EECS 16B Designing Information Devices and Systems II
Fall 2018 Elad Alon and Miki Lustig

Midterm 2

Name: ________________________________

SID: ________________________________

Discussion Section and TA (Monday): ________________________________

Discussion Section and TA (Wednesday): ________________________________

Lab Section and TA: ________________________________

Name and SID of left neighbor: ____

Name and SID of right neighbor: ____

Instructions

- You have 120 minutes to complete this exam. Check that the exam contains 12 pages total.
- After the exam begins, write your SID in the top right corner of each page of the exam.
- Only the front pages will be scanned and graded; you can use the back pages as scratch paper.
- Do not remove any pages from the exam or unstaple the exam as this disrupts scanning. If needed, cross out any work you do not want to be graded.
- Provide explanation with every answer. Final answers with no explanation will not be given credit.

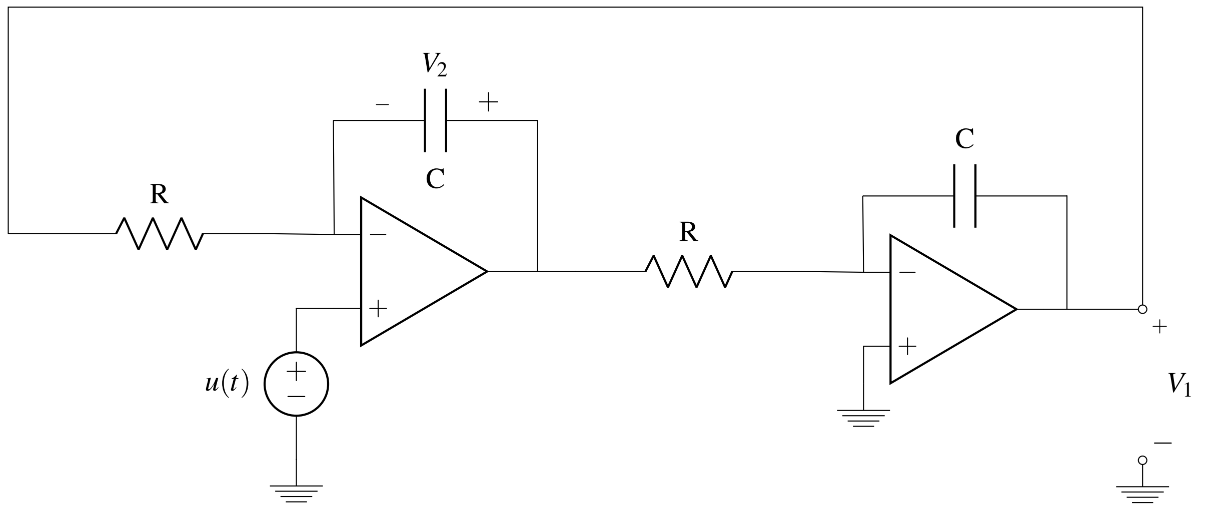
Problem	Points
1	50
2	40
3	30
4	40
5	30

Table of Unit Prefixes

Prefix	M	k	m	µ	n	p	f
Value	10^6	10^3	10^{-3}	10^{-6}	10^{-9}	10^{-12}	10^{-15}

1. Circuit Controls (50 points)

Consider the following circuit with ideal op-amps:



(a) Write a state space model $\frac{d\vec{x}(t)}{dt} = A\vec{x}(t) + Bu(t)$ with $\vec{x}(t) = \begin{bmatrix} V_1(t) \\ V_2(t) \end{bmatrix}$.

You can assume the golden rules of op-amps apply here.

$$A = \begin{bmatrix} \quad & \quad & \quad \\ \\ \end{bmatrix} \qquad B = \begin{bmatrix} \quad \\ \\ \end{bmatrix}$$

(b) Consider the following continuous time system:

$$\frac{d\vec{x}(t)}{dt} = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

Where $0 < |a| < \infty$ and $0 < |b| < \infty$. Is the system stable?

Stable / Not Stable

(c) Let $\vec{y}(t) = C\vec{x}(t)$, where $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$. Is the system observable?

Observable / Not Observable

(d) For the system in part (b), we design a state feedback controller $u(t) = K\vec{x}(t)$, where $K = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$. Find the k_1 and k_2 values which will drive the system to equilibrium with eigenvalues $\lambda_1 = \lambda_2 = -1$.

