Problem 1

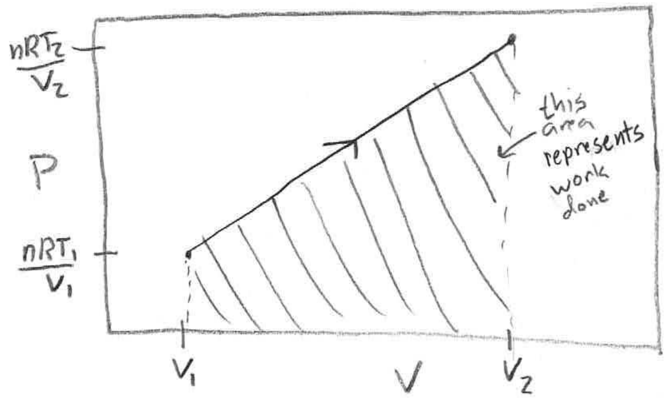

FIGURE 1. *P*-*V* diagram for the thermodynamics process described in Problem 1.

a) To draw this on a *P*-*V* diagram we use the ideal gas law to obtain,

$$\frac{T}{V^2} = \frac{P}{nRV} \quad \rightarrow \quad P = \frac{P_1}{V_1} V.$$

The process thus appears as a straight line with slope P_1/V_1 on the *P*-*V* diagram as shown in Figure 1. This is not an isobaric, isovolumetric, isothermal, or adiabatic process.

b) Work is represented on the PV diagram by the area under the curve as indicated in Figure 1. This area is easily calculated geometrically or from the integral

$$W = \int P dV = \int_{V_1}^{V_2} \frac{P_1}{V_1} V dV = \frac{1}{2} \frac{P_1}{V_1} (V_2^2 - V_1^2) = \frac{nRT_1}{V_1^2} \left(\frac{T_2}{T_1} V_1^2 - V_1^2 \right)$$

$$\boxed{W = nR(T_2 - T_1)}$$

c) We can use equipartition to calculate the change in energy,

$$\Delta U = \frac{d}{2} nR\Delta T.$$

Since a diatomic gas has 5 degrees of freedom, we find for our case that

$$\boxed{\Delta U = \frac{5}{2} nR(T_2 - T_1)}$$

The first law of thermodynamics gives us the heat transfer,

$$\Delta U = Q - W$$

$$\boxed{W = \frac{7}{2} nR(T_2 - T_1)}$$

d) Since this is a reversible process,

$$\Delta S = \int \frac{dQ}{T}.$$

From part (c) we know that $dQ = \frac{7}{2}nRdT$ which allows us to integrate,

$$\Delta S = \frac{7}{2}nR \int_{T_1}^{T_2} \frac{dT}{T} = \boxed{\frac{7}{2}nR \ln T2/T_1.}$$

Problem 2

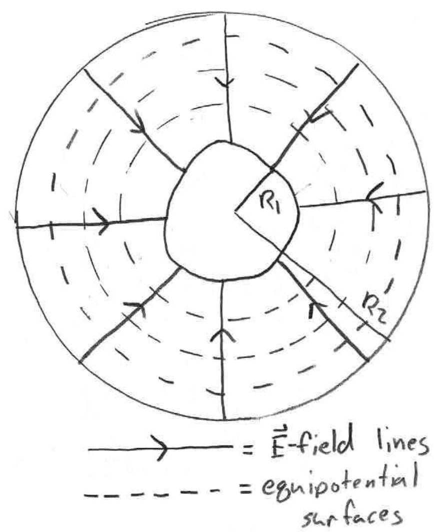

FIGURE 2. Cross-sectional view showing electric field lines and equipotential surfaces. Note that $\vec{E} = 0$ for $r < R_1$ and $r > R_2$.

a) Because we can easily find the electric field using Gauss's law, the best approach is to find $\Delta V = -\int \vec{E} \cdot d\vec{l}$. From symmetry considerations, an infinite cylindrical charge distribution must have a field of the form $\vec{E} = E(r)\hat{r}$ in cylindrical coordinates. By considering a cylindrical Gaussian surface of radius $R_1 < r < R_2$, length L, and the same symmetry axis as the charge distributions, Gauss's law tells us that

$$\int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

Replacing these quantities appropriately,

$$\begin{aligned} \int \vec{E} \cdot d\vec{A} &= 2\pi r L E(r) \\ q_{\text{enclosed}} &= -\lambda L \\ \rightarrow \quad \vec{E}(r) &= -\frac{\lambda}{2\pi\epsilon_0 r}\hat{r} \end{aligned}$$

