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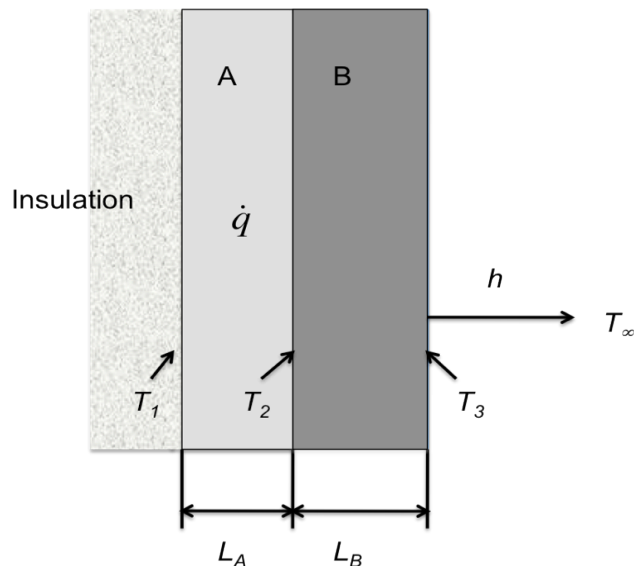
Midterm #1 (3 problems – 45 points)

3/2/15

Problem 1 (15 pts)

The figure below depicts a plane wall composed of two materials of infinite extent. Material A has thickness $L_A = 5$ cm, thermal conductivity $k_A = 30$ W/mK and internal heat generation rate $\dot{q} = 10^5$ W/m³. Material B has thickness $L_B = 10$ cm, thermal conductivity $k_B = 20$ W/mK, but no internal heat generation. The left face of material A is perfectly insulated, and the right face of material B is exposed to a fluid of temperature $T_\infty = 20^\circ\text{C}$ with $h = 100$ W/m²K. Determine the temperatures of

- the exposed surface of material B, T_3 , (5 pts)
- the junction of the two materials, T_2 , (5 pts)
- the insulated surface, T_1 (5 pts)



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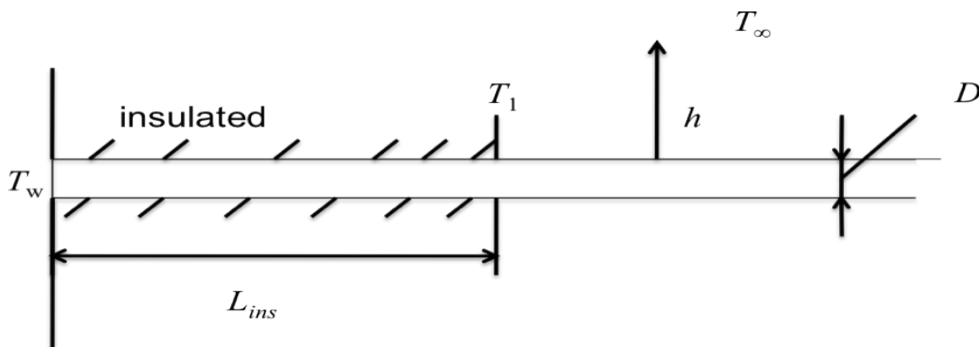
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Problem 2 (15 pts)

A very long rod of diameter $D=0.02$ m, and thermal conductivity $k = 40$ W/mK protrudes from a furnace wall that is at $T_w=300^\circ\text{C}$ and covered by insulation of length $L_{ins}=0.3$ m. The exposed part of the rod is subjected to convection to a fluid of temperature $T_\infty = 20^\circ\text{C}$ with $h = 20$ W/m²K.

- a) Find the temperature T_1 at the end of the insulation (10 pts)
- b) Find the heat transfer from the wall through the fin (5pts)



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Problem 3 (15pts)

A cylinder of, radius $r_o=10\text{cm}$, thermal diffusivity $\alpha=10^{-6}\text{ m}^2/\text{s}$, thermal conductivity $k = 10\text{ W/mK}$ is initially at uniform temperature $T_i=400^\circ\text{C}$. The cylinder surface of the plate is suddenly exposed to a fluid stream of temperature $T_\infty=50^\circ\text{C}$ and a heat transfer coefficient $h=100\text{W/m}^2\text{K}$.

- a) Determine whether the lumped capacitance approximation is valid or not. (2 pts)
- b) Calculate the elapsed time for the exposed surface temperature to be 100°C . (8 pts)
- c) Find the axis temperature ($r=0$) at the time determined in (b). (5 pts)

Coefficients and the table of the Bessel functions are given in the next pages.

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TABLE 5.1 Coefficients used in the one-term approximation to the series solutions for transient one-dimensional conduction

