Phys 7B Lec 03 Midterm 2

TOTAL POINTS

83.5 / 100

QUESTION 1

1 Problem 1 15 / 20

+ **0 pts** Null: no credit
+ **20 pts** Full marks
+ **4 pts** FIRST method: correct starting equation for calculating electric potential
+ **5 pts** FIRST method: correct electric potential
+ **2.5 pts** FIRST method: partial credit for potential calculation
+ **3 pts** FIRST method: correctly applying the binomial approximation with x small
+ **2 pts** FIRST method: binomial approximation written down correctly
+ **2 pts** FIRST method: converting electric potential to potential energy appropriately
+ **4 pts** FIRST method: correct force from F = -dU/dx or E = -dV/dx OR explaining that U has the potential energy of a simple harmonic oscillator.
+ **4 pts** FIRST method: showing or explaining that it satisfies simple harmonic motion
+ **2 pts** FIRST method: correctly showing that it satisfies simply harmonic motion, but having an x^2 in the denominator of the frequency still.

✓ + **4 pts** **SECOND method: Coulomb's Law written down correctly**

✓ + **3 pts** **SECOND method: all components cancel except for the x component**

✓ + **2 pts** **SECOND method: correct trig term to get the x component of the force (or field)**

✓ + **3 pts** **SECOND method: correct force (or field) calculation aside from the trig component**

+ **3 pts** SECOND method: partial credit for the Coulomb's Law calculation of the field or force

✓ + **1 pts** **SECOND method: correct force**

+ **3 pts** SECOND method: correct approximation with x being small
+ **4 pts** SECOND method: showing or explaining how the calculated force means simple harmonic motion

✓ + **2 pts** **SECOND method: showing or explaining it's simple harmonic motion BUT still having an x term in the omega^2 coefficient**

QUESTION 2

2 Problem 2 16 / 20

+ **10 pts** (a) Full marks
+ **0 pts** (a) No marks

✓ + **1 pts** **(a) Mentioned using superposition**

✓ + **1 pts** **(a) Used Gauss' Law (or attempt at Coulomb's law solution)**

✓ + **2 pts** **(a) Correct simplification of flux integral (or correct distance if using Coulomb's law)**

✓ + **1 pts** **(a) Used a gaussian surface with spherical symmetry, and with varaible radius r (ie. Uniform E-field on the surface) (or correct integral bounds if using Coulomb's law)**

✓ + **1 pts** **(a) Correct enclosed charge (or correct dq if using Coulomb's law)**

✓ + **1 pts** **(a) Correct electric field magnitude on gaussian surface (given answer for Q_enc and simplification of flux integral or dq, distance, and integral bounds if using Coulomb's law)**

✓ + **1 pts** **(a) Correct electric field direction on gaussian surface (must hold for ANY position on the sphere)**

+ **1 pts** (a) Correct electric field solution (given solution for a single sphere; must be properly added as vectors)
+ **1 pts** (a) Correct solution in terms of d, eliminating r (given electric field solution)
+ **10 pts** (b) Full marks

+ **0 pts** (b) No marks

✓ + **2 pts** **(b) No electric field inside conductor**

✓ + **2 pts** **(b) Charge on cavity surface is equal and opposite to interior charge**

✓ + **2 pts** **(b) Charge on outer surface must equal sum of interior charges**

+ **2 pts** (b) Charge distributes uniformly on outer surface (must mention that it is uniform or that the effect of the point charges is cancelled by the inner cavity walls)

✓ + **2 pts** **(b) Field outside is that of a point charge Q_1+Q_2 (The above logical steps must all be present, not just the solution)**

QUESTION 3

3 Problem 3 **19 / 20**

✓ + **20 pts** **All Correct**

+ **12 pts** Part (a): All Correct

+ **8 pts** Part (b): All Correct

+ **6 pts** Part (a): r>R, all correct

+ **6 pts** Part (a): r<R, all correct

+ **4 pts** Part (a): Know Gauss's Law 4/12pts

+ **3 pts** Part (a): r>R, correct equation for charge enclosed 3/12pts

+ **3 pts** Part (a): r<R, correct equation for charge enclosed 3/12pts

+ **1 pts** Part (a): r>R, correct answer 1/12pts

+ **1 pts** Part (a): r<R, correct answer 1/12pts

+ **3 pts** Part (b): Know energy conservation 3/8pts

+ **2 pts** Part (b): Electrical potential, correct formula 2/8pts

+ **2 pts** Part (b): Electrical potential, correct answer 2/8pts

+ **1 pts** Part (b): Correct answer 1/8pts

+ **0 pts** No point

- 1 **Point adjustment**

Slight mistake in the calculation

QUESTION 4

4 Problem 4 **16.5 / 20**

+ **20 pts** All correct

✓ + **1 pts** **a) Correct direction from symmetry**

✓ + **2 pts** **a) Recognition that Gauss's law is applicable (including attempting to set up a surface)**

