EE 120: Signals and Systems
Department of Electrical Engineering and Computer Sciences — MIDTERM 1
UC BERKELEY — 25 September 2018

FIRST Name ____________ LAST Name ____________

Discussion Section Time: ____________ SID (All Digits): ____________

- **(20 Points)** Print your *official* name (not your e-mail address) and *all* digits of your student ID number legibly, and indicate your lab time, on *every* page.
- This exam should take up to 100 minutes to complete. However, you may use up to a maximum of 110 minutes *in one sitting*, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided $8.5" \times 11"$ sheet of handwritten, original notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- We will provide you with scratch paper. Do not use your own.
- **The exam printout consists of pages numbered 1 through 12.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the twelve numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't evaluate it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

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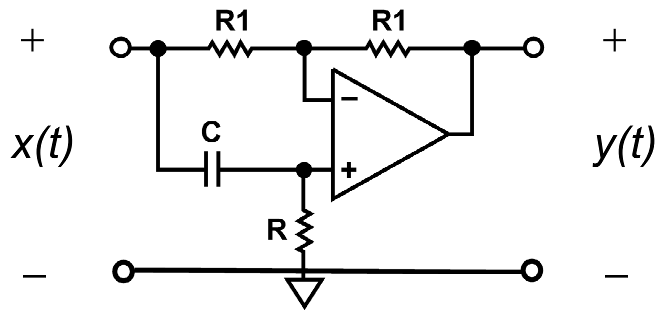
MT1.1 (50 Points) Consider the circuit H having input x and output y, as shown below.[1]



After some analysis, you learn that the circuit is a BIBO stable filter whose frequency response is

$$\forall \omega \in \mathbb{R}, \quad H(\omega) = \frac{i\omega RC - 1}{i\omega RC + 1}.$$

Do NOT attempt to show here the analysis that led to the expression for $H(\omega)$. Take the expression as a given throughout this problem.

(a) (10 Points) Determine a linear, constant-coefficient differential equation governing the input-output behavior of the filter. Also, provide an integrator-adder-gain block diagram implementation of the filter. Your implementation must use the minimal number of integrators needed.

[1] Adapted from a *Mini Tutorial* by Analog Devices, Inc. (ADI).

(b) (20 Points) Determine a reasonably simple expression, and provide a well-labeled plot, for each of the magnitude response $|H(\omega)|$ and the phase response $\angle H(\omega)$.

(c) (10 Points) Show that if a continuous-time LTI system F has impulse response $f(t)$ and frequency response $F(\omega)$, then the LTI system G whose impulse response is $g(t) = \dfrac{df(t)}{dt}$ has frequency response $G(\omega) = i\omega F(\omega)$.
Hint: Depending on how you choose to tackle this part, you may or may not find it useful to recall the formula for integration by parts: $\int u\, dv = uv - \int v\, du$. It's entirely possible to proceed without this formula.

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(d) (10 Points) Use the result(s) of the previous part(s) to show that the impulse response of the filter H is given by

$$\forall t \in \mathbb{R}, \quad h(t) = \delta(t) - \frac{2}{RC}\, e^{-t/RC} u(t).$$

Also, provide a well-labeled plot of the impulse response $h(t)$.
Hint: You may or may not find it useful to know that if the impulse response of a BIBO stable continuous-time LTI system is

$$\beta\, e^{-\alpha t} u(t), \text{then its frequency response is } \frac{\beta}{i\omega + \alpha},$$

where α and β are, in general, complex scalars, with the value of α consistent with BIBO stability.

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MT1.2 (60 Points) The input-output behavior of a causal, discrete-time LTI filter F is described by the following linear, constant-coefficient difference equation:

$$\forall n \in \mathbb{Z}, \quad y(n) - \alpha\, y(n-1) = x(n) - x(n-1), \tag{1}$$

where $0 < \alpha < 1$.

(a) (10 Points) Provide a delay-adder-gain block diagram implementation of the filter. Your implementation must use the minimal number delay blocks needed.

(b) (10 Points) Determine a reasonably simple expression for, and provide a well-labeled plot of, $f(n)$, the impulse response of the filter. You may proceed entirely in the time domain. But it's also fine if you choose to tackle this part based on the frequency response expression given to you in part (c).

