ME 132, Spring 2014, Quiz # 1

# 1	# 2	# 3	# 4	#5	NAME
8	14	10	8	10	

Rules:

1. 1 sheet of notes allowed, 8.5×11 inches. Both sides can be used.
2. Calculator is allowed. **Keep it in plain view on the desk next to you.**
3. **No laptops, phones, headphones, pads, tablets, or any other such device may be out. If such a device is seen after 9:10AM, your test will be confiscated, and you will get a 0 for the exam.**
4. **Sit with at least one open space between every student**
5. **The exam ends promptly at 10:00 AM**.
6. **Stop working, and turn in exams when notified.**

For complex numbers N and D

$$|ND| = |N| \cdot |D|, \quad \angle(ND) = \angle N + \angle D$$

Furthermore, if $D \neq 0$,

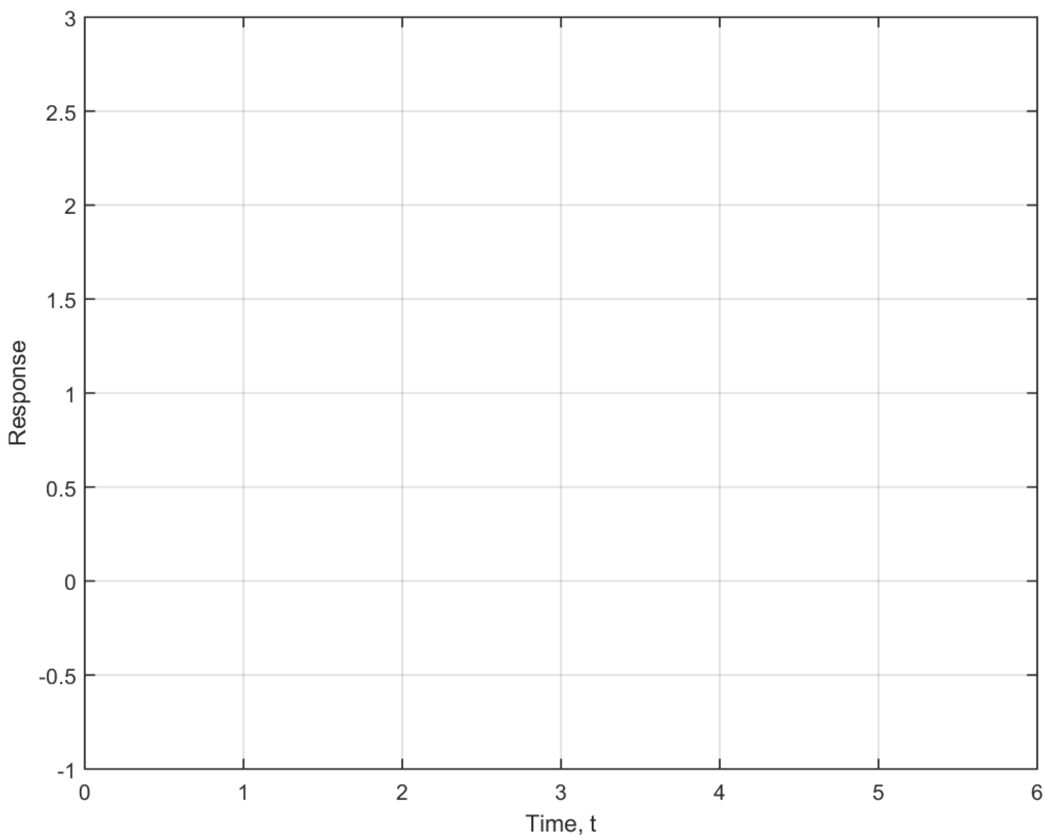
$$\angle \frac{1}{D} = -\angle D.$$

If D is real, and $D > 0$, then $\angle D = 0$.

1. Consider the following Matlab code:

```
wH = @(t) t>4;
f = @(x,u) -x+2*u;
f45 = @(t,xt) f(xt,wH(t));
[tSol,xSol] = ode45(f45,[0 6],1);
plot(tSol, xSol, '--', tSol, wH(tSol));
```

In the axes below, sketch the result produced by the `plot` command above.



2. A plant is described by a simple proportional model relating the control input (u) and disturbance input (d) to the output, namely $y(t) = \alpha u(t) + \beta d(t)$. An integral controller is implemented, of the form

$$\dot{x}(t) = r(t) - y(t), \qquad u(t) = K_I x(t)$$

(a) In terms of α, β and K_I, what are the conditions such that closed-loop system is stable?

