Phys 7B Lec 03 Final Exam

TOTAL POINTS

123 / 140

QUESTION 1

1 Problem 1 14 / 20

✓ + **5 pts** **Part A) Correct**

✓ + **5 pts** **Part B) Correct**

+ **5 pts** Part C) Correct
+ **5 pts** Part D) Correct
+ **1 pts** A) Q = mcΔT
+ **1 pts** A) Sum of Q = 0 / Net heat transferred = 0
+ **1 pts** A) Correct final temperature calculation
+ **2 pts** A) Sufficient Explanation given calculation of temperature
+ **2 pts** B) Equation for entropy dS= dQ/T
+ **2 pts** B) Equation for net entropy change of the system (iron with water)
+ **1 pts** B)Correct calculation of entropy (If a previous quantity was calculated wrong it will be taken into account)
+ **3 pts** C) Thorough explanation involving the second law of thermodynamics (Points may be awarded if ΔS was calculated to be < 0 but explained)
+ **2 pts** C) Correct/Consistent calculation of ΔS > 0 (SHOW EXPLICITLY)

✓ + **1 pts** **D) R_new = 1.5 R at melting point condition (or 2.5 R)**

✓ + **1 pts** **D) ΔR equation**

✓ + **2 pts** **D) Correct symbolic answer (Either using ΔR = 0.5 or ΔR = 1.5)**

+ **1 pts** D) Correct/Consistent numerical value for T (or consistent with derived equation)
+ **0 pts** Click here to replace this description.

QUESTION 2

2 Problem 2 20 / 20

+ **2 pts** Part (a): Writing down the correct formula for efficiency
+ **1 pts** Part (a): Partially writing down the correct formula for efficiency, but not making it clear what Q is, (or some other incomplete expression)
+ **1 pts** Part (a): In calculating heat along 1->2, set heat equal to net work (or other partial credit for heat output 1->2 calculation). Note, if you just calculated the net work, this and the following 3 rubric items do not apply (you can either calculate net work, or heat output)
+ **1 pts** Part (a): Correct calculation of heat output from 1 -> 2
+ **1 pts** Part (a): In calculating heat along 4->1, used Q = C_P delta T (or other partial credit for heat output 4->1 calculation).
+ **1 pts** Part (a): Correctly calculating heat output 4 -> 1.
+ **1 pts** Part (a): Correct calculation of work along 1->2. Again, disregard this and the following 3 rubric items if you got credit/attempted the rubric items above
+ **1 pts** Part (a): Correct calculation of work along 2->3
+ **1 pts** Part (a): Correct calculation of work along 3->4
+ **1 pts** Part (a): Correct calculation of work along 4->1
+ **1 pts** Part (a): In calculating heat input from 2->3, used Q = C_p delta T (or some other partial credit)
+ **1 pts** Part (a): Correctly calculated the heat input from 2->3
+ **1 pts** Part (a): In calculating the heat input in 3->4, set the heat equal to work done (or other partial credit for 3->4 heat calculation)
+ **1 pts** Part (a): Correctly calculated the heat input from 3 -> 4
+ **1 pts** Part (a): Partial credit for attempting to write

quantities in terms of common pressure and volume (usually P_1 V_1). The calculation is either wrong or incomplete

- **- 2 pts** Part (a): Incorrect substitution of temperature in terms of P_1 and V_1 (gives correct final answer, but units of heat are off)
- **- 2 pts** Part (a): Plugging in all heats as opposed to just heats put in to the system into the efficiency formula, or switching q_in and q_out, or otherwise incorrect substitution into efficiency equation (you may have calculated all the heats correctly, but then only chose to plug one of them in)
- **+ 3 pts** Part (a): Writing net work/heat in terms of common reference pressures and volumes (usually P_1 V_1) that cancel out
- **+ 1 pts** Part (a): Correct numerical value
- **+ 3 pts** Part (b): Correct Carnot efficiency
- **+ 1 pts** Part (b): Partial credit for 1 - T_L/T_H
- **+ 2 pts** Part (b): Partial credit if 1 - T_L/T_H given and the wrong numerical value is obtained because temperatures are calculated incorrectly in part (a)
- **+ 1 pts** Part (b): Partial credit for commenting on the ordering between the two values, but not offering any explanation as to why this ordering is the case, or explanation not a good one
- **+ 2 pts** Part (b): Attempted explanation but not quite there
- **+ 3 pts** Part (b): Correct explanation (something along the lines of Carnot engine is the most efficient engine, or that since the Ericsson cycle does not have all heat exchange at fixed temperatures, it will not saturate the Carnot efficiency)
- **+ 0 pts** no points acquired
- ✓ **+ 20 pts Full credit**

QUESTION 3

3 Problem 3 **20 / 20**

- ✓ **+ 20 pts Full credit**
- **+ 0 pts** Null: zero credit
- **+ 14 pts** (a) all correct
- **+ 3 pts** (a) recognize spherical symmetry to say E is radial
- **+ 2 pts** (a) recognize spherical symmetry to say E is a constant at a given radius
- **+ 2 pts** (a, partial) almost there with the symmetry
- **+ 2 pts** (a) correctly write down Gauss' Law
- **+ 2 pts** (a) correct setup of Gauss' Law in this problem and simplification given the symmetries
- **+ 2 pts** (a) correct integral setup for total charge enclosed by Gaussian sphere
- **+ 2 pts** (a) correct charge enclosed by a sphere of radius r
- **+ 1 pts** (a, partial) partial credit for charge enclosed
- **+ 1 pts** (a) correct E magnitude
- **+ 6 pts** (b) all correct
- **+ 2 pts** (b) recognize that the total charge of the electron cloud must be -e for the electron's charge
- **+ 2 pts** (b) setup the integral to find the total charge of the cloud correctly
- **+ 2 pts** (b) correct answer for the constant A
- **+ 1 pts** (b, partial) partial credit for the integral setup and/or arithmetic in solving for A

