Name: ____________ SID: ____________

Midterm

Nuclear Engineering 180

Thursday, 11/18/04

This is a take-home examination. Please turn the test in by noon, Fri. 11/19/04 at 4116 Etcheverry Hall.

1.	______ /20
2.	______ /45
3.	______ /15
4.	______ /20
T.	______ /100

1. A point charge q_0 is inserted into a pure electron plasma with a temperature of $k_BT_e = 100\ eV$, and a density n. The electrostatic potential 0.5 mm away from the point charge is measured to be 2.72 V, and the potential 1 mm away from the point charge is 1.00 V. Find the electron density n and the charge q_0. **(20 Points)**

$$\phi = \left(\frac{q_0}{4\pi\varepsilon_0 r}\right) e^{-\frac{r}{\lambda_D}}$$

$$\begin{cases} \phi_1 = \left(\frac{q_0}{4\pi\varepsilon_0 r_1}\right) e^{-\frac{r_1}{\lambda_D}}, & \phi_1 = 2.72\ V, \quad r_1 = 0.5\times10^{-3}\ m \\ \phi_2 = \left(\frac{q_0}{4\pi\varepsilon_0 r_2}\right) e^{-\frac{r_2}{\lambda_D}}, & \phi_2 = 1.00\ V, \quad r_2 = 1\times10^{-3}\ m \end{cases}$$

$$\frac{r_1\phi_1}{r_2\phi_2} = e^{\frac{r_2-r_1}{\lambda_D}}$$

$$\therefore \quad \frac{r_2-r_1}{\lambda_D} = \ln\left(\frac{r_1\phi_1}{r_2\phi_2}\right)$$

$$\lambda_D = \frac{r_2-r_1}{\ln\left(\frac{r_1\phi_1}{r_2\phi_2}\right)} = \frac{1.0\times10^{-3} - 0.5\times10^{-3}}{\ln\left[\frac{(0.5\times10^{-3})(2.72)}{(1\times10^{-3})(1)}\right]}$$

$$= 1.63\times10^{-3}\ m$$

$$\lambda_D = \sqrt{\frac{\varepsilon_0 kT_e}{ne^2}} \Rightarrow n = \frac{\varepsilon_0 kT_e}{\lambda_D^2 e^2}$$

$$= \frac{(8.85\times10^{-12})(100)(1.6\times10^{-19})}{(1.63\times10^{-3})^2(1.6\times10^{-19})^2}$$

$$\simeq 2.08\times10^{15}\ m^{-3}$$

$$q_0 = \left[(4\pi\varepsilon_0 r_1)\, e^{\frac{r_1}{\lambda_D}}\right]\phi_1$$

$$= \left[(4\pi)(8.85\times10^{-12})(0.5\times10^{-3}\ m)\, e^{\frac{0.5\times10^{-3}}{1.63\times10^{-3}}}\right] 2.72$$

$$= 2.06\times10^{-13}\ C$$

2. An electron, with total kinetic energy $W = 1\ KeV$, is in motion in a mirror magnetic field, where the minimum field strength of $0.8\ T$ at the center increases *linearly* to a maximum field strength of $2\ T$ over a distance of $1\ m$. Assume the magnetic field lines are straight over the mirror region (or see Dolan Fig. 7D1 where the field lines are straight near the axis).

(a) In the region of minimum magnetic field strength ($0.8\ T$), the electron has parallel (with respect to the magnetic field) kinetic energy $W_{\|} = 0.33\ KeV$. Calculate the gyro-radius (Larmor radius), the gyro-frequency and and the magnetic moment of the electron. Is the magnetic moment constant over the mirror region (i.e. is the condition that allows the magnetic moment to be constant satisfied?)? Explain. **(20 Points)**

(i) $\omega_{ce} = \dfrac{eB}{m_e} = \dfrac{(1.6\times10^{-19})(0.8)}{9.11\times10^{-31}} = 1.41\times10^{11}$ rad/sec