✓ + **2 pts** **a) Valid implementation of Gauss's law**

+ **1 pts** a) Partial credit for writing down infinite sheet formula

✓ + **1 pts** **b) Recognition that spring force is zero**

+ **1 pts** b) Statement that gravitational and electrostatic forces are equal or that net force (including electrostatic) equals zero on bottom plate

✓ + **2 pts** **b) Correct electrostatic force**

+ **1 pts** b) Partial credit: Electrostatic force off by factor of 2

✓ + **0.5 pts** **c) Correct capacitance formula**

✓ + **1 pts** **c) Correct RC charge exponential**

✓ + **1.5 pts** **c) Correct use of zero initial charge condition**

✓ + **1.5 pts** **c) Correct use of 90% charge condition**

+ **1 pts** d) Correct prediction of force direction

✓ + **1.5 pts** **d) Valid relationship between force and energy**

✓ + **2 pts** **d) Recognition that the capacitor may be considered as two capacitors in parallel**

✓ + **1 pts** **d) Correct capacitance formulas for the two sides of the divide**

✓ + **1 pts** **d) Correct expressions for energy on each side of the divide**

- 1.5 **Point adjustment**

In part a), what is A? It seems like you're using it to refer to both the face of the rectangle and the area of the plate, so you need to be careful regarding your definitions. We need sigma defined in terms of the given quantities, so I'm deducting a point for that.

In b), sigma is proportional to Q, so you can't have it in your expression. You lose half a point here.

QUESTION 5

5 Problem 5 **17 / 20**

+ **15 pts** Part A Correct

✓ + **5 pts** **Part B Correct**

✓ + **2 pts** **KCL/KVL Equation or equivalent**

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✓ + **2 pts** **KCL/KVL Equation or equivalent**

✓ + **2 pts** **KCL/KVL Equation or equivalent**

+ **3 pts** Part A Correct Answer subject to system of equations

+ **1 pts** Ohm's Law

+ **2 pts** Ohm's Law for Equivalent Circuit

+ **2 pts** Correct Answer corresponding to part a

+ **0 pts** Click here to replace this description.

PHYSICS 7B, Lecture 3 – Spring 2018
Midterm 2, C. Bordel
Monday, April 2nd, 2018
7pm-9pm

- Student name:
- Student ID #:
- Name of your GSI: Caleb Eades
- Discussion section #: 301

Math Information Sheet

$$\int \frac{dx}{x^2} = -\frac{1}{x} \qquad \int \frac{dx}{x} = Ln\, x \qquad \int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{(x^2+a^2)}}$$

$$\frac{\partial(\tan\theta)}{\partial\theta} = \frac{1}{\cos^2\theta} \qquad \vec{F} = -\vec{\nabla}U \qquad (1+x)^{\alpha} \sim 1 + \alpha x \text{ when } x \to 0$$

* Cylindrical coordinate system

$$\overrightarrow{dr} = dr\,\overrightarrow{u_r} + r\,d\theta\,\overrightarrow{u_\theta} + dz\,\overrightarrow{u_z} \qquad \vec{\nabla}f = \frac{\partial f}{\partial r}\overrightarrow{u_r} + \frac{1}{r}\frac{\partial f}{\partial\theta}\overrightarrow{u_\theta} + \frac{\partial f}{\partial z}\overrightarrow{u_z}$$

* Spherical coordinate system

$$\overrightarrow{dr} = dr\,\overrightarrow{u_r} + r\,d\theta\,\overrightarrow{u_\theta} + r\sin\theta\,d\varphi\,\overrightarrow{u_\varphi} \qquad \vec{\nabla}f = \frac{\partial f}{\partial r}\overrightarrow{u_r} + \frac{1}{r}\frac{\partial f}{\partial\theta}\overrightarrow{u_\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\varphi}\overrightarrow{u_\varphi}$$

Make sure you show all your work and justify your answers in order to get full credit!