QUESTION 4

4 Problem 4 **14 / 20**

- **+ 20 pts** Response completely correct
- **+ 12 pts** Part (a): Response totally correct (shortcut for grader)
- ✓ **+ 2 pts Part (a): Identifying the magnitude of the acceleration from the electric field**
- ✓ **+ 1 pts Part (a): Identifying the direction of the electric force (reporting a positive deflection)**
- ✓ **+ 1 pts Part (a): Correctly using the horizontal velocity to find the time spent moving through the system**
- **+ 2 pts** Part (a): breaking up the trajectory into two regions, within and outside the field, and recognizing a qualitatively different contribution to to the deflection from both
- ✓ **+ 2 pts Part (a): Correct kinematic approach for region 1 (within the field)**
- **+ 1 pts** Part (a): Partial credit for region 1
- **+ 1 pts** Part (a): Correct delta y from region 1
- **+ 2 pts** Part (a): Correct kinematic approach for

region 2 (outside the field)

+ **1 pts** Part (a): Partial credit for region 2
+ **1 pts** Part (a): Correct delta y from region 2
+ **1 pts** Part (b): Realizing that electric and magnetic forces cancel out
+ **1 pts** Part (b): Writing down the electric force
+ **1 pts** Part (b): Writing down the magnetic force
+ **1 pts** Part (b): Finding the right velocity such that they cancel

✓ + **4 pts** **Part (b): Response totally correct (shortcut for grader)**

+ **2 pts** Part (c): Description of phase 1
+ **1 pts** Part (c): Partial credit for phase 1
+ **2 pts** Part (c): Description of phase 2
+ **1 pts** Part (c): Partial credit for phase 2

✓ + **4 pts** **Part(c): Response totally correct (shortcut for grader)**

+ **0 pts** No points accrued

QUESTION 5

5 Problem 5 16 / 20

+ **1 pts** (a) Correct charge on ring: $$dq = \sigma 2 \pi r dr = (2Q/R^2) r dr$$
+ **2 pts** (a) Correct expression for the current on the ring: $$dI = f dq = \frac{\omega Q}{\pi R^2} r dr$$

✓ + **1 pts** **(a) Deduction for messed up current**

✓ + **1 pts** **(a) Noting $$d\mu = dI A$$ where $$A$$ is area *enclosed***

+ **1 pts** (a) Final answer $$d\mu = \frac{\omega Q r^3}{R^2} dr$$

✓ + **3 pts** **(b) To find total magnetic moment, need to integrate previous answer from $$r = 0$$ to $$r = R$$.**

✓ + **2 pts** **(b) Obtaining correct answer of $$\mu = \frac{\omega QR^2}{4}$$ (full credit if procedure is right, unless answer makes no sense)**

✓ + **1 pts** **(c) Recognizing need for Biot-Savart**

✓ + **1 pts** **(c) Using $$dl = r d\theta$$ (line element of current), $$r_{sep} = \sqrt{r^2+x^2}$$ (separation vector magnitude)**

✓ + **1 pts** **(c) Recognizing only the $$x$$-component of $$B$$ survives and multiplying overall result by $$\frac{r}{\sqrt{r^2+x^2}}$$**

✓ + **2 pts** **(c) Integrate over $$\theta$$ to get a factor of $$2\pi$$ and plug in the current from part (a) to get $$dB_x = \frac{\mu_0 \omega Q r^3 dr}{2\pi R^2 (r^2+x^2)^{3/2}}$$ (full credit if process is correct)**

✓ + **2 pts** **(c) For $$x\gg R$$, drop terms quadratic in $$r/x$$ to get $$dB_x \approx \frac{\mu_0 \omega Q r^3 dr}{2\pi R^2 x^3}$$**

- **0.5 pts** (c) Very slight errors
- **1.5 pts** (c) Slight errors

+ **2 pts** (c) Questionable work with some semblance of correctness

✓ + **2 pts** **(d) Integrate previous answer from $$0$$ to $$R$$.**

✓ + **1 pts** **(d) Obtain final answer $$B_x = \frac{\mu_0 \mu}{2\pi x^3}$$ after simplifying in terms of magnetic moment (full credit if procedure correct, unless answer makes no sense)**

- **0.5 pts** (d) Not simplifying in terms of $$\mu$$

✓ - **1 pts** **Not indicating that *all* quantities in this problem point in the $$x$$-direction**

+ **0 pts** No points awarded

QUESTION 6

6 Problem 6 20 / 20

✓ + **4 pts** **Part (a): All correct**

✓ + **6 pts** **Part (b): All correct**

✓ + **5 pts** **Part (c): All correct**

✓ + **5 pts** **Part (d): All correct**

+ **2 pts** Part (a): Lenz's law
+ **2 pts** Part (a): Correct direction
+ **3 pts** Part (b): Biot-Sawart Law
+ **2 pts** Part (b): Correct result
+ **1 pts** Part (b): Correct direction
+ **2 pts** Part (c): Correct flux
+ **2 pts** Part (c): Correct emf
+ **1 pts** Part (c): Correct direction
+ **2 pts** Part (d): Know Kirchoff's Law
+ **3 pts** Part (d): Correct equation
+ **0 pts** No point

QUESTION 7

7 Problem 7 **19 / 20**

- \+ **10 pts** A) Correct
- ✓ + **5 pts** **B) Correct**
- ✓ + **5 pts** **C) Correct**
- ✓ + **2 pts** **A) Ampère's Law (Justification + Choice of Loop)**
- ✓ + **1 pts** **A) Constant Magnitude of B at constant radii**
- \+ **1 pts** A) Correct Direction of B Field
- ✓ + **2 pts** **A) Correct B Field r < a**
- ✓ + **2 pts** **A) Correct B Field a < r < b**
- ✓ + **2 pts** **A) Correct B field b < r**
- \+ **2 pts** B) Correct equation for energy stored in a spatially dependent magnetic field
- \+ **1 pts** B) Using correct choice of B field from A)
- \+ **2 pts** B) Correct calculation of energy w.r.t B field found and correct equation
- \+ **2 pts** C) Equation for relationship between energy and inductance
- \+ **1 pts** C) Correct energy or energy from part b
- \+ **2 pts** C) Correct calculation w.r.t energy found
- \+ **0 pts** Click here to replace this description.

