Nuclear Engineering 101
Fall Semester 2005
Final Examination

THREE HOURS. CLOSED BOOK WITH TWO: $8\frac{1}{2}'' \times 11''$ SHEETS OF NOTES.

1. Use the Semi-Empirical Mass Formula,

$$B = a_V A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{sym}\frac{(A-2Z)^2}{A} + \delta$$

And the equation to convert binding energy B into the atomic mass M_A:

$$m_A = Zm_H + (A-Z)m_n - B$$

   (a) Find an expression for the mass difference for "mirror nuclei" such as ${}^{17}_{8}\mathrm{O}$ and ${}^{17}_{9}\mathrm{F}$, which have the same atomic number and have the number of protons and neutrons (which differ by one) interchanged.

   (b) Evaluate this expression to predict $M({}^{17}_{8}\mathrm{O}) - M({}^{17}_{9}\mathrm{F})$. Use $a_c = 0.72\mathrm{MeV}$, and $m_n - m_H = 0.782$ MeV.

2. The Gamow tunneling constant for α-decay can be written as

$$2G \approx \alpha z_\alpha Z' \left[\pi \sqrt{\frac{2m_\alpha c^2}{Q}} - 4\sqrt{\frac{2m_\alpha c^2}{B}} \right],$$

where $m_\alpha c^2$ is the alpha mass-energy = 3727.409 MeV, B is the barrier energy, z_α is the charge of the alpha and Z' is the charge of the daughter nucleus, both in units of the electron charge, and $\alpha = e^2/\hbar c$ is the fine structure constant (= 1/137.036). The decay constant $\lambda \propto e^{-2G}$.

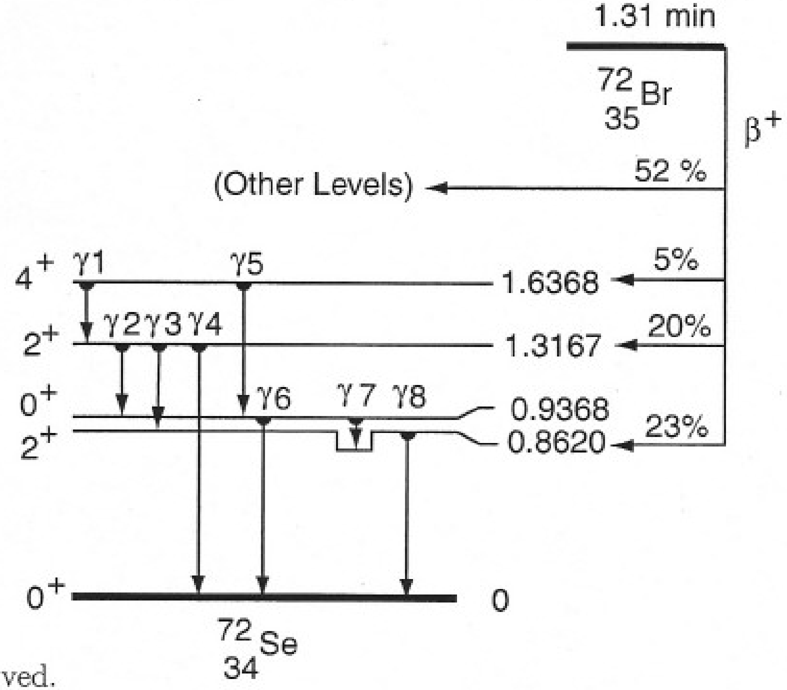
(a) Given the following information from the mass table: $m({}^{210}_{84}\text{Po}) =$ 209.982848 amu, $m({}^{206}_{82}\text{Pb}) = 205.974440$ amu, $m({}^{4}\text{He}) = 4.002603$ amu, and $m(e^-) = 0.5485803 \times 10^{-3}$ amu, and 1 amu = 931.502 MeV, find the energy release for the α decay of ^{210}Po.

(b) ^{210}Po has a half-life of 138.4 days. Use the Gamow formula to estimate the energy release of ^{208}Po, if it is known that the half-life of this isotope to α decay is 2.90 years.