Problem 1 – Charged particle interacting with a charged ring (20 pts)

An electron of charge *-e* and mass *m* is placed at the center of a circular ring of radius *R*, which carries a uniformly distributed positive charge *Q*.

If the electron is displaced from the center a <u>small</u> distance *x* as pictured in Figure 1, show that it undergoes simple harmonic motion when released.

Hint: Simple harmonic oscillator verifies

$$\frac{d^2x}{dt^2}+\omega^2 x=0$$

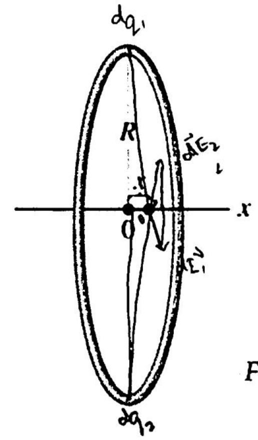


Figure 1

Electric field:

By symmetry only $\cos\theta$ is importan

$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cos\theta$ $\quad r^2 = x^2+R^2$

$\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+R^2}}$

$\lambda = \frac{Q}{L} = \frac{Q}{2\pi R}$ $\quad l = R\theta$

$dq = \lambda dl = \lambda R d\theta$ $\quad dl = R d\theta$

$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x dx}{(x^2+R^2)^{\frac{3}{2}}}$ $\quad E = \int_0^x \frac{\lambda}{4\pi\varepsilon_0}\frac{x dx}{(x^2+R^2)^{3/2}} = \frac{\lambda}{4\pi\varepsilon_0}\frac{x}{R^2\sqrt{x^2+R^2}}$

$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R}{r^2}\cos\theta d\theta$

$\vec{E} = \int_0^{2\pi}\frac{1}{4\pi\varepsilon_0}\frac{\lambda R}{x^2+R^2}\cos\theta d\theta = \frac{\lambda R}{4\pi\varepsilon_0(x^2+R^2)}\int_0^{2\pi}\cos\theta d\theta = \left[-\sin\theta\right]_{2\pi}^{0} = 0$

$\vec{E} = \frac{kQx}{R^2\sqrt{x^2+R^2}}\hat{x}$ $\quad F = q\vec{E} = -\frac{ekQx}{R^2\sqrt{x^2+R^2}}$

Problem 2– Spheres with cavities (20 pts)

A solid dielectric sphere of radius R_1 carries uniform volume charge density ρ (ρ>0). A spherical hole of radius R_2 is made in the larger sphere such that the 2 centers are separated by a distance d verifying $d+R_2<R_1$. See Figure 2.1.

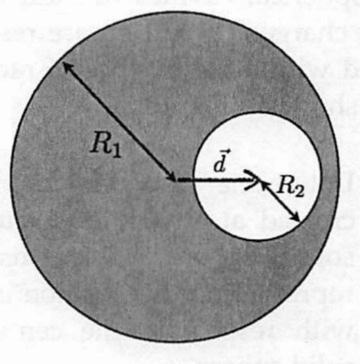


Figure 2.1

a. Calculate the electric field created at any point inside the spherical cavity, in terms of the vector $\vec{d}$ joining the two centers.

Superposition Principle:

$\oint E\cdot dA = \frac{Q_{enc}}{\varepsilon_0}$

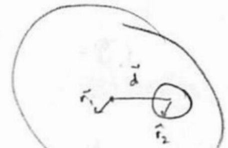
$E\oint dA = \frac{Q_{enc}}{\varepsilon_0} = \frac{\rho(\frac{4}{3}\pi r^3)}{\varepsilon_0} = \vec{E}A = \vec{E}(r)\cdot 4\pi r^2$

$\vec{E}(r) = \frac{\rho r_1}{3\varepsilon_0}\hat{r}_1$

$\oint \vec{E}\cdot dA = \frac{Q_{enc}}{\varepsilon_0}$

By the same logic

$\vec{E}(r) = \frac{\rho r_2}{3\varepsilon_0}\hat{r}_2$



$E_{cavity} = \frac{\rho R_1}{3\varepsilon_0}\hat{r}_1 - \frac{\rho R_2}{3\varepsilon_0}\hat{r}_2 = \boxed{\frac{\rho}{3\varepsilon_0}(R_1-R_2)\,\vec{d}}$

Now let's consider an uncharged solid conducting sphere of radius r_0 containing two spherical cavities of radii r_1 and r_2. Point charges Q_1 and Q_2 are respectively placed within the cavities of radii r_1 and r_2, as shown in Figure 2.2.