PHYSICS 7B, Lecture 3 – Spring 2018
Final Exam, C. Bordel
Tuesday May 8th, 8-11 am

- Student name:
- Discussion section #:
- Student ID #:
- Name of your GSI:

Make sure you show all your work and justify your answers in order to get full credit!

Math Information Sheet

sin2x = 2 sinx cosx $\qquad \int \frac{dx}{x^2} = -\frac{1}{x} \qquad \int x \exp(-\frac{x}{a})dx = -a \exp(-\frac{x}{a})(a+x)$

cos2x = 2cos²x -1 $\qquad \int \frac{dx}{x} = Ln\, x \qquad \int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{(x^2+a^2)}}$

$\frac{\partial(\tan\theta)}{\partial\theta} = \frac{1}{\cos^2\theta} \qquad \vec{F} = -\vec{\nabla}U \qquad (1+x)^\alpha \sim 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 \text{ when } x \to 0$

* Cylindrical coordinate system

$$\overrightarrow{dr} = dr\,\overrightarrow{u_r} + r\,d\theta\,\overrightarrow{u_\theta} + dz\,\overrightarrow{u_z} \qquad \vec{\nabla}f = \frac{\partial f}{\partial r}\overrightarrow{u_r} + \frac{1}{r}\frac{\partial f}{\partial\theta}\overrightarrow{u_\theta} + \frac{\partial f}{\partial z}\overrightarrow{u_z}$$

* Spherical coordinate system

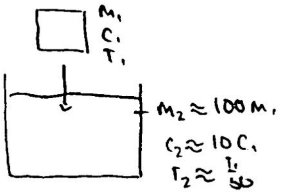
$$\overrightarrow{dr} = dr\,\overrightarrow{u_r} + r\,d\theta\,\overrightarrow{u_\theta} + r\sin\theta\,d\varphi\,\overrightarrow{u_\varphi} \qquad \vec{\nabla}f = \frac{\partial f}{\partial r}\overrightarrow{u_r} + \frac{1}{r}\frac{\partial f}{\partial\theta}\overrightarrow{u_\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\varphi}\overrightarrow{u_\varphi}$$

Problem 1 - Calorimetry and Thermometry (20 pts)

Part 1:

After being forged, a hot iron horseshoe of mass m_1, specific heat C_1 and initial temperature T_1 is dropped into a large bucket of cold water of mass m_2 ($m_2 \approx 100\, m_1$), specific heat C_2 ($C_2 \approx 10\, C_1$) and initial temperature T_2 ($T_2 \approx T_1/50$). You may assume that water and horseshoe do not experience any significant heat exchange with their surroundings.

a- Explain why no phase transition occurs.



$Q_L = $ heat lost by horseshoe $= m_1 C_1 (T_f - T_1)$

$Q_G = $ heat gained by water $= m_2 C_2 (T_f - T_2) = (100 m_1)(10 C_1)(T_f - \frac{T_1}{50})$

No significant heat exchange with surroundings:

$$Q_G = -Q_L$$

$$1{,}000\, m_1 C_1 (T_f - \frac{T_1}{50}) = m_1 C_1 (T_1 - T_f)$$

$$1{,}000\, T_f - \frac{1{,}000\, T_1}{50} = T_1 - T_f$$

$$1{,}001\, T_f = 21 T_1$$

$$T_f = \frac{21 T_1}{1{,}000}$$

For there to have been a phase transition, $T_f > 373$ K

$$373 = \frac{21 T_1}{1{,}000} \qquad T_1 = \frac{373 (1{,}000)}{21} \sim \frac{400}{20}(1{,}000) \sim 20{,}000 \text{ K}$$

The temperature of the horseshoe would have had to be on the order of 10^4, at which it would likely not be solid.

b- Determine the change in entropy of the system from the moment the horseshoe gets dropped into the water to a few hours later.

$$\Delta S = \frac{Q}{T} = \frac{-m_1 C_1 (T_f - T_1)}{T_1} = \frac{-m_1 C_1 (\frac{21 T_1}{1{,}000} - T_1)}{T_1} = \frac{-m_1 C_1 (\frac{21 T_1}{1{,}000} - \frac{1{,}000 T_1}{1{,}000})}{T_1} = \frac{m_1 C_1 (\frac{1{,}000 - 21}{1{,}000} T_1)}{T_1}$$

$$= \frac{979}{1{,}000} m_1 C_1$$

$$\Delta S_H = \int_A^B \frac{m C_1\, dT}{T} = m C_1 \ln\left(\frac{T_F}{T_1}\right)$$

$$T_f / T_1 = \frac{\frac{21 T_1}{1{,}000}}{T_1} = \frac{21}{1{,}000} \sim \frac{1}{50}$$

$$\Delta S_W = \int_A^B \frac{m_2 10 C_1\, dT}{T} = m_2 10 C_1 \ln\left(\frac{T_F}{T_2}\right) = m_2 (10 C_1) \ln\left(\frac{50 T_F}{T_1}\right)$$

$$\Delta S_{sys} = \Delta S_H + \Delta S_W = m_1 C_1 \ln\left(\frac{T_F}{T_1}\right) + m_2 (10 C_1) \ln\left(\frac{50 T_F}{T_1}\right)$$

$$= m_1 C_1 \ln(T_F/T_1) + m_2 (10 C_1)\left[\ln 50 + \ln(T_F/T_1)\right]$$

$$\approx m_1 C_1 \ln\left(\frac{1}{50}\right) + m_2 (10 C_1)\underbrace{\left[\ln 50 + \ln\left(\frac{1}{50}\right)\right]}_{0}$$

$$\boxed{\approx -m_1 C_1 \ln 50 \approx -4 m_1 C_1}$$

c- Explain the sign of your result, given that $Ln50 \approx 4$.