The potential difference is therefore

$$V(R_2) - V(R_1) = -\int \vec{E} \cdot d\vec{l} = \frac{\lambda}{2\pi\epsilon_0} \int_{R_1}^{R_2} \frac{1}{r} dr$$

$$\boxed{V(R_2) - V(R_1) = \frac{\lambda}{2\pi\epsilon_0} \ln R_2/R_1}$$

Note that $V_2 > V_1$, as expected when going from a negative charge distribution towards a positive one.

b) We have derived the electric field in part (a) and seen that it points radially inward. The lines of constant equipotential are perpendicular to the electric field lines and form circles of constant radii (this can also be seen by generalizing the result of the previous part). See Figure 2 for a sketch.

c) An electron of charge $q = -e$ in this electric field feels a force $\vec{F} = q\vec{E} = \frac{e\lambda}{2\pi\epsilon_0 r}\hat{r}$, i.e. it will be pushed radially outward. By conservation of energy, we can find the final velocity:

$$E_i = E_f$$

$$PE_i = KE_f + PE_f$$

$$q(V(R_1) - V(R_2)) = e(V(R_2) - V(R_1)) = \frac{1}{2}mv_f^2$$

This can be expressed either in terms of V_1 and V_2,

$$\boxed{v_f = \left[\frac{2e}{m}(V_2 - V_1)\right]^{1/2}}$$

or in terms of the actual potential difference calculated in part (a):

$$\boxed{v_f = \left[\frac{e\, m\, \lambda}{\pi\, \epsilon_0} \ln R_2/R_1\right]^{1/2}}$$

3

Applying Kirchoff's second rule on a clockwise loop encircling all 4 resistors,

$-I_3(R_3 + R_x) + I_1(R_1 + R_2) = 0$
$\Rightarrow \frac{I_1}{I_3} = \frac{R_3+R_x}{R_1+R_2} \quad (1)$

On the other hand, drawing a loop circling I_3, I_x and the ammeter,

$-I_3 R_3 + I_1 R_1 = 0$
$\Rightarrow \frac{I_1}{I_3} = \frac{R_3}{R_1} \quad (2)$

Combining (1) and (2),
$R_x = \frac{R_2 R_3}{R_1}$

4

Let the z-axis be pointing out of the paper.

For the contribution due to the wire, $\mathbf{B_{wire}}$,
From the right hand rule, $\mathbf{B_{wire}}$ points in the $\hat{\mathbf{z}}$ direction.
Drawing a circle with radius a, centered on the wire and passing through the center of the loop, by Ampere's law,
$\int \mathbf{B} \cdot \mathbf{dl} = (B)(2\pi a) = \mu_0 I \Rightarrow \mathbf{B_{wire}} = \frac{\mu_0 I}{2\pi a}\hat{\mathbf{z}}$

For the contribution due to the circular loop, $\mathbf{B_{loop}}$,
From the right hand rule, $\mathbf{B_{loop}}$ also points in the $\hat{\mathbf{z}}$ direction.
By Biot-Savart law,
$\mathbf{B} = \frac{\mu_0 I}{4\pi}\int \frac{\mathbf{dl}\times\mathbf{r}}{r^2} = \frac{\mu_0 I}{4\pi}\int_0^{2\pi} \frac{(ad\theta)(a)\hat{\mathbf{z}}}{a^2}$
$\Rightarrow \mathbf{B_{loop}} = \frac{\mu_0 I}{2a}\hat{\mathbf{z}}$

Combining the two, $\mathbf{B} = \frac{\mu_0 I}{2a}(1 + \frac{1}{\pi})\hat{\mathbf{z}}$

5

a)
As the electrons flow through the set-up, they experience a magnetic force into the paper. The accumulated electrons at the "back" side of the slab (with dimensions $t \times b$) create a net negative charge on that side and a net positive charge on the "front" side (also with dimensions $t \times b$).
This creates a electric potential difference, i.e. the Hall voltage, between the back side and front side of the slab, and the corresponding electric force balances the magnetic force exactly at equilibrium.

b)
Let v_d denote the drift velocity, j denote the current density.
$\Rightarrow \mathcal{E} = v_d B w$
$\Rightarrow K_H = \frac{\mathcal{E}}{IB} = \frac{v_d}{I}$

On other hand, note that $j = nev_d = \frac{I}{wt}$
$\Rightarrow K_H = \frac{v}{I} = \frac{1}{net}$

c)
The semiconductor candidate is better, as K_H is inversely proportional to n.

d)
The "front" side (see explanation in part a)).