$k_1 =$ $k_2 =$

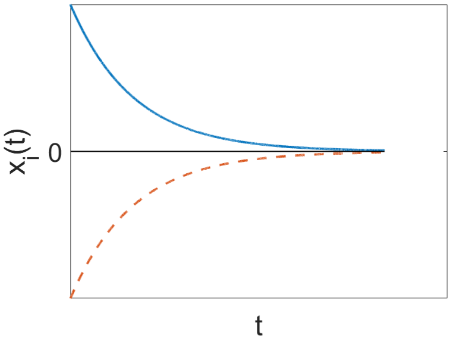
2. System Responses (40 points)

Consider again the system:

$$\frac{d\vec{x}(t)}{dt} = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} \vec{x}(t)$$

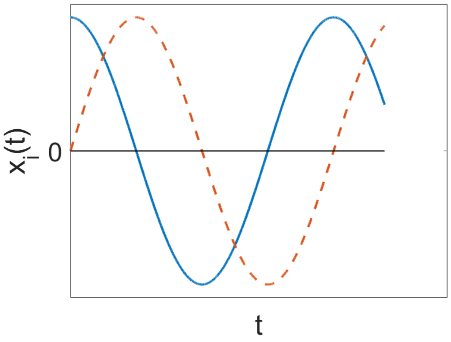
Where $0 < |a| = |b| < \infty$, i.e $a = b$ or $a = -b$. For each of the following plots, state if the plot could be a possible system response for some initial state $\vec{x}(0)$. Provide a sufficient explanation to your answer. The solid line is $x_1(t)$ and the dashed line is $x_2(t)$.

(a)



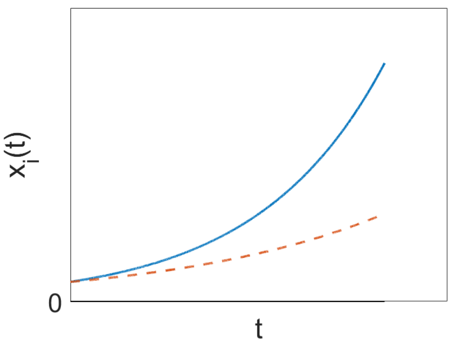
Possible? Yes / No
Explanation:

(b)



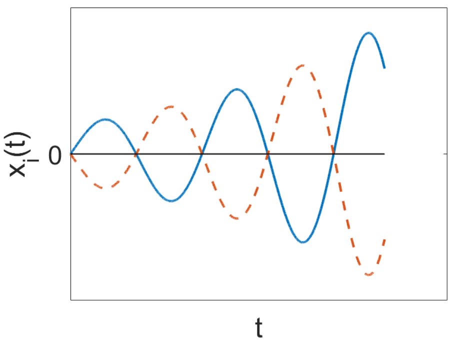
Possible? Yes / No
Explanation:

(c)



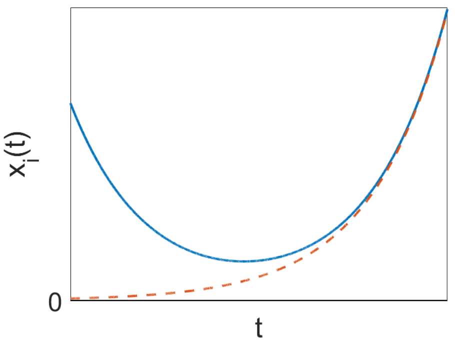
Possible? Yes / No
Explanation:

(d)



Possible? Yes / No
Explanation:

(e)



Possible? Yes / No
Explanation:

3. Discrete Time System (30 points)

Consider the discrete-time system

$$x(t+1) = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

Where $-\infty < a < \infty, -\infty < b < \infty$

(a) Under what conditions on a, b is the system stable?

Stable if ____

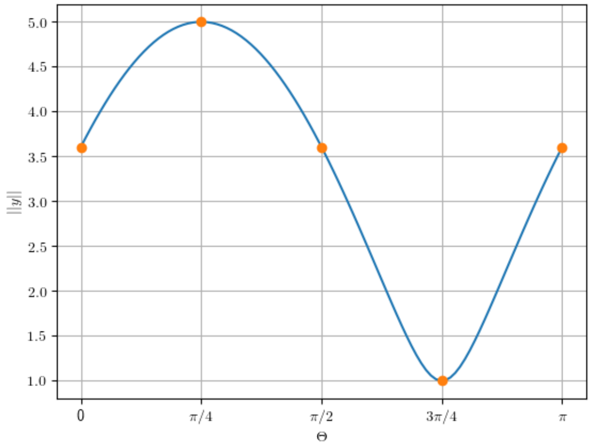
(b) Determine the inputs of an open-loop controller that will take the system from $\vec{x}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ to $\vec{x}(2) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$u(0) =$

$u(1) =$

4. SVD (40 points)

(a) Let $A \in \mathbb{R}^{2\times 2}$ and $\vec{x} = \begin{bmatrix} \sin(\theta) \\ \cos(\theta) \end{bmatrix}$, $||\vec{x}|| = 1$. Now let $\vec{y} = A\vec{x}$. Below is the plot of $||\vec{y}||$ vs θ.



What can we learn of the SVD of A? In the space provided below, complete the matrices that can be determined with the above information, and explain what's missing.

