ME 132, Spring 2014, Quiz # 2

# 1	# 2	# 3	# 4	Total	NAME
10	20	8	12	50	

Rules:

1. 2 sheet of notes allowed, 8.5×11 inches. Both sides, on both sheets, can be used.
2. Calculator is allowed. **Keep it in plain view on the desk next to you.**
3. **No laptops, phones, headphones, pads, tablets, or any other such device may be out. If such a device is seen after 9:10AM, your test will be confiscated, and you will get a 0 for the exam.**
4. **Sit with at least one open space between every student**
5. **The exam ends promptly at 10:00 AM**.
6. **Stop working, and turn in exams when notified.**

A general linear system, with input u and output y of the form $\dot{x}(t) = Ax(t) + Bu(t), \;\; y(t) = Cx(t) + Du(t)$ is stable if all the eigenvalues of A have negative real-parts. In that case, the steady-state gain from u to y is $D - CA^{-1}B$ and the instantaneous gain from u to y is D.

1. Consider the following Matlab code:

```
P = ss(1,2,3,0);
P.u = 'uP';   P.y = 'yP';
C = ss(0,1,2,1);
C.u = 'e';   C.y = 'ucmd';
S1 = sumblk('e=r-ym');
S2 = sumblk('uP = ucmd + d');
S3 = sumblk('ym = yP + n');
H = connect(P,C,S1,S2,S3,{'r','d','n'},{'yP','ucmd'});
H.a
H.d
```

(without worrying about the format) Write down, separately, the two matrices that will be displayed in the Command Window, due to the last two commands.

2. An unstable plant is described by a 1st-order linear differential equation

$$\begin{aligned} \dot{x}(t) &= x(t) + d(t) + u(t) \\ y(t) &= x(t) \end{aligned}$$

The variables are: x is the state of the plant, d is the disturbance input, u is the control input, and y is the plant output. There is a measurement noise, n, so that the measured output y_m is defined by $y_m(t) = y(t) + n(t)$.

Design a 1st-order linear control system with the following properties:

- There are two inputs to the controller: a reference input r, and the measured plant output, y_m; The controller has one state, you can call it z, or whatever your favorite letter is for a controller state; There is one output of the controller, u, which becomes an input to the plant (ie., the "control input")
- The 2nd-order closed-loop system is stable. In fact, expanding on closed-loop stability, the eigenvalues of the closed-loop "A" matrix should be at -2 and -3.
- The steady-state gain from r to y should be 1. The steady-state gain from d to y should be 0. The instantaneous-gain from r to u should be 0.

3. The matrix A is

$$A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$$

(a) What are the eigenvalues of A?

(b) Find a 2×2 diagonal matrix Λ such that $AV = V\Lambda$, where V is given by

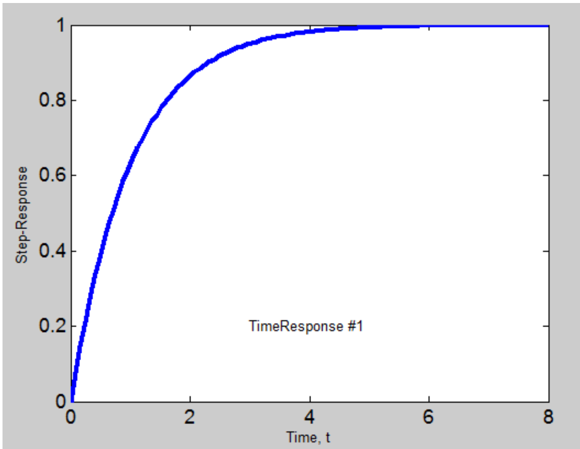
$$V = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

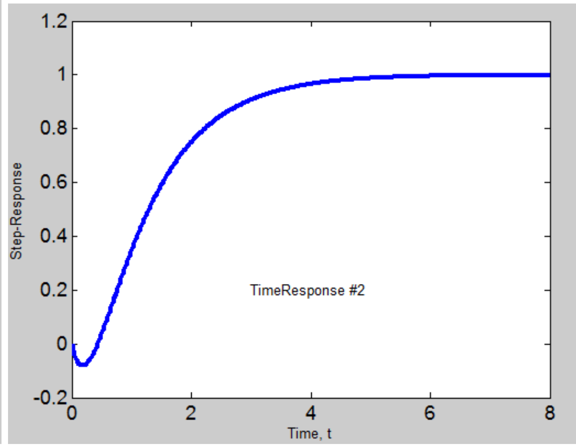
(c) Confirm that the inverse matrix of V is

