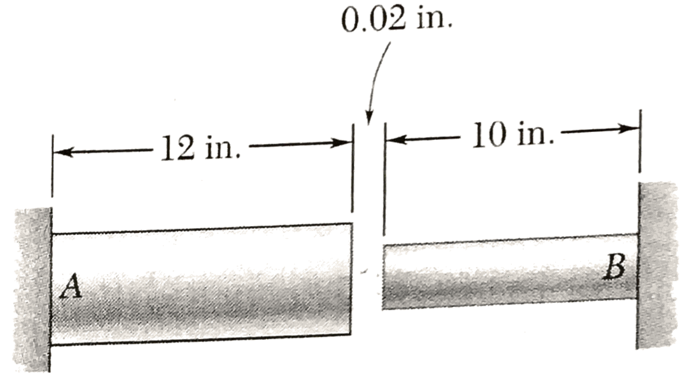
or

$$\sigma^{max} = \frac{0.6875\cdot 2000\ N}{4\times10^{-4}\,m^2} = 3.44\times10^6\ Pa$$

$$= 3.44\ MPa$$

PROBLEM 5: 20 pts total

1. Find the change in temperature (same in each cylinder) that will cause the cylinders to expand just enough to fill the 0.02-inch gap. **(10 points)**
2. If the temperature were then increased another 50 °C, what would be the ratio of compressive stresses generated in part A to part B? Explain your answer. **(10 points)**



Aluminum	Stainless steel
$A = 2.8\text{ in}^2$	$A = 1.2\text{ in}^2$
$E = 10.4 \times 10^6$ psi	$E = 28.0 \times 10^6$ psi
$\alpha = 13.3 \times 10^{-6}/°C$	$\alpha = 9.6 \times 10^{-6}/°C$

1. We want:

$$\delta_{rodA} + \delta_{rodB} = 0.02 \Rightarrow \alpha_1 \Delta T l_1 + \alpha_2 \Delta T l_2 = 0.02\text{ in} \Rightarrow$$

$$\Delta T(13.3\times10^{-6}\cdot 12 + 9.6\times10^{-6}\cdot 10) = 0.02\text{ in} \Rightarrow$$

$$\boxed{\Delta T = 78.25\ °C}$$

2. If the temperature were increased another 50°C, then the two rods are in contact, exerting force P one to the other.

FBD of A

FBD of B

Remember Newton's 3rd law: For every action there is an equal and opposite reaction. Thus, the forces each cylinder exerts on the other must be equal.

$$\sigma_A = \frac{P}{A_A} \qquad \sigma_B = \frac{P}{A_B}$$

$$\text{Ratio} = \frac{\sigma_A}{\sigma_B} = \frac{\frac{\cancel{P}}{A_A}}{\frac{\cancel{P}}{A_B}} = \frac{A_B}{A_A} = \frac{1.2\text{ in}^2}{2.8\text{ in}^2} = 0.428$$