The change in entropy is negative, which means "disorder" is decreasing. This is because the order of the water/horseshoe system decreased as a result of cooling. In exchange, the entropy of the surroundings increased. This is because

$$\Delta S_{univ} = \Delta S_{sys} + \Delta S_{sur} \geq 0.$$

Part 2:

A resistance thermometer, made of a platinum wire of constant cross-sectional area, is used to determine the melting point of indium. The resistance of the platinum wire is R_0 at room temperature (T_0) and increases by a factor of 1.5 as indium starts to melt (T_m). You may assume that the change in length is negligible and that the temperature coefficient of resistivity α is constant in the temperature range $[T_0, T_m]$.

d- Determine symbolically the melting point of indium, then find an approximate numerical value (in °C or K) for T_m, given that the temperature coefficient of resistivity is on the order of 4×10^{-3} /K for platinum at room temperature.

$$\rho(T) = \rho_0(1 + \alpha(T - T_0)) \qquad R_0 = \frac{\rho_0 L}{A} \qquad \rho_0 = \frac{AR_0}{L}$$

$$\rho(T) = \frac{AR_0}{L}(1 + \alpha(T - T_0))$$

$$\rho(T_0) = \frac{AR_0}{L}(1 + \alpha(T_0 - T_0)) = \frac{AR_0}{L} = \rho_0$$

$$\rho(T_m) = \frac{AR_0}{L}(1 + \alpha(T_m - T_0)) = \frac{3AR_0}{2L} = \frac{3\rho_0}{2}$$

$$= \rho_0(1 + \alpha(T_m - T_0)) = \frac{3\rho_0}{2}$$

$$1 + \alpha(T_m - T_0) = 3/2$$

$$\alpha(T_m - T_0) = 3/2 - 1 = 1/2$$

$$T_m - T_0 = \frac{1}{2\alpha}$$

$$\boxed{T_m = T_0 + \frac{1}{2\alpha}}$$

$$T_m = 300\text{ K} + \frac{1}{8 \cdot 10^{-3}} = 300 + 8{,}000\text{ K} = \boxed{8{,}300\text{ K}}$$

Problem 2 – Thermodynamic cycle (20 pts)

Largely forgotten over the course of the 20th century, the Ericsson engine is regaining some interest due the development of new technologies. *n* moles of a monatomic ideal gas undergo an Ericsson cycle, which is described as follows:

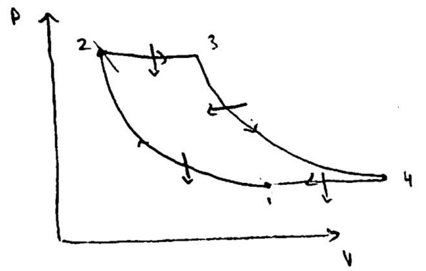
1→2: isothermal compression from pressure P_1 and volume V_1 to pressure $P_2=3\,P_1$

2→3: isobaric expansion with a twofold volume increase

3→4: isothermal expansion

4→1: isobaric compression

a- Determine the efficiency of the engine, symbolically and numerically, assuming $Ln3 \approx 1$.



$1:\ P_1,\ V_1$

$2:\ 3P_1,\ \frac{V_1}{3}$

$3:\ 3P_1,\ \frac{2V_1}{3}$

$4:\ P_1,\ 2V_1$

$P_1V_1 = P_2V_2$

$P_1V_1 = 3P_1V_2$

$V_2 = V_1/3$

$3P_1 \cdot \frac{2V_1}{3} = P_4V_4 \qquad P_4 = P_1$

$3\cancel{P_1} \cdot \frac{2V_1}{3} = \cancel{P_1}V_4$

$2V_1 = V_4$

$$e = \frac{W_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

$1 \rightarrow 2:\quad \Delta E_{int} = Q - W = 0$

$$Q = W = \int_1^2 P\,dV = nRT \int_1^2 \frac{1}{V}\,dV = nRT_1 \ln\left(\frac{V_2}{V_1}\right) = nRT_1 \ln\left(\frac{V_1/3}{V_1}\right) = nRT_1 \ln(1/3) = -nRT_1 \ln 3 = -nR\left(\frac{P_1V_1}{nR}\right)\ln 3$$
$$= -P_1V_1 \ln 3$$

$2 \rightarrow 3:\quad Q = n C_P \Delta T = n C_P (T_3 - T_2)$

$$T_2 = \frac{3P_1(V_1/3)}{nR} = \frac{P_1V_1}{nR} \qquad T_3 = \frac{3P_1\left(\frac{2V_1}{3}\right)}{nR} = \frac{2P_1V_1}{nR}$$

$$Q = nC_P\left(\frac{P_1V_1}{nR}\right) = \frac{C_P(P_1V_1)}{R} = \frac{\frac{5R}{2}(P_1V_1)}{R} = \frac{5P_1V_1}{2}$$

$3 \rightarrow 4:\quad \Delta E_{int} = Q - W = 0$

$$Q = W = \int_3^4 P\,dV = nRT_3 \ln\left(\frac{V_4}{V_3}\right) = nR\left(\frac{2P_1V_1}{nR}\right)\ln\left(\frac{2V_1}{2V_1/3}\right) = 2P_1V_1 \ln 3$$