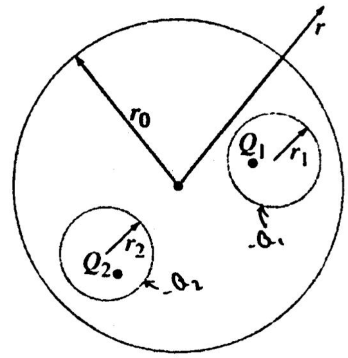
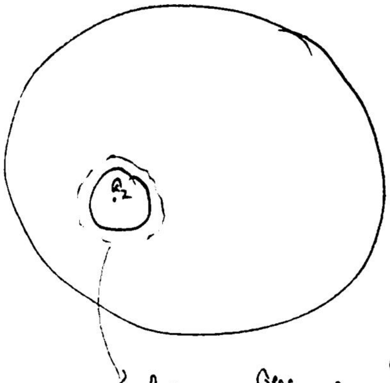


Figure 2.2

b. Determine the electric field $\vec{E}$ created at any point P outside the solid sphere in terms of $\vec{r}$, representing the position of point P with respect to the center of the solid sphere.

The electric field inside a conductor is 0.

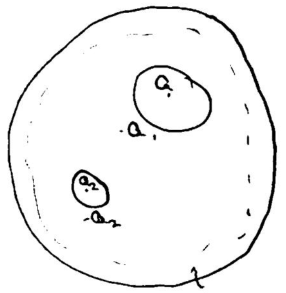


$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} = 0$

$Q_{enc} = 0 = Q_2 + Q_{surface}$

↳ charge of $-Q_2 = Q_{surface}$ accumulated on surface

By the same logic, a charge of $-Q_1$ accumulates on inner surface of the other cavity



net charge = 0 on conductor

charge on surface of capacitor = $Q_2 + Q_1$

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$

↓ spherical shell ↓

$E \oint d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$

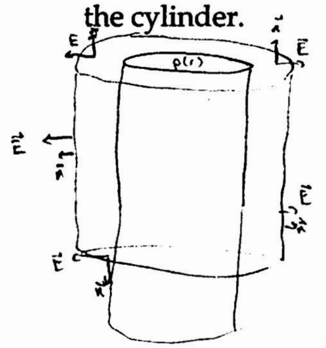
$E(4\pi r^2) = \frac{Q_{enc}}{\varepsilon_0}$

$$\boxed{\vec{E}(r) = \frac{Q_1 + Q_2}{4\pi \varepsilon_0 r^2}\,\hat{r}}$$

Problem 3 – Charged particle and infinite charged cylinder (20 pts)

An infinitely long cylinder of radius R carries positive charge with volume density $\rho(r) = \rho_0\,(1 - \frac{r^2}{R^2})$, where ρ_0 is a positive constant and r the radial distance measured from the symmetry axis of the cylinder.

a. Determine the electric field created by the charge distribution both inside and outside the cylinder.



$r > R$

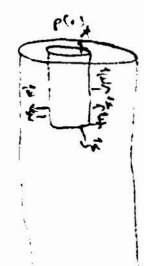
$$\Phi = \oint \vec{E}\cdot d\vec{A} = \vec{E}\cdot\oint d\vec{A} = EA = \frac{Q_{enc}}{\varepsilon_0} = \frac{\int dq}{\varepsilon_0} = \frac{\int_0^R \rho\, dV}{\varepsilon_0}$$

