Physics 7B Midterm 2 - Fall 2018
Professor R. Birgeneau

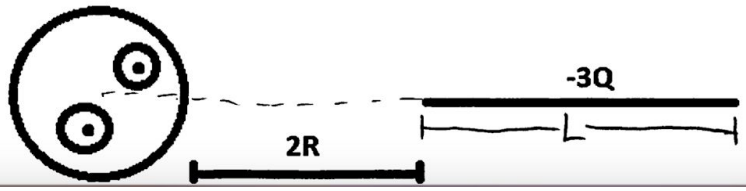
Total Points: 100 (6 Problems)

This exam is out of 100 points. Show all your work and take particular care to explain your steps. Partial credit will be given. Use symbols defined in problems and define any new symbols you introduce. If a problem requires that you obtain a numerical result, first write a symbolic answer and then plug in numbers. Label any drawings you make. **Good luck!**

Problem 1 (15 pts.) Consider the diagram below, which depicts a spherical conductor of radius R with two cavities. Two points charges, with charges 4Q and -2Q, are placed inside the conductor with one in each cavity. At a distance 2R away, an insulating rod of charge -3Q and length L is oriented radially from the center of the sphere. Assume that the electric field of the rod does not affect the charges on the conductor and that the charge is uniformly distributed on the rod.

(a) (5 pts.) Describe the charges on the inner and outer surfaces of the conductor.

(b) (10 pts.) Calculate the force on the rod.



Problem 2 (20 pts.)
An infinitely long insulating cylinder of radius a has a charge density within the cylinder that is given by $\rho(r) = \rho_0(r/a)^2$

(a) (10 pts.) Calculate the electric field everywhere inside and outside the cylinder.

(b) (5 pts.) Sketch the magnitude of the electric field as a function of r.

(c) (10 pts.) Find the electric potential everywhere inside and outside the cylinder. Take the zero of the potential to be at $r = 0$

(d) (5 pts.) Sketch the electric potential as a function of r.

Problem 3 (10 pts.)

(a) (5 pts.) Calculate the capacitance of a spherical capacitor, which consists of two concentric spherical conducting shells with a dielectric filling the space between them. Take the outer radius of the small shell to be r_1 and the inner radius of the larger shell to be r_2. Assume the dielectric has a dielectric constant K.

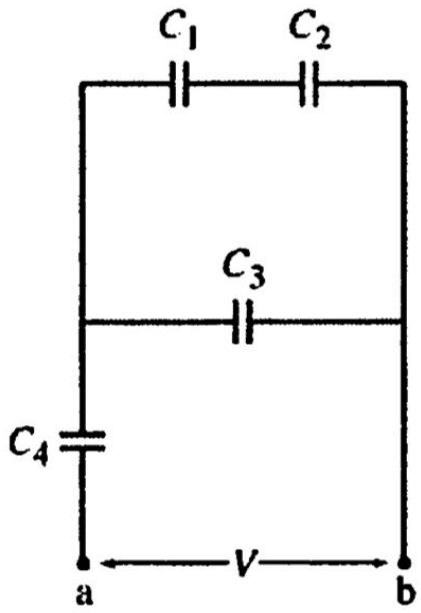
(b) (5 pts.) Calculate the capacitance per unit length of a cylindrical capacitor, which consists of an infinitely long conducting cylinder of radius r_1 surrounded by a coaxial cylindrical shell of inner radius r_2. The space between the cylinder and shell is filled with a dielectric with dielectric constant K.

Problem 4 (15 pts.)
Determine a formula for the total resistance of a spherical shell made of material whose conductivity is σ and whose inner and outer radii are r_1 and r_2. Assume the current flows radially outward. Hint: Apply $R = \rho\frac{l}{A}$ to spherical geometry.

Problem 5 (20 pts.)

(a) (10 pts.) In the figure below, suppose that $C_1 = C_2 = C_3 = C_4 = C$. Determine the equivalent capacitance between points a and b. Also determine the charge on each capacitor and the potential difference across each in terms of V.

(b) (10 pts.) For the same diagram below, now assume that a battery has been connected to the circuit and there is a potential difference V_{ab} between points a and b after equilibrium has been established. Assume that $C_1 = C_2 = C_3 = 2\ \mu$F and $C_4 = 5\ \mu$F. If the charge on C_2 is $Q_2 = 10\ \mu$C, determine the charge on each of the other capacitors, the voltage across each capacitor, and the voltage V_{ab} across the entire combination.



Problem 6 (20 pts.)
Consider an infinite plane of charge lying in the x-y plane with uniform surface charge density σ that has a circular hole of radius R cut out from it.

(a) (10 pts.) Calculate the electric field a distance z above the center of the circular hole.

(b) (5 pts.) Determine the electric field in the limit $z \gg R$, including terms quadratic in z. Explain the physical interpretation of your result.

(c) (5 pts.) Determine the electric field in the limit $z \ll R$. Explain the physical interpretation of your result.