(ii) $\rho_e = \dfrac{v_\perp}{\omega_{ce}} = \dfrac{\sqrt{2W_\perp/m_e}}{\omega_{ce}} = \dfrac{\sqrt{2(0.67)(1000)(1.6\times10^{-19})/9.11\times10^{-31}}}{1.41\times10^{11}}$

$= 1.09\times10^{-4}\ m$

(iii) $\mu_m = \dfrac{W_\perp}{B} = \dfrac{(0.67)(1000)(1.6\times10^{-19})}{(0.8)} = 1.34\times10^{-16}\ \dfrac{J}{T}$

(iv) $B_{min} = 0.8T$ linearly increases to $B_{max} = 2T$

over $1\ m$

$\therefore\ \nabla B = \dfrac{(2.0-0.8)}{1} = 1.2\ T/m$

$\therefore\ \rho_e\dfrac{\nabla B}{B} = (1.09\times10^{-4})\ \dfrac{1.2}{0.8} = 1.6\times10^{-4} \ll 1$

$\therefore$ Condition $\left|\rho_e\dfrac{\nabla B}{B}\right| \ll 1$ is satisfied,

so μ_m is conserved over the mirror region.

(b) What is the force acting on the electron when the electron moves to a region with a magnetic field strength $B = 1\ T$. In what direction is the force (i.e. toward increasing field strength or toward decreasing field strength)? What is the electron parallel kinetic energy $W_{||}$ at that point? **(15 Points)**

(i) $\vec{F} = -\mu_m \vec{\nabla} B$

$F = \mu_m \nabla B = (1.34 \times 10^{-16})(1.2)$

$= 1.61 \times 10^{-16}\ N$

$\vec{F}$ points in the direction of decreasing B

(ii) μ_m is conserved

$W = W_{||} + W_{\perp}$ is conserved

$\therefore \quad \mu_m = \dfrac{W_{\perp o}}{B_{min}} = \dfrac{W_{\perp}}{B}$

$\therefore \quad W_{\perp} = B\left(\dfrac{W_{\perp o}}{B_{min}}\right) = 1\left(\dfrac{0.67}{0.80}\right) = 0.838\ KeV$

$W_{||} = W - W_{\perp} = 1 - 0.838 = 0.162\ KeV$

(c) Starting from the region of minimum magnetic field strength (0.8 T) with a parallel kinetic energy $W_{\|} = 0.33\ KeV$ (same as part (a)), how far along the straight magnetic field line could the electron move before being reflected? **(10 Points)**

(·) At reflection, $W_{\|} = 0$ ...

$\vec{F} = -\mu_m \nabla B$ is constant since B is linearly increasing from B_{min} to B_{max}

$$\therefore \quad \Delta W_{\|} = \int_0^S \vec{F} \cdot d\vec{s} = -FS$$

$\vec{s}$ points from B_{min} to B_{max}
$\vec{F}$ points from B_{max} to B_{min}

$$\therefore \quad 0 - W_{\|_0} = -FS$$

$$S = \frac{W_{\|_0}}{F} = \frac{(0.33)(1000)(1.6 \times 10^{-19})}{1.61 \times 10^{-16}}$$

$$= 0.328 \ m$$

3. (a) Captain Kirk of the Enterprise wants to use the ship's laser to fire at a Klingon warship hiding in a nebula (a plasma). He found that green-light (an electromagnetic wave) laser could propagate only some distance into a non-uniform plasma (i.e. the plasma density is increasing in the propagation direction). Should he switch to red-light (an electromagnetic wave with longer wavelength than green light) laser or blue-light (an electromagnetic wave with shorter wavelength than green light) laser to penetrate deeper into the plasma? Explain. **(5 Points)**

$$\omega^2 = \omega_{pe}^2 + k^2 c^2$$

cut-off freq. $\omega = \omega_{pe} = \sqrt{\dfrac{n_e(x)\, e^2}{m_e\, \varepsilon_0}}$ $\quad (\omega < \omega_{pe})$

cut-off wave length $\lambda = \dfrac{c}{f} = \dfrac{2\pi c}{\omega}$

$$= 2\pi c \sqrt{\frac{m_e \varepsilon_0}{n_e(x) c^2}} \qquad \lambda > \lambda_c$$

$$\therefore \quad \lambda_c \propto \frac{1}{\sqrt{n_e}}$$

Thus, a shorter wave length light $\boxed{\text{(blue)}}$ would penetrate the plasma to a higher density, thus deeper into the plasma,