$(\rho V = Q$; $dq = \rho\, dV$; $V = \pi r^2 h$ $\quad dV = 2\pi r h\, dr$)

$$Q_{enc} = \int_0^R \rho\, dV = \int_0^R \rho_0\left(1 - \frac{r^2}{R^2}\right)\cdot 2\pi r h\, dr = \rho_0 2\pi h \int_0^R r - \frac{r^3}{R^2}\, dr = \rho_0 2\pi h\left[\frac{1}{2}r^2 - \frac{1}{4R^2}r^4\right]_0^R$$

$$= \rho_0 2\pi h\left[\frac{1}{2}R^2 - \frac{R^4}{4R^2}\right] = \rho_0 2\pi h\left[\frac{3}{4}R^2\right]$$

$$EA = \frac{\rho_0 2\pi h\left[\frac{3}{4}R^2\right]}{\varepsilon_0} = \vec{E}(r)(2\pi r h) = \frac{\rho_0 2\pi h\left[\frac{3}{4}R^2\right]}{\varepsilon_0} \qquad \boxed{E(r) = \frac{3R^2\rho_0}{4\varepsilon_0 r}\hat{r}}$$



$r < R$

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \vec{E}\cdot\oint d\vec{A} = EA = \frac{Q_{enc}}{\varepsilon_0} = \frac{\int dq}{\varepsilon_0} = \frac{\int_0^r \rho\, dV}{\varepsilon_0} = \frac{\int_0^r \rho_0\left(1 - \frac{r^2}{R^2}\right)\cdot 2\pi r h\, dr}{\varepsilon_0}$$

$$= \frac{\rho_0 2\pi h \int_0^r \left(r - \frac{r^3}{R^2}\right)dr}{\varepsilon_0} = \frac{\rho_0 2\pi h}{\varepsilon_0}\left[\frac{1}{2}r^2 - \frac{1}{4R^2}r^4\right]_0^r = \frac{\rho_0 2\pi h}{\varepsilon_0}\left[\frac{1}{2}r^2 - \frac{r^4}{4R^2}\right]$$

$$EA = \vec{E}(r)(2\pi r h) = \frac{\rho_0 2\pi h}{\varepsilon_0}\left[\frac{1}{2}r^2 - \frac{r^4}{4R^2}\right] \qquad \frac{1}{2}r - \frac{r^3}{4R^2}$$

$$\vec{E}(r) = \frac{\frac{\rho_0}{\varepsilon_0}\left[\frac{1}{2}r^2 - \frac{r^4}{4R^2}\right]}{r} = \boxed{\frac{\rho_0}{\varepsilon_0}\left[\frac{1}{2}r - \frac{r^3}{4R^2}\right]\hat{r}}$$

b. If an electron of electric charge *-e* and mass *m* is released from rest at a distance *r=10R*, determine the speed of the electron when it reaches the surface of the cylinder.

$$\vec{E} = \frac{3R^2\rho_0}{4\varepsilon_0 r}\hat{r}$$

$$dV = -\vec{E}\cdot d\vec{\ell} = -\frac{3R^2\rho_0}{4\varepsilon_0 r}\hat{r}\cdot d\vec{\ell} = -\frac{3R^2\rho_0}{4\varepsilon_0 r}dr$$

$$V_R - V_{10R} = -\int_{10R}^{R}\frac{3R^2\rho_0}{4\varepsilon_0 r}dr = \frac{3R^2\rho_0}{4\varepsilon_0}\int_R^{10R}\frac{1}{r}dr = \frac{3R^2\rho_0}{4\varepsilon_0}\left[\ln r\right]_R^{10R} = \frac{3R^2\rho_0}{4\varepsilon_0}\ln\left(\frac{10R}{R}\right) = \frac{3R^2\rho_0\ln(10)}{4\varepsilon_0}$$

$$\Delta U = qV = \frac{e\,3R^2\rho_0\ln(10)}{4\varepsilon_0} = KE = \frac{1}{2}mv^2$$

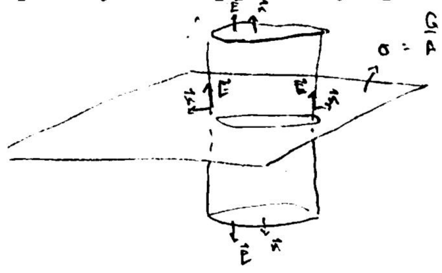
$$v^2 = 2mqV = \frac{6meR^2\rho_0\ln(10)}{4\varepsilon_0} = \frac{3meR^2\rho_0\ln(10)}{2\varepsilon_0}$$

$$\boxed{v = \sqrt{\frac{3meR^2\rho_0\ln(10)}{2\varepsilon_0}}}$$

Problem 4 - Capacitor (20 pts)

In the entire problem, consider that the plates are infinitely thin large square sheets of side length ℓ

a. Determine the magnitude and direction of the electric field created at any point in space by such a plate carrying some charge Q.