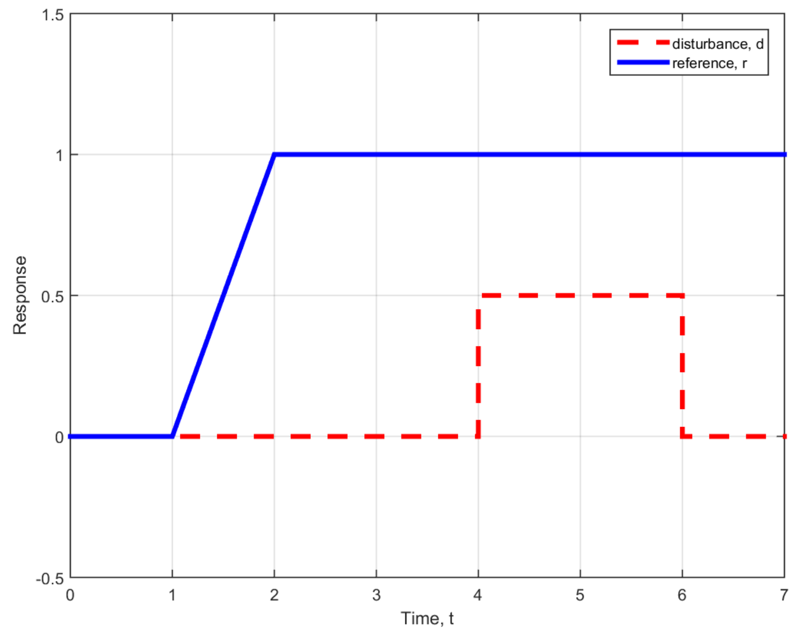
(b) Assuming the the closed-loop is stable, in terms of α, β and K_I, what is the steady-state gain from r to y?

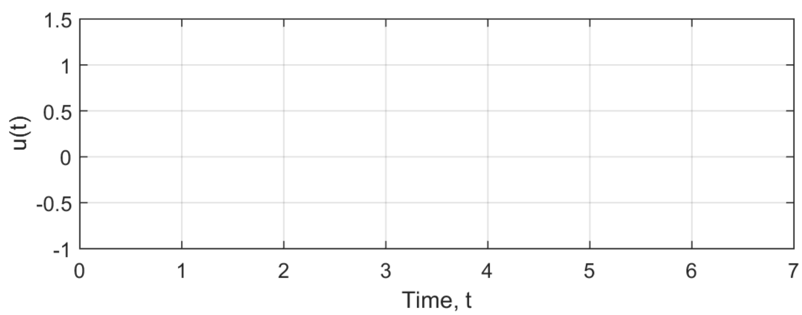
(c) Assuming the the closed-loop is stable, in terms of α, β and K_I, what is the steady-state gain from d to y?

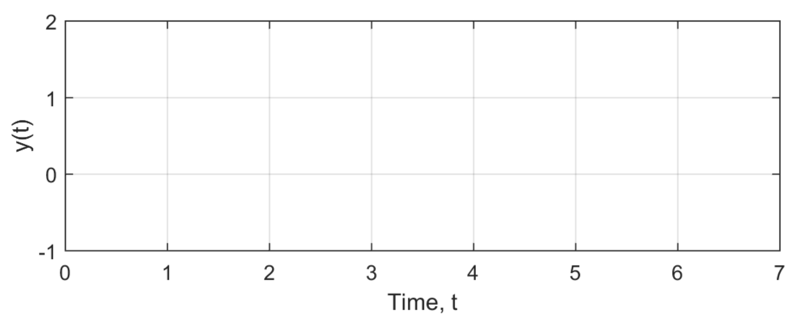
(d) Assuming the the closed-loop is stable, in terms of α, β and K_I, what is the steady-state gain from d to u?

(e) Explain how the answer in part 2d is consistent with the answer in part 2c.

(f) Suppose $\alpha = 2$ and $\beta = 1$ are the parameters of the plant. Design K_I so that the closed-loop time-constant is $\frac{1}{4}$. Assuming an initial condition of $x(0) = 0$, a reference input r and disturbance d, shown below are applied. On the two graphs provided, accurately sketch the response (u and y).







3. What is the smallest value of $T > 0$ such that for some real-valued ω, the function $z(t) = \sin(\omega t)$ satisfies the differential equation

$$\dot{z}(t) = -4z(t - T)$$

4. Consider our standard feedback interconnection, consisting of a 1st-order linear controller and proportional plant described by

$$\text{controller}: \quad \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + B_1 r(t) + B_2 y_m(t) \\ u(t) & = & Cx(t) + D_1 r(t) \end{array}$$

$$\text{plant}: \quad y(t) = \alpha u(t) + \beta d(t)$$

The measurement equation is $y_m(t) = y(t) + n(t)$.

(a) Under what conditions (on the parameters $A, B_1, \ldots, D_1, \alpha, \beta$) is the closed-loop system stable? Carefully justify your answer.

(b) Is it possible that the controller, as a system by itself, is unstable, but the closed-loop system is stable. If your answer is "No", please give a careful explanation. If your answer is "Yes", please give a concrete example.

5. A stable linear system of the form $\dot{z}(t) = Az(t) + Bu(t)$ is forced with a sine-wave input, $u(t) = \sin 3t$.

   (a) What is the frequency of the resulting sinusoidal steady-state response $z(t)$?

   (b) The resulting sinusoidal steady-state response $z(t)$ has a magnitude of $\sqrt{2}$, and "lags" the input u by $\frac{1}{8}$ of a period. Determine A and B.

firstOrderSystemID