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(c) (10 Points) Show that the frequency response of the filter is given by

$$F(\omega) = \frac{e^{i\omega} - 1}{e^{i\omega} - \alpha}$$

.

(d) (10 Points) Assume for this part that $\alpha = 0.99$. Provide a well-labeled *approximate* plot of $|F(\omega)|$, the magnitude response of the filter. Use the frequency interval $-\pi \leq \omega \leq \pi$ for your plot.

(e) (20 Points) Let the input to the filter be

$$\forall n \in \mathbb{Z}, \quad x(n) = \frac{1 + (-1)^n}{2}.$$

(i) (10 Points) Determine a reasonably simple expression for the corresponding output $y(n)$.

(ii) (10 Points) Assume $\alpha = 0.99$. Provide a well-labeled plot of the *approximate* response $y(n)$.

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MT1.3 (20 Points) Consider a BIBO stable discrete-time LTI system F whose impulse response $f(n)$ is *unknown* to you. However, you do know a particular input-output pair of signals consistent with the behavior of the system. In particular, you know that if the input signal is

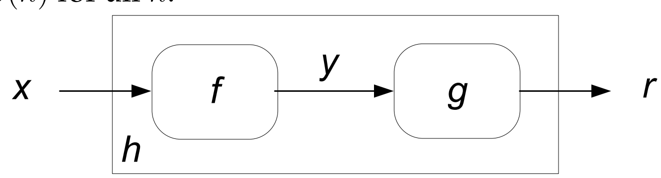
$$\forall n \in \mathbb{Z}, \quad x(n) = \delta(n) + \delta(n-1),$$

then the corresponding output is

$$\forall n \in \mathbb{Z}, \quad y(n) = \begin{cases} \alpha^{n/N} & \text{if } n \mod N = 0 \text{ and } n \geq 0, \\ 0 & \text{if } n \mod N \neq 0 \text{ or } n < 0, \end{cases}$$

for some $N \in \{3, 4, 5, \ldots\}$.

Determine the impulse response $g(n)$ and frequency response $G(\omega)$ of a system G, such that the system G is the inverse of the system F. In particular, determine $g(n)$ and $G(\omega)$ such that if the system G is placed in cascade (series) with the system F, as shown below, the output signal r will be equal to the input signal x, for all n. In other words, determine the system G such that the overall system H has impulse response $h(n) = \delta(n)$ for all n.



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MT1.4 (50 Points) A continuous-time LTI system H has impulse response

$$h(t) = \begin{cases} \dfrac{1}{T} & |t| \leq T/2 \\ 0 & |t| > T/2, \end{cases}$$

where $T > 0$ is a known time duration parameter.

(a) (10 Points) Show that the instantaneous output $y(t)$ can be expressed in terms of the input x as follows:

$$\forall t \in \mathbb{R}, \quad y(t) = \frac{1}{B} \int_{t-A}^{t+A} x(\tau)\, d\tau,$$

for appropriate scalar parameters A and B. Determine, in a reasonably simple form, each of A and B in terms of T.

(b) (10 Points) For the limiting case $T \to 0$, determine the instantaneous output $y(t)$ in terms of the input x. That is, determine $\lim\limits_{T \to 0} y(t)$.

(c) (10 Points) Show that the frequency response can be expressed in the form

$$\forall \omega \in \mathbb{R}, \quad H(\omega) = \frac{\sin(C\omega)}{D\omega},$$

for appropriate scalar parameters C and D. Determine, in a reasonably simple form, each of C and D in terms of T. Also, provide a well-labeled plot of the magnitude response $|H(\omega)|$.

(d) (20 Points) For each input signal below, determine a reasonably simple expression for, and provide a well-labeled plot of, the corresponding output $y(t)$:

(i) (5 Points) $x(t) = h(t)$ for all t.

(ii) (5 Points) $x(t) = \cos\left(\frac{4\pi}{T}t\right)$ for all t.

(iii) (5 Points) $x(t) = \sum_{n=-\infty}^{+\infty} \delta(t - 2Tn)$, where $\delta(\cdot)$ denotes the Dirac delta.

(iv) (5 Points) $x(t) = e^{-t}u(t)$ for all t.

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With the exception of your identifying information above,
no other writing on this page will be graded.