$\sigma = \frac{Q}{A}$

$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$

$\Phi_{top} + \Phi_{bottom} = \frac{Q_{enc}}{\varepsilon_0}$

$\Phi_{top} = \Phi_{bottom} = 2\Phi_{top}$

$\Phi_{top} = \frac{Q_{enc}}{2\varepsilon_0} = \oint \vec{E} \cdot d\vec{A} = E \oint_A d\vec{A} = \frac{\sigma A}{2\varepsilon_0} = E \cdot A = \frac{\sigma A}{2\varepsilon_0}$

$\boxed{\vec{E}(z) = \frac{\sigma}{2\varepsilon_0}}$

A horizontal parallel plate capacitor is made of two such large conducting sheets of mass m, separated by a small distance d. The lower sheet is resting on a massless spring of stiffness constant k and resistance R, and the upper sheet hangs down held by a thin metallic cable, as shown in Fig. 3.1.

b. Determine the amount of electric charge Q that needs to be accumulated on the plates of this capacitor in order for the spring to keep its equilibrium length (neither compressed nor stretched).

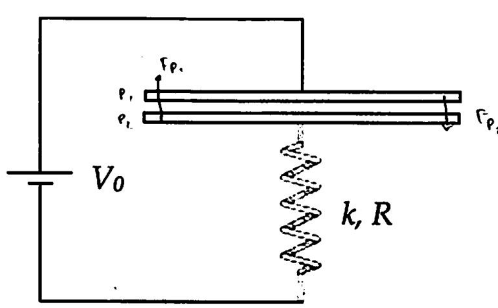


Figure 3.1

$E_{between} = E_{plate\,1} + E_{plate\,2} = 2\left(\frac{\sigma}{2\varepsilon_0}\right)$

$= \frac{\sigma}{\varepsilon_0}$

$F_{plate\,1} = Q\vec{E} = \frac{Q\sigma}{2\varepsilon_0}$

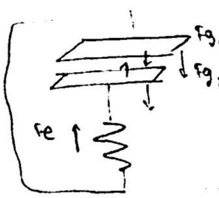
$F_{plate\,2} = -QE = -\frac{Q\sigma}{2\varepsilon_0}$

$F = kx$

$V = IR$

$V_0 - \frac{Q}{C} - IR = 0$

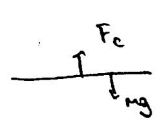
$V_0 = \frac{Q}{C} + IR$



$Fg_1 + Fg_2 + F_{plate\,1} - F_{plate\,2} = 0$

$\frac{Q\sigma}{2\varepsilon_0} - 2mg = 0$

$Q = \frac{4mg\varepsilon_0}{\sigma}$



You may consider that the lower plate is static in the rest of the problem.

c. Assuming that the capacitor is initially uncharged, how long do you need to wait after you connect it to a battery that supplies a voltage V_0 if you want the capacitor to reach 90% of its full charge?

$V_0 - \frac{Q}{C} - IR = 0$

$V_0 = \frac{Q}{C} + IR = \frac{Q}{C} + \frac{dQ}{dt}R$

$\frac{dQ}{dt}R + \frac{Q}{C} = 0$

$\frac{dQ}{dt}RC = -Q$

$dQ\,RC = -Q\,dt$

$\int \frac{1}{Q}\,dQ = \int -\frac{1}{RC}\,dt$

$\ln(Q) = -\frac{t}{RC} + \alpha$

$\underbrace{Q = \beta e^{-t/RC}}_{\text{general solution}}$

$\left.\begin{array}{l} Q \text{ constant} \\ \frac{dQ}{dt} = 0 \end{array}\right\}$ particular

$V_0 = \frac{Q}{C}$ $\quad Q = CV_0$

$Q(t) = \beta e^{-t/RC} + CV_0$

$Q(0) = 0 = \beta + CV_0$

$\beta = -CV_0$

$Q(t) = -CV_0 e^{-t/RC} + CV_0$

$= CV_0(1 - e^{-t/RC}) = Q_{max}(1 - e^{-t/RC})$

at $t = \infty$, $Q(t) = CV_0$

$\cancel{CV_0} \cdot \frac{9}{10} = \cancel{CV_0}(1 - e^{-t/RC})$

$9 = 10(1 - e^{-t/RC})$

$9 = 10 - 10e^{-t/RC}$

$-1 = -10e^{-t/RC}$

$\frac{1}{10} = e^{-t/RC}$

$\ln\left(\frac{1}{10}\right) = -t/RC = -\ln(10)$

$\boxed{t = RC\ln(10)}$

Now the fully charged parallel-plate capacitor, initially carrying charge Q, is disconnected from the battery. Then a slab of dielectric material of dielectric constant K, thickness d and side length ℓ is slowly introduced between the plates. At time t, x represents the length of the slab that has been inserted in the capacitor. See Fig. 3.2.

d. Determine the direction and magnitude of the electric force exerted on the slab.

