Solutions to Midterm 1

Problem 1. (a) Set up an absolute (x, y) frame with the x-axis directed horizontally to the right. When P is applied, the force acting on block A is $2P$. If $P = 60$,

$$2P = 120 > \mu N_A = 0.5(20g) = 98.1$$

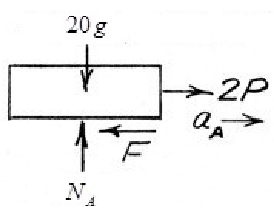
Thus slipping between block A and cart B occurs. For block A,

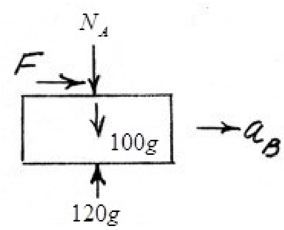
$$\sum F_x = ma_x$$
$$\Rightarrow \quad 2P - \mu N_A = 20a_A$$
$$\Rightarrow \quad a_A = 1.095 \text{ m/s}^2$$

For block B,

$$\sum F_x = ma_x$$
$$\Rightarrow \quad \mu N_A = 100a_B$$
$$\Rightarrow \quad a_B = 0.1g = 0.981 \text{ m/s}^2$$

Both A and B move to the right but $a_A > a_B$ because of slipping.





(b) If $P = 40$,

$$2P = 80 < \mu N_A = 98.1$$

There is no slipping. For blocks A and B combined,

$$\sum F_x = ma_x$$
$$\Rightarrow \quad 2P = (20 + 100)a$$
$$\Rightarrow \quad a_A = a_B = 0.667 \text{ m/s}^2$$

Problem 2. Set up an absolute (x, y) frame with the x-axis directed to the right. When the system moves from an initial rest configuration with the spring stretched by 2 m to a final configuration with the spring being unstretched,

$$U_{1-2} = \Delta T = T_2$$
$$\Rightarrow \quad \frac{1}{2}k(2^2) = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$
$$\Rightarrow \quad 2(180) = \frac{1}{2}(40)v_A^2 + \frac{1}{2}(60)v_B^2 \qquad (1)$$

The spring forces are action and reaction internal forces and the net external force on the system is zero. Thus

$$\Delta G = G_2 - G_1 = 0$$
$$\Rightarrow \quad m_A v_A + m_B v_B = 0$$
$$\Rightarrow \quad 40v_A + 60v_B = 0 \qquad (2)$$

There are 2 unknowns v_A, v_B in 2 equations. Simultaneous solution gives

$$v_A = 3.29 \text{ m/s}$$

$$v_B = -2.19 \text{ m/s}$$

Block A moves to the right and block B to the left.

Problem 3. In this oblique central impact, the tangential direction is parallel to the stationary surface and the normal direction is perpendicular to the stationary surface. For the moving particle,

$$\Delta G_t = 0$$

$$\Rightarrow \qquad v\cos\theta = v'\cos(\theta/2) \tag{1}$$

Restitution in n-direction yields

$$e = \frac{v'\sin(\theta/2)}{v\sin\theta} \tag{2}$$

Eliminate v from Eqs. (1) and (2),

$$e = \cot\theta\tan(\theta/2)$$

For $\theta = 40°$,

$$e = 0.434$$