| Bi^a | Plane Wall | | Infinite Cylinder | | Sphere | |
|----------|--------------------|--------|--------------------|--------|--------------------|--------|
| | ζ_1 (rad) | C_1 | ζ_1 (rad) | C_1 | ζ_1 (rad) | C_1 |
| 0.01 | 0.0998 | 1.0017 | 0.1412 | 1.0025 | 0.1730 | 1.0030 |
| 0.02 | 0.1410 | 1.0033 | 0.1995 | 1.0050 | 0.2445 | 1.0060 |
| 0.03 | 0.1723 | 1.0049 | 0.2440 | 1.0075 | 0.2991 | 1.0090 |
| 0.04 | 0.1987 | 1.0066 | 0.2814 | 1.0099 | 0.3450 | 1.0120 |
| 0.05 | 0.2218 | 1.0082 | 0.3143 | 1.0124 | 0.3854 | 1.0149 |
| 0.06 | 0.2425 | 1.0098 | 0.3438 | 1.0148 | 0.4217 | 1.0179 |
| 0.07 | 0.2615 | 1.0114 | 0.3709 | 1.0173 | 0.4551 | 1.0209 |
| 0.08 | 0.2791 | 1.0130 | 0.3960 | 1.0197 | 0.4860 | 1.0239 |
| 0.09 | 0.2956 | 1.0145 | 0.4195 | 1.0222 | 0.5150 | 1.0268 |
| 0.10 | 0.3111 | 1.0161 | 0.4417 | 1.0246 | 0.5423 | 1.0298 |
| 0.15 | 0.3779 | 1.0237 | 0.5376 | 1.0365 | 0.6609 | 1.0445 |
| 0.20 | 0.4328 | 1.0311 | 0.6170 | 1.0483 | 0.7593 | 1.0592 |
| 0.25 | 0.4801 | 1.0382 | 0.6856 | 1.0598 | 0.8447 | 1.0737 |
| 0.30 | 0.5218 | 1.0450 | 0.7465 | 1.0712 | 0.9208 | 1.0880 |
| 0.4 | 0.5932 | 1.0580 | 0.8516 | 1.0932 | 1.0528 | 1.1164 |
| 0.5 | 0.6533 | 1.0701 | 0.9408 | 1.1143 | 1.1656 | 1.1441 |
| 0.6 | 0.7051 | 1.0814 | 1.0184 | 1.1345 | 1.2644 | 1.1713 |
| 0.7 | 0.7506 | 1.0919 | 1.0873 | 1.1539 | 1.3525 | 1.1978 |
| 0.8 | 0.7910 | 1.1016 | 1.1490 | 1.1724 | 1.4320 | 1.2236 |
| 0.9 | 0.8274 | 1.1107 | 1.2048 | 1.1902 | 1.5044 | 1.2488 |
| 1.0 | 0.8603 | 1.1191 | 1.2558 | 1.2071 | 1.5708 | 1.2732 |
| 2.0 | 1.0769 | 1.1785 | 1.5994 | 1.3384 | 2.0288 | 1.4793 |
| 3.0 | 1.1925 | 1.2102 | 1.7887 | 1.4191 | 2.2889 | 1.6227 |
| 4.0 | 1.2646 | 1.2287 | 1.9081 | 1.4698 | 2.4556 | 1.7202 |
| 5.0 | 1.3138 | 1.2402 | 1.9898 | 1.5029 | 2.5704 | 1.7870 |
| 6.0 | 1.3496 | 1.2479 | 2.0490 | 1.5253 | 2.6537 | 1.8338 |
| 7.0 | 1.3766 | 1.2532 | 2.0937 | 1.5411 | 2.7165 | 1.8673 |
| 8.0 | 1.3978 | 1.2570 | 2.1286 | 1.5526 | 1.7654 | 1.8920 |
| 9.0 | 1.4149 | 1.2598 | 2.1566 | 1.5611 | 2.8044 | 1.9106 |
| 10.0 | 1.4289 | 1.2620 | 2.1795 | 1.5677 | 2.8363 | 1.9249 |
| 20.0 | 1.4961 | 1.2699 | 2.2881 | 1.5919 | 2.9857 | 1.9781 |
| 30.0 | 1.5202 | 1.2717 | 2.3261 | 1.5973 | 3.0372 | 1.9898 |
| 40.0 | 1.5325 | 1.2723 | 2.3455 | 1.5993 | 3.0632 | 1.9942 |
| 50.0 | 1.5400 | 1.2727 | 2.3572 | 1.6002 | 3.0788 | 1.9962 |
| 100.0 | 1.5552 | 1.2731 | 2.3809 | 1.6015 | 3.1102 | 1.9990 |
| ∞ | 1.5708 | 1.2733 | 2.4050 | 1.6018 | 3.1415 | 2.0000 |

^a $Bi = hL/k$ for the plane wall and hr_o/k for the infinite cylinder and sphere. See Figure 5.6.

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B.4 *Bessel Functions of the First Kind*

| x | $J_0(x)$ | $J_1(x)$ |
|-----|----------|----------|
| 0.0 | 1.0000 | 0.0000 |
| 0.1 | 0.9975 | 0.0499 |
| 0.2 | 0.9900 | 0.0995 |
| 0.3 | 0.9776 | 0.1483 |
| 0.4 | 0.9604 | 0.1960 |
| 0.5 | 0.9385 | 0.2423 |
| 0.6 | 0.9120 | 0.2867 |
| 0.7 | 0.8812 | 0.3290 |
| 0.8 | 0.8463 | 0.3688 |
| 0.9 | 0.8075 | 0.4059 |
| 1.0 | 0.7652 | 0.4400 |
| 1.1 | 0.7196 | 0.4709 |
| 1.2 | 0.6711 | 0.4983 |
| 1.3 | 0.6201 | 0.5220 |
| 1.4 | 0.5669 | 0.5419 |
| 1.5 | 0.5118 | 0.5579 |
| 1.6 | 0.4554 | 0.5699 |
| 1.7 | 0.3980 | 0.5778 |
| 1.8 | 0.3400 | 0.5815 |
| 1.9 | 0.2818 | 0.5812 |
| 2.0 | 0.2239 | 0.5767 |
| 2.1 | 0.1666 | 0.5683 |
| 2.2 | 0.1104 | 0.5560 |
| 2.3 | 0.0555 | 0.5399 |
| 2.4 | 0.0025 | 0.5202 |
