**Math 1B, Second Midterm Examination**

2:00-3:00pm, N.Reshetikhin, October 24, 2016

Student's Name:

GSI's name:

Student's i.d. number:

| *Problem* | 1 | 2 | 3 | 4 | 5 | *Total* |
|---|---|---|---|---|---|---|
| *Points* | 20 | 20 | 20 | 20 | 20 | 100 |
| *Grade* | | | | | | |

1.(*20 points*) For each of the following series determine whether the series is divergent, conditionally convergent, or absolutely convergent. Indicate which tests you used.

a) (*10 points*)

$$\sum_{n=1}^{\infty}(-1)^n \sin(\frac{1}{n})$$

b) (*10 points*)

$$\sum_{n=1}^{\infty}\frac{n\sin(n)}{(n^3+1)}$$

2.(*20 points*)These are True-False questions. If the answer is True, you should explain why (concisely). If the answer is False, you should give a counter-example.

a) (*5 points*) If the series $\sum_{n=1}^{\infty} a_n$ converges absolutely and $\sum_{n=1}^{\infty} b_n$ converges conditionally, then $\sum_{n=1}^{\infty} a_n b_n$ converges absolutely.

b) (*5 points*) If the sequence $\{b_n\}$ is convergent and the sequence $\{a_n\}$ is monotonically decreasing, then the sequence $\{a_n b_n\}_{n=1}^{\infty}$ converges.

c) (*5 points*) If the series $\sum_{n=1}^{\infty} a_n$ is conditionally convergent and the sequence $\{b_n\}$ is bounded and nonnegative, then the series $\sum_{n=1}^{\infty} a_n b_n$ converges.

d) (*5 points*) If the series $\sum_{n=1}^{\infty} a_n$ converges, then the series $\sum_{n=1}^{\infty} a_n^4$ converges.

3.(*20 points*)Find the first three non-zero terms of the the Taylor series about $x = 0$ for

$$f(x) = \frac{e^x}{(1 + x)}$$

4.(*20 points*)Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(1-x)^n}{n+5}$$

.

5.(*20 points*)Answer True or False. You do not have to show your work.

a) (*4 points*) If $\sum_{n=1}^{\infty} a_n(x-2)^n$ converges at $x=0$ , then it converges at $x=3$.

b) (*4 points*) If a series $\sum_{n=1}^{\infty} a_n 2^n$ diverges, then $\sum_{n=1} a_n(-2)^n$ diverges.

c) (*4 points*) If the series $\sum_{n=1}^{\infty} a_n$ converges conditionally, then the radius of convergence of $\sum_{n=1}^{\infty} a_n(x-5)^n$ is 1.

d) (*4 points*) It is possible that the series $\sum_{n=1}^{\infty} a_n x^n$ has infinite radius of convergence, but the series $\sum_{n=1}^{\infty} a_n^2 x^n$ has finite radius of convergence.

e) (*4 points*) If the series $\sum_{n=1}^{\infty} a_n$ absolutely converges by the root test, then the radius of convergence of $\sum_{n=1}^{\infty} a_n^2 x^n$ is at least 1.