EE 120: Signals and Systems
Department of Electrical Engineering and Computer Sciences — MIDTERM 2
UC BERKELEY — 30 October 2018

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Discussion Section Time: ____________ SID (All Digits): ____________

- **(10 Points)** Print your *official* name (not your e-mail address) and *all* digits of your student ID number legibly, and indicate your lab time, on *every* page.
- This exam should take up to 100 minutes to complete. However, you may use up to a maximum of 110 minutes *in one sitting*, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except two double-sided $8.5" \times 11"$ sheets of handwritten, original notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- We will provide you with scratch paper. Do not use your own.
- **The exam printout consists of pages numbered 1 through 10.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the ten numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't evaluate it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

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MT2.1 (75 Points) [Dereverberation] In this problem we'll explore aspects of the phenomenon of *reverberation* and the signal processing of *dereverberation*.

Reverberation of a signal x is the superposition of the signal with delayed and weighted copies of itself. Reverberation is used by the music industry to enhance the voices of vocal artists. Performance halls, too, are often acoustically designed to reverberate the voices of artists and the sounds of musical instruments during performances.

In this problem we'll stay entirely in the discrete-time realm. In particular, we'll consider the reverberation model

$$\begin{aligned} \forall n \in \mathbb{Z}, \quad y(n) &= x(n) + \alpha\, x(n-N) + \alpha^2\, x(n-2N) + \alpha^3\, x(n-3N) + \cdots \\ &= \sum_{\ell=0}^{\infty} \alpha^{\ell}\, x(n-\ell N), \end{aligned}$$

where $0 < \alpha < 1$ represents an attenuation factor that could be due to reflection of the input signal x from a barrier; $N \in \{1, 2, 3, \ldots\}$ denotes the fundamental delay in samples; and y is the "output" signal representing the reverberated version of x.

The reverberation model is well-represented by a causal, BIBO-stable DT-LTI system G having frequency response $G(\omega)$ and impulse response $g(n)$.

(a) (10 Points) Determine a reasonably simple expression for, and provide a well-labeled plot of, the impulse response $g(n)$.

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(b) (20 Points) Determine a reasonably simple expression for the frequency response $G(\omega)$, and provide a well-labeled plot of the magnitude response $|G(\omega)|$. You may tackle this part independently of the previous one.

(c) (15 Points) The input-output behavior of the system G is described by the following linear, constant-coefficient difference equation:

$$y(n) = \beta\, y(n-\gamma) + x(n-\mu),$$

where $\beta \in \mathbb{R}$ and $\gamma, \mu \in \mathbb{Z}$. Determine β, γ, and μ in terms of the known parameters α and N.

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(d) (30 Points) In some contexts reverberation is undesirable—for example, if the delay N is too long. In this and other scenarios we want to devise a system to eliminate reverberation. In particular, we want to design a DT-LTI system H such that when it's placed in series with G, we can recover the original signal x from y.

(i) (15 Points) Determine a reasonably simple expression for, and provide a well-labeled plot of, the impulse response $h(n)$ of the system H.

(ii) (15 Points) Determine a reasonably simple expression for the frequency response $H(\omega)$, and provide a well-labeled plot of the magnitude response $|H(\omega)|$ of the inverse system H.

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MT2.2 (95 Points) The frequency response of a CT-LTI filter H is given by

$$\forall \omega \in \mathbb{R} \text{ and } \exists\, \sigma > 0, \quad H(\omega) = e^{-\sigma|\omega|}.$$

(a) (15 Points) Show that the impulse response $h(t)$ of the filter is of the form

$$\forall t \in \mathbb{R}, \text{and } \exists\, A, B > 0, \quad h(t) = \frac{A}{B^2 + t^2},$$

and determine reasonably simple expressions for the constant parameters A and B in terms of σ.

(b) (15 Points) Provide a well-labeled plot of $h(t)$ and determine the values of t at which $h(t) \leq \frac{1}{2} h(0)$.

(c) (5 Points) Select the strongest true statement from the following:

(I) The filter H *must* be causal.

(II) The filter H *cannot* be causal.

(III) We have insufficient information to determine whether the filter H is causal.

Provide a succinct, but clear and convincing, explanation for your selection.

(d) (15 Points) Select the strongest true statement from the following:

(I) The filter H *must* be BIBO stable.

(II) The filter H *cannot* be BIBO stable.

(III) We have insufficient information to determine whether the filter H is BIBO stable.

Provide a succinct, but clear and convincing explanation for your selection. If you choose (I) or (II), then you must, as part of your explanation, evaluate

$$\int_{-\infty}^{+\infty} |h(t)|\, dt.$$

(e) (20 Points) For this part only, suppose we apply the input signal

$$\forall t \in \mathbb{R}, \quad x(t) = \frac{\sin t}{\pi t}$$

to the filter H. Determine a reasonably simple expression for

$$E_y = \int_{-\infty}^{+\infty} |y(t)|^2 \, dt,$$

the energy of the corresponding output signal y.

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(f) (25 Points) For this part only, suppose we apply the standard impulse train as the input signal to the filter H. That is, let

$$\forall t \in \mathbb{R} \text{ and } \exists\, T > 0, \quad x(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT).$$

(i) (15 Points) Provide a well-labeled plot of $Y(\omega)$, the spectrum of the corresponding output signal y.

(ii) (10 Points) Since the input x to the filter is periodic, so is the output y. Determine a reasonably simple expression for the coefficients Y_k in the CTFS expansion of y:

$$y(t) = \sum_{k=-\infty}^{+\infty} Y_k\, e^{i2\pi kt/T}.$$

You may tackle this part independently of part (i), but your results in the two parts must be consistent.

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MT2.3 (20 Points) Consider a BIBO stable DT-LTI filter H that has frequency response $H(\omega)$ and a *real-valued* impulse response $h(n)$, each of which is known. We apply the input signal

$$\forall n \in \mathbb{Z}, \quad x(n) = \cos(\omega_0 n)$$

to the filter. Show that the corresponding response is

$$\forall n \in \mathbb{Z}, \quad y(n) = |H(\omega_0)| \cos\Big(\omega_0 n + \angle H(\omega_0)\Big).$$

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