$4 \rightarrow 1:\quad Q = n C_P \Delta T = n C_P (T_1 - T_4)$

$$T_4 = \frac{2P_1V_1}{nR} \qquad T_1 = \frac{P_1V_1}{nR}$$

$$Q = nC_P\left(-\frac{P_1V_1}{nR}\right) = -\frac{C_P(P_1V_1)}{R} = -\frac{5P_1V_1}{2}$$

$$Q_{in} = \frac{5P_1V_1}{2} + 2P_1V_1 \ln 3$$

$$Q_{out} = \frac{5P_1V_1}{2} + P_1V_1 \ln 3$$

$$e = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{P_1V_1\left(\frac{5}{2} + \ln 3\right)}{P_1V_1\left(\frac{5}{2} + 2\ln 3\right)} = 1 - \frac{\left(\frac{5}{2}+1\right)}{\left(\frac{5}{2}+2\right)} = 1 - \frac{7}{9} = \boxed{\frac{2}{9}}$$

b- Calculate numerically the efficiency of the Carnot engine operating between the same two temperatures. Explain the ranking between the two values.

$$e = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{nRT_L \ln\left(\frac{V_C}{V_D}\right)}{nRT_H \ln\left(\frac{V_B}{V_A}\right)} = 1 - \frac{T_L}{T_H} = \frac{T_H - T_L}{T_H} \qquad \ln\frac{V_C}{V_D} = \ln\frac{V_B}{V_A}$$

$$T_H = T_3 = \frac{2P_1V_1}{nR}$$

$$T_L = T_1 = \frac{P_1V_1}{nR}$$

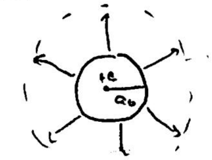
$$e = \frac{\frac{2P_1V_1}{nR} - \frac{P_1V_1}{nR}}{\frac{2P_1V_1}{nR}} = \frac{\frac{P_1V_1}{nR}}{\frac{2P_1V_1}{nR}} = \boxed{\frac{1}{2}}$$

The Carnot engine is the most efficient engine between any two temperatures because it utilizes adiabats between its isotherms. This is why $e_{carnot} = \frac{1}{2} > e_{ericsson} = \frac{2}{9}$.

Problem 3 – Hydrogen atom (20 pts)

Neutral hydrogen can be modeled as a positive point charge +*e*, located at *r=0*, surrounded by a distribution of negative charge with volume density $\rho_n(r) = -A/r^2 \exp(-2r/a_0)$ beyond the radial distance a_0 , called the Bohr radius. *r* is the radial distance measured from the nucleus, *A* is a positive constant to be determined, and *exp* is the exponential function.

a- Determine the electric field, in magnitude and direction, at a distance $r>a_0$ from the nucleus.



$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$

$\vec{E}\cdot\oint d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$

$\vec{E}(r)\cdot 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0}$

$Q_{enc} = e - \int \frac{A}{r^2} e^{-\frac{2r}{a_0}} dr$

$= e - A\int_{a_0}^{r} \frac{1}{r^2} e^{-\frac{2r}{a_0}} dr$

$\int r e^{-\frac{r}{a}} dr = -a e^{-\frac{r}{a}}(a+r)$

$$\boxed{\vec{E}(r) = \frac{e - A\int_{a_0}^{r}\frac{1}{r^2}e^{-\frac{2r}{a_0}}dr}{4\pi r^2 \varepsilon_0}}$$ radially

$\frac{d}{dr}\left[-a e^{-\frac{r}{a}}(a+r)\right]$

$-a\left[-\frac{1}{a}e^{-\frac{r}{a}}(a+r) + e^{-\frac{r}{a}}\right]$

$\left[e^{-\frac{r}{a}}(a+r) - a e^{-\frac{r}{a}}\right]$

$\cancel{a e^{-\frac{r}{a}}} + r e^{-\frac{r}{a}} - \cancel{a e^{-\frac{r}{a}}}$

$Q_{enc} = e + \int \rho\, dV$ $\quad V = \frac{4}{3}\pi r^3$

$dV = 4\pi r^2 dr$

$-a\left[e^{\frac{r}{a}}(a + \frac{1}{r^2})\right]$

$= e - \int \frac{A}{r^2} e^{-\frac{2r}{a_0}} 4\pi r^2 dr = e - A4\pi\int_{a_0}^{r} e^{-\frac{2r}{a_0}}dr$

$\vec{E}(r) = \frac{e - 4A\pi\int_{a_0}^{r} e^{-\frac{2r}{a_0}}dr}{4\pi r^2}$

$= e - 4A\pi\left[-\frac{a_0}{2}e^{-\frac{2r}{a_0}}\right]_{a_0}^{r} = e - 4A\pi\left[-\frac{a_0}{2}e^{-\frac{2r}{a_0}} - (-\frac{a_0}{2}e^{-2})\right]$

$= e - 4A\pi\left[-\frac{a_0}{2}e^{-\frac{2r}{a_0}} + \frac{a_0}{2}e^{-2}\right]$ euler's #

$$\boxed{\vec{E}(r) = \frac{\overset{\text{charge}}{e} - 4A\pi\left[\frac{a_0}{2}e^{-2} - \frac{a_0}{2}e^{-\frac{2r}{a_0}}\right]}{4\pi r^2}}$$

b- Determine the constant *A*.

$0 = e - A\int_{a_0}^{\infty}\frac{1}{r^2}e^{-\frac{2r}{a_0}}dr$

$A\int_{a_0}^{\infty}\frac{1}{r^2}e^{-\frac{2r}{a_0}}dr = e$

$$\boxed{A = \frac{e}{\int_{a_0}^{\infty}\frac{1}{r^2}e^{-\frac{2r}{a_0}}dr}}$$

$A = \frac{e}{\int_{a_0}^{\infty}4\pi e^{-\frac{2r}{a_0}}dr}$

If hydrogen is neutral, then the total charge over all space is 0.