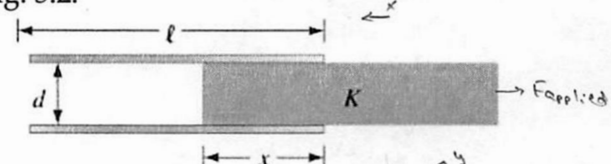
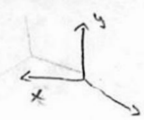


Figure 3.2

Capacitors in parallel



$C(x) = \varepsilon_0 \frac{\ell(\ell - x)}{d} + \varepsilon \frac{\ell x}{d}$

$= \varepsilon_0 \frac{\ell(\ell - x)}{d} + K\varepsilon_0 \frac{\ell x}{d} = \varepsilon_0\left(\frac{\ell(\ell - x)}{d} + \frac{K\ell x}{d}\right) = \varepsilon_0\left(\frac{\ell^2 - \ell x + K\ell x}{d}\right) = \varepsilon_0\left(\frac{\ell^2 - \ell x(K-1)}{d}\right)$

$U(x) = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{\varepsilon_0\left(\frac{\ell^2 - \ell x(K-1)}{d}\right)} = \frac{dQ^2}{2\varepsilon_0(\ell^2 - \ell x(K-1))}$

$\vec{F} = -\nabla U = -\left\langle \frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z} \right\rangle$

$F_x = -\frac{\partial U}{\partial x} = -\frac{\partial}{\partial x}\left[\frac{dQ^2}{2\varepsilon_0(\ell^2 - \ell x(K-1))}\right] = \frac{-dQ^2}{2\varepsilon_0}\frac{\partial}{\partial x}\left[\frac{1}{\ell^2 - \ell x(K-1)}\right] = -\frac{dQ^2}{2\varepsilon_0}\frac{\partial}{\partial x}\left(\left[\ell^2 - \ell x(K-1)\right]^{-1}\right)$

$= -\frac{dQ^2}{2\varepsilon_0}\left(-1\left(\ell^2 - \ell x(K-1)\right)^{-2} \cdot -\ell(K-1)\right)$

$= -\frac{dQ^2\ell(K-1)}{2\varepsilon_0} \cdot \frac{1}{(\ell^2 - \ell x(K-1))^2}$

$F_y = -\frac{\partial U}{\partial y} = 0$

$F_z = -\frac{\partial U}{\partial z} = 0$

$\boxed{\vec{F}_x = -\frac{dQ^2\ell(K-1)}{2\varepsilon_0(\ell^2 - \ell x(K-1))^2}\,\hat{x}}$

Problem 5 - DC circuit (20 pts)

A network of five identical resistors of resistance R is connected to a battery supplying a voltage V_0, as shown in Figure 4.

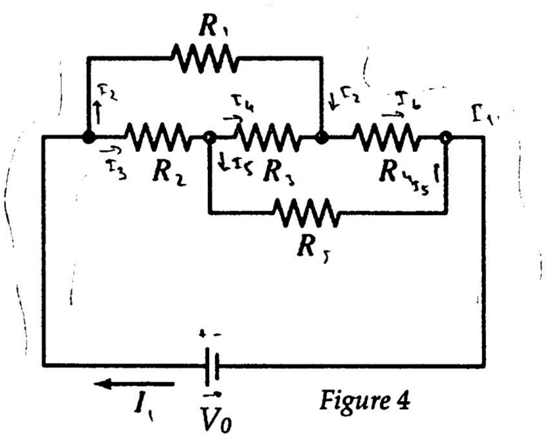
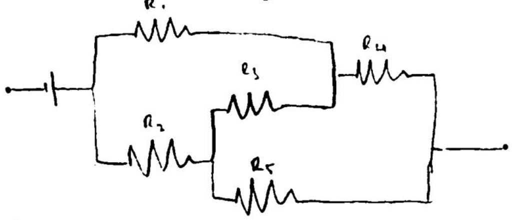


Figure 4

$R_1 = R_2 = R_3 = R_4 = R_5 = R$

a. Using Kirchhoff's rules exclusively, determine the current I that flows out of the battery.