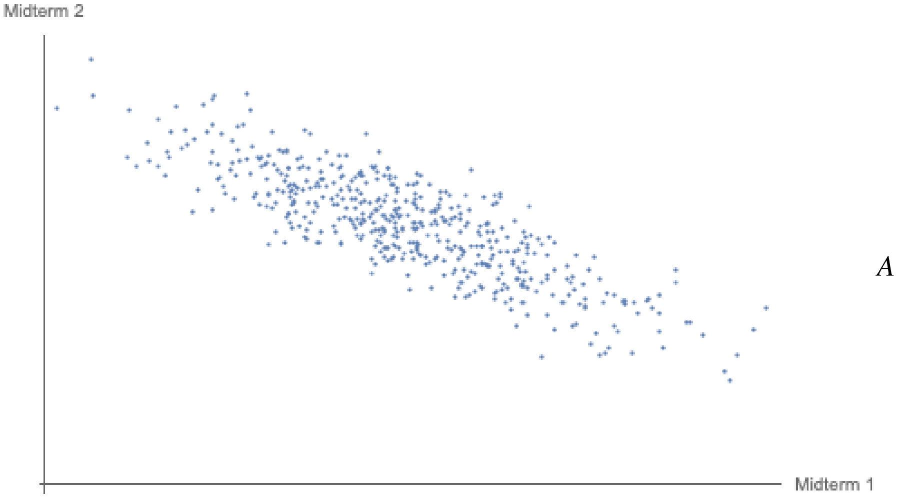
$U = \begin{bmatrix} & \\ & \end{bmatrix}$ $S = \begin{bmatrix} & \\ & \end{bmatrix}$ $V = \begin{bmatrix} & \\ & \end{bmatrix}$

(b) Let $A \in \mathbb{R}^{N \times N}, B \in \mathbb{R}^{N \times N}$ be full rank matrices and let $\vec{x} \in \mathbb{R}^N$ have $||\vec{x}|| = 1$. We compute $\vec{y} = A \cdot B \cdot \vec{x}$. Find the upper bound for $||\vec{y}||$ in terms of the singular values of A and B. Explain your answer

$||y|| \leq$

5. Data Science (30 pts)

After midterm 2, we conducted a survey in which we asked students to rate the difficulty of midterms 1 and 2 on a continuous scale from 0 to 10. The results of the scatter plots are below.



$$A = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} \text{mid}_1 & \text{mid}_2 \\ \end{array}$$

$$A = \left[\begin{array}{cc} a_1 & b_1 \\ a_2 & b_2 \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ a_n & b_n \end{array}\right]$$

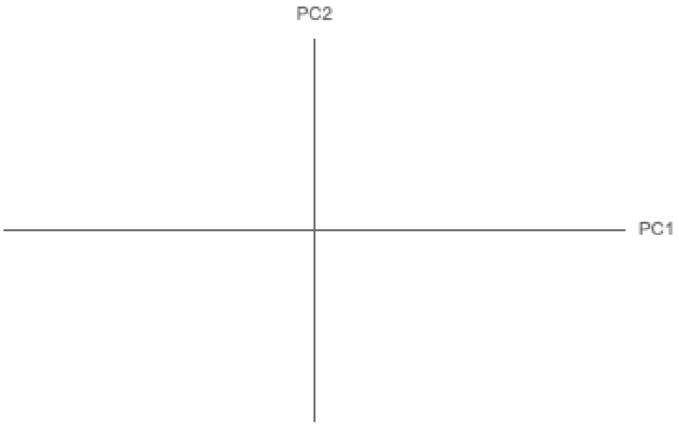
You perform PCA on the data by:

(1) Subtract the mean of each column and store the demeaned data in $\tilde{A}$

(2) Compute the SVD: $\tilde{A} = \sigma_1 \vec{u}_1 \vec{v}_1^T + \sigma_2 \vec{u}_2 \vec{v}_2^T$

(3) $\tilde{A}^T u_1 = \begin{bmatrix} -41.7 \\ 46.9 \end{bmatrix}, \quad \tilde{A}^T u_2 = \begin{bmatrix} 8.6 \\ 7.6 \end{bmatrix}$

(a) Draw a scatter plot of the projected $\tilde{A}v_1$, $\tilde{A}v_2$ points on the PCA basis. Explain your answer.



(b) Let $[a_i, b_i]$ be the rating of the ith student, and $\vec{v}_2$ be the second principal component vector. What would it mean for $\begin{bmatrix} a_i & b_i \end{bmatrix} \vec{v}_2 > \begin{bmatrix} a_j & b_j \end{bmatrix} \vec{v}_2$? Circle the answer within each set of slashes. Choose the most complete assertion. Explain your answer.

Student i / j found midterm(s) 1 / 2 / 1&2

more / less difficult than student i / j found midterm(s) 1 / 2 / 1&2.