$0 = e + \int_{a_0}^{\infty}\rho_n(r)dV = e - \int_{a_0}^{\infty}\rho(r)4\pi r^2 dr = e - 4A\pi\int_{a_0}^{\infty}e^{-\frac{2r}{a_0}}dr$

$= e - 4A\pi\left[-\frac{a_0}{2}e^{-\frac{2r}{a_0}}\right]_{a_0}^{\infty} = e - 4A\pi\left[0 - (-\frac{a_0}{2}e^{-\frac{2a_0}{a_0}})\right]$

$= e - 4\pi(\frac{a_0}{2}e^{-2})A = 0$

$e^{-\infty} = \frac{1}{e^{\infty}} = 0$

$$\boxed{A = \frac{\overset{\text{charge}}{e}}{\frac{4\pi a_0}{2e^{2}}}}$$ euler's #

Problem 4 - Charge trajectory (20 pts)

The apparatus shown in figure 1 is set up to reproduce Thomson's experiment. In a highly evacuated glass tube, a beam of electrons (mass m and charge $-e$), all moving in the same direction with speed v_0, passes between two parallel plates of length d (also parallel to the initial velocity of the electrons) and strikes a screen at a distance L from the end of the plates and perpendicular to them. Δy is the distance between the point where the beam strikes the screen when there is no electric field between the plates and the point where the beam strikes the screen when a uniform electric field of magnitude E_0 is established between the plates. You may assume that the electric field is zero outside the region between the plates and that v_0 is large enough that all the particles systematically hit the screen.

$-eE_0 = ma_y$

$a_y = \frac{-eE_0}{m}$

$\Delta x = v_0 t + \cancel{\frac{1}{2}a_x t^2} \to 0$

$(d + L) = v_0 t$

$t = \frac{d+L}{v_0}$

$\Delta y = \cancel{v_0 t}^{\,0} + \frac{1}{2} a_y t^2$

$\boxed{\Delta y = \frac{eE_0}{2m}\left(\frac{d+L}{v_0}\right)^2}$

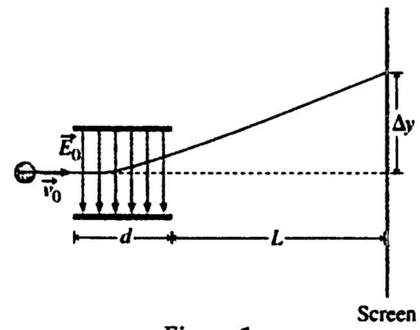


Figure 1

a- Determine the distance Δy.

Now the entire apparatus is placed inside a region of magnetic field of magnitude B_0. The magnetic field is perpendicular to the electric field and directed straight into the plane of the figure, as shown in figure 2. The value of B_0 is adjusted so that no deflection of the electron beam is observed on the screen.

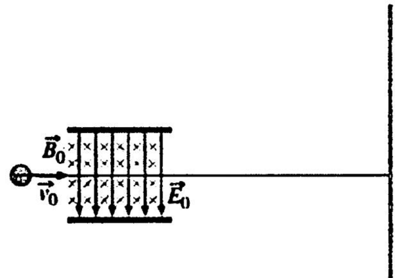


Figure 2

Screen

b- Determine the speed v_0 of the electrons.

$\vec{F}_E = q E_0 = -e E_0$

$\vec{F}_B = q v B = -e v_0 B_0$

$F_B + F_E = -eE_0 + -e v_0 B_0 = \overset{0 \text{ since no deflection}}{\cancel{m a_y}}$

$-e E_0 = e v_0 B_0$

$v_0 = -\frac{E_0}{B_0}$

$\boxed{|v_0| = \frac{E_0}{B_0}}$

Now suppose you carry out a second experiment with a different beam that contains two types of particles. Both types have the same mass m, but one has charge q and the other has charge $2q$. The beam is filtered, such that both types of particle have the same speed. As in the previous experiment, initially only the electric field is imposed; then, in the second phase of the experiment, the magnetic field is tuned in order to exactly cancel the effect of the electric field. Assume that both types of particle reach the screen in each case.

c- Explain qualitatively what would be observed on the screen in each phase of this experiment.

$\Delta y = \frac{q E_0}{2m} \left(\frac{d+L}{v_0}\right)^2$

Phase 1: There would be two areas of detection, since all the particles of charge q would hit at a certain Δy, and all the particles of charge $2q$ would hit at a different Δy, approximately double the Δy of the particles of charge q

Phase 2: All the particles would hit the screen starting at the same time, since v_0 is independent of charge. The particles would also not be deflected, so they would all hit the same spot.

Problem 5 – Spinning wheel (20 pts)

A non-conducting circular disk of negligible thickness and radius R carries a uniformly distributed electric charge Q. The disk spins about its symmetry axis with angular speed ω, as shown in Fig.3.

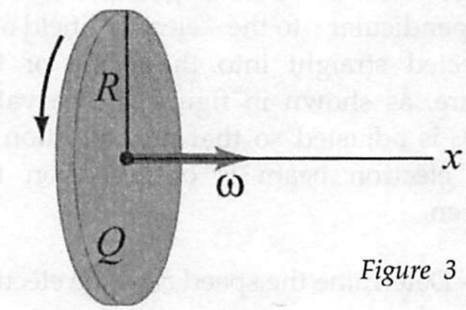
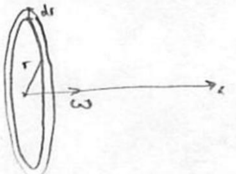


Figure 3

a- Determine the infinitesimal magnetic dipole moment of a thin circular ring of inner radius r and width dr.



