Math 1A (Fall 2017) Final Exam (Friday December 15, 19:10-22:00)

Name: SID:

General directions:

- Do NOT open the exam before the start time.

- Avoid sitting next to each other. Your immediate left and right seats should be empty.

- You cannot leave the room during the last 15 minutes (21:45–22:00).

- If you are staying until the end, do NOT hand in your exam in person to a proctor. Rather, stop working at 22:00 following the direction from proctors, place the exam on the desk with cover page up, and leave the room. Proctors will walk around to pick it up.

Directions on writing solutions:

- Please write clearly and legibly.

- Problem 1 is worth 40 points. Each of the other problems is worth 20 points.

- For problems 2 through 9, justify your answers.

- Insert scratch paper in the exam (only) if you put solutions there or if you may get partial credit from scratch work. If so, write your name on scratch paper to avoid a mix-up.

1. Mark each of the following True (T) or False (F). No justification is necessary.
(For each sub-problem, correct = 4 pts, no response = 2 pts, wrong = 0 pts.)

(1) (   ) Suppose that $f(g(x))$ is continuous at 0. Then f is continuous at $g(0)$, and g is continuous at 0.

(2) (   ) If $f(x) = -\cos x$ then its 31st derivative $f^{(31)}(x) = \sin x$.

(3) (   ) Let a be a constant. Let f and g be functions that are continuous everywhere. If $\int_a^x f(t)dt = \int_a^x g(t)dt$ as functions of x, then $f(x) = g(x)$.

(4) (   ) For any continuous function f and any real numbers a and b,

$$\int_a^b |f(x)|\, dx = \left| \int_a^b f(x)\, dx \right|.$$

(5) (   ) If the functions f and g have antiderivatives F and G, respectively, then FG is an antiderivative of the function $Fg + fG$.

(6) (   ) Suppose that $\lim_{x\to 0} f(x) = 1$. Then for every number $\epsilon > 0$, there is a number $\delta > 0$ such that if $|x| < \delta$ then $|f(x) - 1| < \epsilon$.

(7) (   ) If $f(x)$ is an even continuous function then $\int_0^x f(t)dt$ is an odd function.

(8) (   ) Suppose that both f is differentiable twice and that f'' is continuous on the domain of f. If f has an inflection point then f' has a local maximum or a local minimum.

(9) (   ) If the line $x = 1$ is a vertical asymptote of $y = f(x)$ then f is not defined at 1.

(10) (   ) Let f and g be functions. Then the domain of the composite function $f \circ g$ is contained in the domain of f.

2. Compute the following. (Say DNE to (1) or (4) if the requested answer does not exist.)

(1) (5 points) $\lim_{x\to 0^+}(1+2x)^{1/x}$

(2) (5 points) $\displaystyle\int_{\pi^2/9}^{\pi^2/4}\frac{\sin\sqrt{x}}{\sqrt{x}}dx$

(3) (5 points) the second derivative of e^{e^x}

(4) (5 points) horizontal asymptotes of $y=\dfrac{3e^x}{e^x-2}$.

3. (1) (10 points) The velocity function of a particle is $v(t) = \dfrac{\sin t}{1+\cos^2 t}$ meters/sec, where t stands for time in seconds. Find the total distance (*not displacement*) the particle traveled from time $t = 0$ to $t = 2\pi$. (Hint: First determine the intervals on which $v(t) \geq 0$ and $v(t) \leq 0$.)
(2) (10 points) Find y' in terms of x and y if $y = \ln(xy + x^2)$.

4. (20 points) A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string has been let out? (The person stays in the same place. The string always makes a straight line.)

5. (20 points) A soda company wants to design a new cylindrical can. If the company wants to design a can with volume $16\pi\ cm^3$, and minimize how much aluminium it takes to make, what should the radius and the height of the can be, in cm?

6. (20 points) Sketch the curve $y = \dfrac{x^2}{x-1}$. Doing so, clearly indicate the following (if they exist): domain, x, y-intercepts, vertical/horizontal/slant asymptotes, intervals where the graph is increasing/decreasing/concave upward/concave downward, local maxima/minima.

7. (20 points) Find the total area enclosed by $x = 3y^3$ and $x = 4y^3 - 2y$.

8. (20 points) For any number x such that $-1 < x < 1$, show that $\sin^{-1} x = \dfrac{\pi}{2} - \cos^{-1} x$.

(Hint: You need not apply the Mean Value Theorem but you may want to use the following fact that follows from the Mean Value Theorem: two functions on an open interval differ by a constant if their derivatives are equal.)

9. (20 points) Find the volume of the solid given by rotating the area bounded between $xy = 1$, $x = 1$, $x = 2$, $y = 0$ about the line $x = -1$.