(b) Suppose the plasma in (a) has $n(x) = n_0 + n'x$ where $n_0 = 0\ m^{-3}$ and $n' = 10^{20}\ m^{-4}$, and green light has wavelength $530 \times 10^{-9}\ m$. Starting at $x = 0$, find the distance green light could propagate into the plasma. **(10 Points)**

$$\lambda_c = 2\pi c \sqrt{\frac{m_e \varepsilon_0}{n_e(x)\, e^2}} = 530 \times 10^{-9}\ m$$

$$\therefore \quad n_e(x) = (4\pi^2 c^2)\ \frac{m_e \varepsilon_0}{\lambda_c^2 e^2}$$

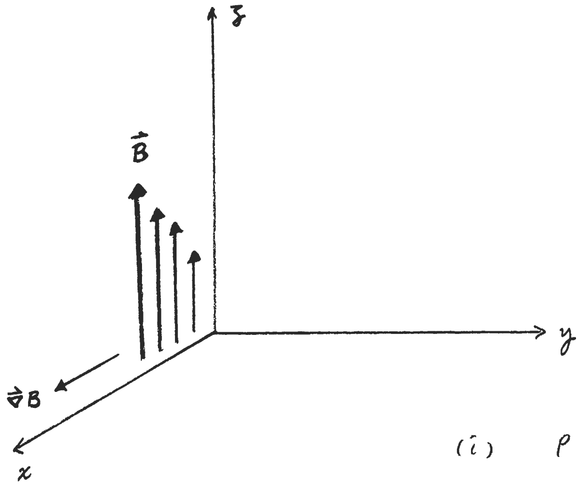
$$= 4\pi^2 (3 \times 10^8)^2\ \frac{(9.11 \times 10^{-31})(8.85 \times 10^{-12})}{(530 \times 10^{-9})^2 (1.6 \times 10^{-19})^2}$$

$$= 3.98 \times 10^{27}\ m^{-3}$$

$$n_e = \underbrace{n_0}_{0} + \underbrace{n'}_{10^{20}} x = 3.98 \times 10^{27}\ m^{-3}$$

$$\therefore \quad x = 3.98 \times 10^{7}\ m$$

4. An electron and an proton are in motion in a non-uniform magnetic field where $\vec{B} = (B_0 + B'x)\hat{z}$ with B_0 and B' positive, shown below. Both the electron and the proton have only motion in the perpendicular direction (with respect to the magnetic field) and both particles have identical total kinetic energy. Sketch the trajectories of both particles (the relative sizes of the trajectories are important but there is no need to draw the exact ratio). Choose any starting and ending points for the trajectories (the proton and the electron do not have to start at the same point). In a same amount of time, which particle's guiding center has drifted a longer distance? **(20 Points)**



(i) $\rho = \dfrac{v_\perp}{\omega_c} = \dfrac{\sqrt{2W_\perp/m}}{eB/m} = \dfrac{\sqrt{2mW_\perp}}{eB}$

$\therefore \quad \rho_i = \dfrac{\sqrt{2m_iW}}{eB} > \rho_e = \dfrac{\sqrt{2m_eW}}{eB}$

(ii) $\vec{v}_{\nabla B} = -\dfrac{(\frac{1}{2}mv_\perp^2)\,\vec{\nabla} B \times \vec{B}}{qB^3} = -\dfrac{W_\perp\,\vec{\nabla} B \times \vec{B}}{qB^3}$

(*) Both guiding centers drift the same distance in the same amount of time. ⟵ $\therefore \quad (v_{\nabla B})_i = (v_{\nabla B})_e$

$(\hat{v}_{\nabla B})_i = -\hat{x} \times \hat{z} = \hat{y}$

$(\hat{v}_{\nabla B})_e = \hat{x} \times \hat{z} = -\hat{y}$