$\mu = NI\vec{A}$

$= Q\omega$

$\lambda = \frac{Q}{2\pi r}$ $\quad L = r\theta \quad Q = \lambda L = \lambda r\theta$

$dQ = \lambda dL = \lambda r d\theta$

$\mu = \int_0^{2\pi} \lambda r d\theta \omega = \omega \lambda r (2\pi) = \omega \frac{Q}{2\pi r} \cdot r = \frac{\omega Q}{2\pi}$

$I = Q\omega$

$A = \pi r^2 \quad dA = 2\pi r dr$

$\mu = IA$

$d\mu = I dA = \boxed{Q\omega 2\pi r dr}$

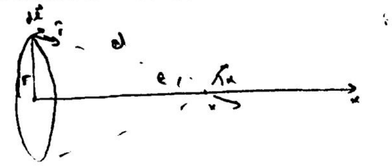
b- Determine the magnetic dipole moment of the entire spinning disk.

$\mu = \int_0^R \mu_{ring} = \int_0^R \frac{\omega Q}{2\pi} dr = \frac{\omega Q R}{2\pi}$

$\mu = \int d\mu = \int_0^R 2\pi Q\omega r dr = 2\pi Q\omega \int_0^R r dr = 2\pi Q\omega \frac{1}{2}R^2 = \boxed{\pi R^2 Q\omega}$

$= IA$ ✓

c- Determine, in terms of Q, ω and other given variables, the magnitude of the infinitesimal magnetic field created by the thin circular ring considered in part (a) at a distance $x>>R>r$.



$$d\vec{B} = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi d^2} \qquad \vec{B} = \vec{B}_x$$

angle α between $d\vec{l}$ & $\hat{r}$

$$d\vec{B}_x = d\vec{B}\cos\alpha = \frac{\mu_0 I dl}{4\pi d^2}\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}-\theta\right) = \frac{\mu_0 I dl}{4\pi d^2}\sin\theta$$

$$\vec{B}(x) = \int_{l=0}^{l=2\pi r} \frac{\mu_0 I\, dl}{4\pi d^2}\sin\theta = \frac{\mu_0 I \sin\theta\, 2\pi r}{4\pi d^2} \qquad \sin\theta = r/d \qquad d^2 = r^2 + x^2$$

$$\vec{B}(x) = \frac{\mu_0 I r^2}{2(r^2+x^2)^{3/2}} \rightarrow \text{Long-distance} \rightarrow \frac{\mu_0 r^2 I}{2x^3} = \frac{\mu_0 Q\omega r^2}{2x^3}$$

$$\boxed{\vec{B}(x) = \frac{\mu_0 Q\omega r^2}{2x^3}}$$

↳ acts as magnetic dipole

d- Determine the total magnetic field created at distance $x>>R$ as a function of the magnetic dipole moment.

$$\vec{B} = \int_0^R \frac{\mu_0 Q\omega r^2}{2x^3}\,dr = \frac{\mu_0 Q\omega}{2x^3}\int_0^R r^2\,dr = \frac{\mu_0 Q\omega}{2x^3}\left[\frac{1}{3}r^3\right]_0^R = \frac{\mu_0 Q\omega R^3}{6x^3} \text{ — whole disc}$$

$$\mu = \pi R^2 Q\omega$$

$$\vec{B} = R^2 Q\omega \cdot \frac{\mu_0 R}{6x^3} = \pi R^2 Q\omega \cdot \frac{\mu_0 R}{6\pi x^3}$$

$$\boxed{\vec{B} = \mu\left[\frac{\mu_0 R}{6\pi x^3}\right]}$$

Problem 6 – Double loop (20 pts)

A large loop, containing a battery that supplies a voltage V_0 and a variable resistor of resistance R, can be considered as a circle of radius r. A much smaller loop of surface area A, containing an uncharged capacitor of capacitance C and a resistor of resistance R_0, is placed at the center of the larger loop (Fig.4). You may ignore the self-inductance in the small loop. After being maintained at resistance R_0 at $t \leq 0$, the variable resistance is ramped up, starting at time $t=0$, from R_0 to $R(t) = R_0(1 + at)$, with $a>0$.

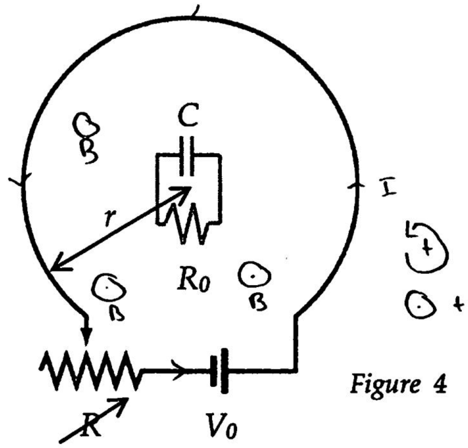


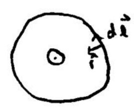
Figure 4

a- Using Lenz's law, predict the direction of the induced current generated in the small loop.

Assuming the current flows counterclockwise, it will decrease for $t \geq 0$ because $V_0 = IR \Rightarrow V_0 = IR_0(1+at)$. As resistance increases, the current will decrease. A decreasing counterclockwise current will create a decreasing B-field out of the page. Lenz's Law states that the induced current will oppose the change in flux (in this case, decreasing), so it will create a B-field out of the page. In order to do that, current must flow counterclockwise inside the small loop

b- Determine the direction and magnitude of the magnetic field created by the outer loop at its center.

$$d\vec{B} = \frac{\mu_0 I \, d\vec{l} \times \hat{r}}{4\pi r^2}$$



$$d\vec{B} = \frac{\mu_0 I \, dl}{4\pi r^2}$$

$$\vec{B} = \int d\vec{B} = \int_0^{2\pi r} \frac{\mu_0 I}{4\pi r^2} \, dl = \frac{\mu_0 I (2\pi r)}{4\pi r^2} = \frac{\mu_0 I}{2r}$$

$$V_0 = IR \Rightarrow V_0 = I(R_0(1+at))$$

$$I = \frac{V_0}{R_0(1+at)}$$

$$\boxed{\vec{B} = \frac{\mu_0 V_0}{2R_0(1+at)r}}$$

c- Calculate the emf induced in the small loop.