Problem 6

a)

The flux into the page is decreasing, so the current will want to create a magnetic field into the page to counteract this decrease. By the right hand rule, this means that the current flows clockwise (when the fingers curl clockwise the thumb points inward).

b)

Since B is constant everywhere, we know that the flux integral is trivial. Thus, when the bottom of the rod is a position s below the $y = 0$ point,

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \vec{B} \cdot \vec{A} = Ba(a - s)$$

Use Faraday's law to write (noting that $v = \frac{ds}{dt} > 0$ since s increases),

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = Bav$$

And thus, the current through the rod is

$$I = \frac{\mathcal{E}}{R} = \frac{Bav}{R} = \frac{Bav\pi d^2}{16\rho a} = \frac{Bv\pi d^2}{16\rho}$$

c)

The magnetic force on the two sides will cancel each other out as their currents are opposite. Thus, the force is due to the top portion. Writing $\vec{L} = a\hat{x}$ from the reasoning in part a, we get that

$$\vec{F} = I\vec{L} \times \vec{B} = I(a\hat{x}) \times (-B\hat{z}) = \frac{aB^2 v\pi d^2}{16\rho}\hat{y}$$

The terminal velocity occurs when this force balances the gravitiational force. Thus,

$$\begin{aligned} \frac{aB^2 v\pi d^2}{16\rho} &= mg \\ v &= \frac{16mg\rho}{\pi a B^2 d^2} \end{aligned}$$

Problem 7

a)

Imagine connecting just the three inductors connected to a battery. Call the currents through each of the inductors I_1, I_2, and I_3, respectively. Kirchoff's rules give

$$\begin{aligned} V - L_1\frac{dI_1}{dt} - L_2\frac{dI_2}{dt} &= 0 \\ V - L_1\frac{dI_1}{dt} - L_3\frac{dI_3}{dt} &= 0 \\ I_1 &= I_2 + I_3 \end{aligned}$$

We want to rewrite this as an equation of the form

$$V - L_{eff}\frac{dI_1}{dt} = 0$$

Now, from the last expression in the Kirchoff rules, we know $I_2 = I_1 - I_3$. By the second, we can get an expression for I_3 to get

$$\frac{dI_2}{dt} = \frac{dI_1}{dt} - \frac{dI_3}{dt} = \frac{dI_1}{dt} - \frac{L_2}{L_3}\frac{dI_2}{dt}$$

This implies

$$\frac{dI_2}{dt} = \frac{dI_1}{dt}\left(1 + \frac{L_2}{L_3}\right)^{-1}$$

Plug this into the first expression in the Kirchoff rules to get

$$\begin{aligned} V - L_1\frac{dI_1}{dt} - L_2\frac{dI_1}{dt}\left(1 + \frac{L_2}{L_3}\right)^{-1} &= 0 \\ V - \left(L_1 + L_2\left(1 + \frac{L_2}{L_3}\right)^{-1}\right)\frac{dI_1}{dt} &= 0 \end{aligned}$$

So, the equivalent inductance is

$$L_{eff} = L_1 + L_2\left(1 + \frac{L_2}{L_3}\right)^{-1}$$

Note that this implies inductors add like resistors.

b)

Now use Kirchoff's loop rule on the circuit given (simplifying the inductors to its equivalent circuit) to write

$$V_0 - IR - L_{eff}\frac{dI}{dt} = 0$$

c)

The time constant here is L_{eff}/R. Initially there is no current, so the form of the growth must be like (looking at the general solution to the differential equation)

$$IR = V_{max}(1 - e^{-Rt/L_{eff}})$$

Thus, we can solve

$$\begin{aligned} 0.9V_{max} &= V_{max}(1 - e^{-Rt/L_{eff}}) \\ e^{-Rt/L_{eff}} &= 0.1 \\ -\frac{R}{L_{eff}}t &= \ln(0.1) \\ t &= \frac{L_{eff}}{R}\ln(10) \end{aligned}$$