Junction Rule:

$I_1 = I_2 + I_3 = I_5 + I_6$

$I_3 = I_4 + I_5$

$I_6 = I_2 + I_4$

Loop Rule:

$-V_0 + I_2 R + I_6 R = 0$

$-V_0 + I_3 R + I_4 R + I_6 R = 0$

$-V_0 + I_3 R + I_5 R = 0$

$V_0 = R(I_2 + I_6) = R(I_3 + I_4 + I_6)$

$= R(I_3 + I_5)$

$I_2 + I_6 = I_3 + I_4 + I_6 = I_3 + I_5 = \frac{V_0}{R}$

$I_2 + I_2 + I_4 = I_4 + I_5 + I_4 + I_2 + I_4 = I_4 + I_4 + I_5$

$2I_2 + I_4 = 3I_4 + I_2 + I_5 = I_4 + 2I_5$

$2I_2 = 2I_4 + I_2 + I_5 = 2I_5$

$I_2 = I_5$

$2I_2 = 2I_4 + 2I_2$

$V_0 = I_2 R + R(I_2 + I_4) = R(2I_2 + I_4)$

$= (I_4 + I_5)R + I_4 R + (I_2 + I_4)R = R(3I_4 + I_2 + I_5)$

$= (I_4 + I_5)R + I_5 R = R(I_4 + 2I_5)$

$\frac{V_0}{R} = 2I_2 + I_4 \qquad I_4 = \frac{V_0}{R} - 2I_2$

$I_5 = \frac{V_0}{R} - 3I_4 - I_2$

$= \frac{V_0}{R} - 3\left[\frac{V_0}{R} - 2I_2\right] - I_2 = \frac{V_0}{R} - \frac{3V_0}{R} + 5I_2 = 5I_2 - \frac{2V_0}{R}$

$I_6 = I_2 + I_4 = I_2 + \frac{V_0}{R} - 2I_2 = \frac{V_0}{R} - I_2$

$I_3 = I_4 + I_5 = \frac{V_0}{R} - 2I_2 + 5I_2 - \frac{2V_0}{R} = 3I_2 - \frac{V_0}{R}$

$I_1 = I_2 + I_3 = I_2 + 3I_2 - \frac{V_0}{R} = 4I_2 - \frac{V_0}{R} = 5I_2 - \frac{2V_0}{R} + \frac{V_0}{R} - I_2 = 4I_2 - \frac{V_0}{R}$

$I_1 = I_2 + I_3 = \frac{3V_0}{8R} + \frac{9V_0}{8R} - \frac{V_0}{R} = \frac{12V_0}{8R} - \frac{8V_0}{8R} = \frac{4V_0}{8R} = \boxed{\frac{V_0}{2R}}$

b. Deduce from part (a) the equivalent resistance R_{eq} of the single resistor that is equivalent to the five-resistor network.

$V_0 = I R_{eq}$ assume I obtained from a

$R_{eq} = \frac{V_0}{I} = \frac{V_0}{V_0/2R} = \boxed{2R}$

$I_2 + I_3 = 4I_2 - \frac{V_0}{R}$

$I_3 = 3I_2 - \frac{V_0}{R}$

$I_1 = I_2 + I_3$

$I_2 = ?$

$I_3 = 3I_2 - \frac{V_0}{R}$

$I_4 = \frac{V_0}{R} - 2I_2$

$I_5 = 5I_2 - \frac{V_0}{R}$

$I_6 = \frac{V_0}{R} - I_2$

$-V_0 + R(3I_2 - \frac{V_0}{R}) + R(5I_2 - \frac{V_0}{R})$

$3I_2 R - V_0 + 5I_2 R - V_0 = V_0$

$8I_2 R = 3V_0$

$I_2 = \frac{3V_0}{8R} \qquad I_3 = 3I_2 - \frac{V_0}{R} = \frac{9V_0}{8R} - \frac{V_0}{R}$