(c) ^{209}Po ($J^\pi = \frac{1}{2}^-$) has an α decay to ^{205}Pb ($J^\pi = \frac{5}{2}^-$). Find the value of ℓ_α required for this decay.

(d) Do you expect that the half-life of ^{209}Po will be longer or shorter than that predicted by the Gamow model ?

3. Consider the nuclear level scheme for the decay of the 1.31 minute half-life isomer of $^{72}_{35}$Br by positron emission shown here. Note that the energies are in MeV, and that no positron decays to the either 0^+ level are observed.



(a) From the shell model, choices for the spin of the $^{72}_{35}$Br isomer are any integer between 1 and 6. Find the most likely spin, given this decay scheme.

(b) All gamma rays are observed (of the ones labeled $\gamma 1$ through $\gamma 8$) except one. Which one should not be seen?

(c) What other sort of process will occur for this non-gamma producing transition?

(d) The parity of the $^{72}_{35}$Br isomer is unknown. Assuming it is positive, are the positron decays Fermi-type, Gamow-Teller type, or a mixture? What are the degrees of forbiddenness?

4. Using the Breit-Wigner formula for compound nucleus radiative capture:

$$\sigma = \frac{\pi}{k^2} \frac{\Gamma_\gamma \Gamma_n}{(E - E_R)^2 + \Gamma^2/4}$$

   (a) For the most important resonance in ^{238}U at $E_0 = 6.67$ eV, the Breit-Wigner parameters are: $\Gamma = \Gamma_\gamma + \Gamma_n$, $\Gamma_n(E_0) = 1.45 \times 10^{-3}$ eV, and $\Gamma_\gamma = 26.0 \times 10^{-3}$ eV. Find the cross section at resonance, i. e. for $E_n = 6.67$eV.

   (b) Assuming that $\Gamma_n(E) = (E/E_0)^{1/2}\Gamma_n(E_0)$ with no variation in Γ_γ as a function of neutron energy, find the contribution to the cross section for radiative capture in ^{238}U from this resonance for thermal neutron energies ($E_n = 0.025$ eV).

   (c) Why does this (n, γ) reaction motivate the design of heterogeneous assemblies of fuel and moderator in low-enrichment light water uranium fueled nuclear reactors ?

   (d) If the temperature of the reactor goes up, how does the absorption of neutrons by this reaction change?

5. $^{160}_{66}$Dy is a highly deformed nucleus. The following gamma rays are observed from the decay of $^{160}_{67}$Tm by electron capture: 86.7 keV, 197.731 keV, and 297.33 keV.

   (a) Find the spins and parities of the ground state and the first, second, third and fourth states, assuming that they are part of a system of rotational states.

   (b) Show an energy level diagram for $^{160}_{66}$Dy using this data. Write down the mulipolarity of each of these gamma rays.

   (c) Find an approximate value of the moment of inertia $\mathcal{I}$ (in units of amu-fm^2) which fits the above data. (Note that $\hbar c = 197.4$ MeVfm.)

6. A shell model diagram is shown here.

   (a) Using this model, explain why ${}^{136}_{54}\mathrm{Xe}$ has a thermal neutron absorption cross section of only 1.0×10^{-3} b, while ${}^{135}\mathrm{Xe}$ has a thermal neutron absorption cross section of 2.7×10^{6} b.

   (b) The spin-parity J^{π} of ${}^{135}\mathrm{Xe}$ is $\frac{3}{2}^{+}$. (This is *not* consistent with the first choice one might arrive at from this shell model diagram.) Use the shell model to show where the single unpaired neutron might be.

   (c) Do the same for ${}^{137}\mathrm{Xe}$, where there $J^{\pi} = \frac{7}{2}^{-}$.

   (d) ${}^{135}\mathrm{Xe}$ has a β^{-} decay to ${}^{135}\mathrm{Cs}$ which has $J^{\pi} = \frac{7}{2}^{+}$. The half-life is relatively long (9.1 h) considering the energy release of 1.16 MeV. Discuss this in terms of the beta decay selection rules.

   (e) What is the significance of this isotope in fission reactor dynamics?