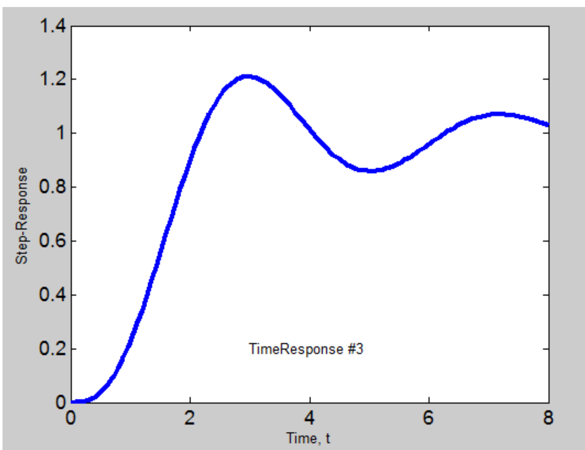
$$V^{-1} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

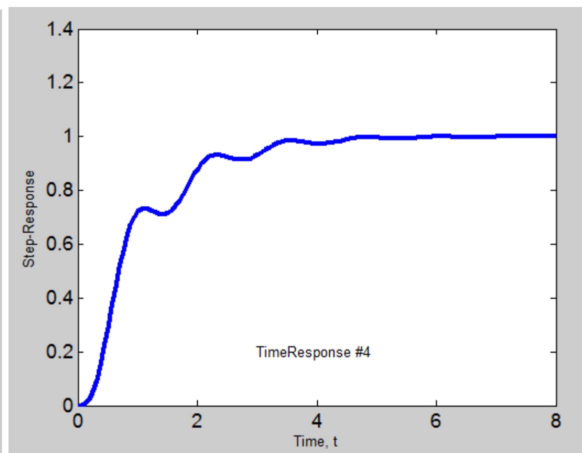
(d) Find e^{At}.

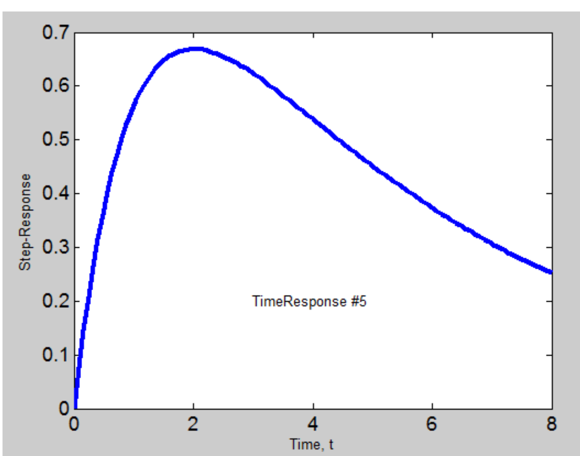
4. The step-response and Bode plot of the system $\dot{x}(t) = -x(t) + u(t), y(t) = x(t)$ is shown below. It is labeled **TimeResponse, #1**. Step-responses of 5 other systems are shown as well (labeled #2 through #6).

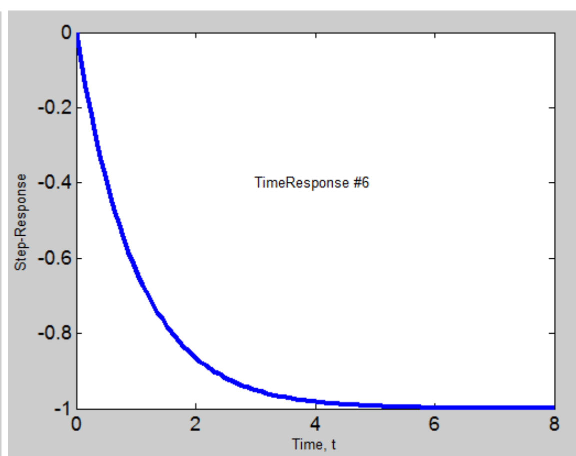












Frequency-response functions of the same 6 systems are shown on the next page. The Frequency-response for System #1 is marked. The others are not marked. **Match (on the next page) each system's Step-response with its associated Frequency-Response.** Within each frequency-response axes boundaries, **give a short (5-10 words) justification for your choice.**

Horizontal axis is frequency, in “rads/time-unit”, consistent with previous page.