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$ ⇒ Assume $\vec{B}$ is uniform over much smaller loop.

$$\frac{d}{dt}(1+at)^{-1} = -1(1+at)^{-2} \cdot a$$

$$= \vec{B} \int d\vec{A} = \vec{B} A$$

$$= \frac{-a}{(1+at)^2}$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -A \cdot \frac{dB}{dt} = -A \frac{d}{dt}\left[\frac{\mu_0 V_0}{2R_0(1+at)r}\right] = \frac{-A\mu_0 V_0}{2R_0 r} \frac{d}{dt}\left[\frac{1}{1+at}\right]$$

$$\mathcal{E} = \frac{-A\mu_0 V_0}{2R_0 r}\left[\frac{-a}{(1+at)^2}\right] = \frac{A\mu_0 V_0 a}{2R_0 r(1+at)^2}$$

d- Establish the differential equation satisfied by the charge accumulated on the capacitor's plates as a function of time.

Capacitor Charging

$$\mathcal{E} - IR - \frac{Q}{C} = 0$$

$$\mathcal{E} = IR + \frac{Q}{C}$$

$I = +\frac{dQ}{dt}$ because of charging

$$= \frac{dQ}{dt}R + \frac{Q}{C}$$

$$\frac{\mathcal{E}}{R} = \frac{dQ}{dt} + \frac{Q}{RC}$$

$$\boxed{\frac{A\mu_0 V_0 a}{2R_0 R r(1+at)^2} = \frac{dQ}{dt} + \frac{Q}{RC}}$$

Problem 7 – Coaxial cable (20 pts)

The coaxial cable shown in Fig.5 is made of two coaxial cylindrical conductors: an inner solid cylinder of radius a and an outer cylindrical shell of radius b ($b > a$) and negligible thickness c ($c << b$). They both carry the same amount of current I, evenly distributed in the conductors, but in opposite directions. You may assume that both conductors are <u>very long</u> compared to their radius ($\ell >> b$).

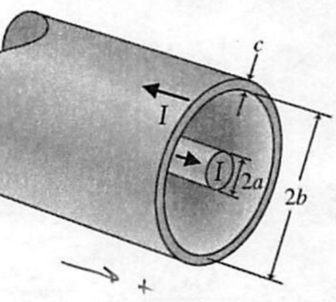


Figure 5

a- Determine the direction and calculate the magnitude of the magnetic field created at any radial distance r from the symmetry axis. *For clarity regarding the direction, assume that you look at the cable from the right-hand side of the figure (current coming toward you at the center and away from you in the outside conductor).*

$r < a$:

$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$ $\qquad I_{enc} = \frac{\pi r^2}{\pi a^2} I = \frac{r^2 I}{a^2}$

$\vec{B}\oint d\vec{\ell} = \mu_0 I_{enc}$

$B(2\pi r) = \mu_0 \frac{r^2 I}{a^2}$

$\boxed{\vec{B}(r) = \frac{\mu_0 I r}{2\pi a^2}}$

$a \leq r \leq b$

$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$ $\qquad I_{enc} = I$

$B\oint d\vec{\ell} = \mu_0 I_{enc} = \mu_0 I$

$B(2\pi r) = \mu_0 I$

$\boxed{B = \frac{\mu_0 I}{2\pi r}}$

$b \leq r \leq b+c$

$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$ $\qquad I_{enc} = -\left(\frac{\pi r^2 - \pi b^2}{\pi(b+c)^2 - \pi b^2}\right) I + I$

$B(2\pi r) = \mu_0 I_{enc}$

$\boxed{B = \frac{\mu_0 I_{enc}}{2\pi r} = \frac{\mu_0\left[I - \left(\frac{\pi r^2 - \pi b^2}{\pi(b+c)^2 - \pi b^2}\right) I\right]}{2\pi r}}$

$r \geq b+c$

$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$ $\qquad I_{enc} = 0$

$\boxed{\vec{B} = \vec{0}}$

b- Based on the energy density stored in a magnetic field, determine the total energy stored per unit length in the magnetic field created in the gap between the two coaxial cylinders.

$$u = \frac{B^2}{2\mu_0} \qquad B = \frac{\mu_0 I}{2\pi r}$$

$$E = \int u\, dV \qquad V = l(\pi r^2 - \pi a^2)$$

$$dV = l(2\pi r)\,dr$$

$$E = \int_a^b \frac{B^2}{2\mu_0} \cdot 2\pi r\, l\, dr = \frac{2\pi l}{2\mu_0}\int_a^b B^2 r\, dr = \frac{\pi l}{\mu_0}\int_a^b B^2 r\, dr$$

$$\frac{E}{l} = \frac{\pi}{\mu_0}\int_a^b B^2 r\, dr = \frac{\pi}{\mu_0}\int_a^b \frac{\mu_0^2 I^2}{4\pi^2 r^2} r\, dr = \frac{\mu_0^2 I^2 \pi}{\mu_0 4\pi^2}\int_a^b \frac{1}{r}\, dr = \frac{\mu_0 I^2}{4\pi}\left[\ln r\right]_a^b = \frac{\mu_0 I^2}{4\pi}\ln\left(\frac{b}{a}\right)$$

$$\boxed{\frac{E}{l} = \frac{\mu_0 I^2}{4\pi}\ln\left(\frac{b}{a}\right)}$$

c- Calculate the self-inductance per unit length of this coaxial cable.

$$\frac{1}{2}\frac{L}{l} I^2 = \frac{E}{l} = \frac{\mu_0 I^2}{4\pi}\ln\frac{b}{a}$$

$$\frac{L}{l} = \frac{E}{l}\cdot\frac{2}{I^2} = \frac{\mu_0 \cancel{I^2}}{\cancel{2}4\pi}\cdot\frac{\cancel{2}}{\cancel{I^2}}\ln\frac{b}{a} = \boxed{\frac{\mu_0}{2\pi}\ln\frac{b}{a}}$$