CE93 Fall 2018
MIDTERM 1

09/27/2018

Name:__________________________________

Problem 1: _____ / 40 pts

Problem 2: _____ / 30 pts

Problem 3: _____ / 30 pts

Total: _____ / 100 pts

1. Hurricane Florence was a powerful and long-lived Cape Verde Hurricane, as well as the wettest tropical cyclone on record in the Carolinas and the ninth-wettest tropical cyclone to affect the contiguous United States. In order to evaluate damages on buildings caused by future hurricanes, a Type A building has been studied. It is estimated that an impending hurricane in the region might be strong (S), moderate (M), or weak (W) with probabilities P(S) = 0.1, P(M) = 0.30, and P(W) = 0.6. The probabilities of failures of a Type A building are 0.3, 0.1, and 0.01 for S, M and W hurricanes, respectively.

a. Draw a Venn diagram to display the information available. [5 points]

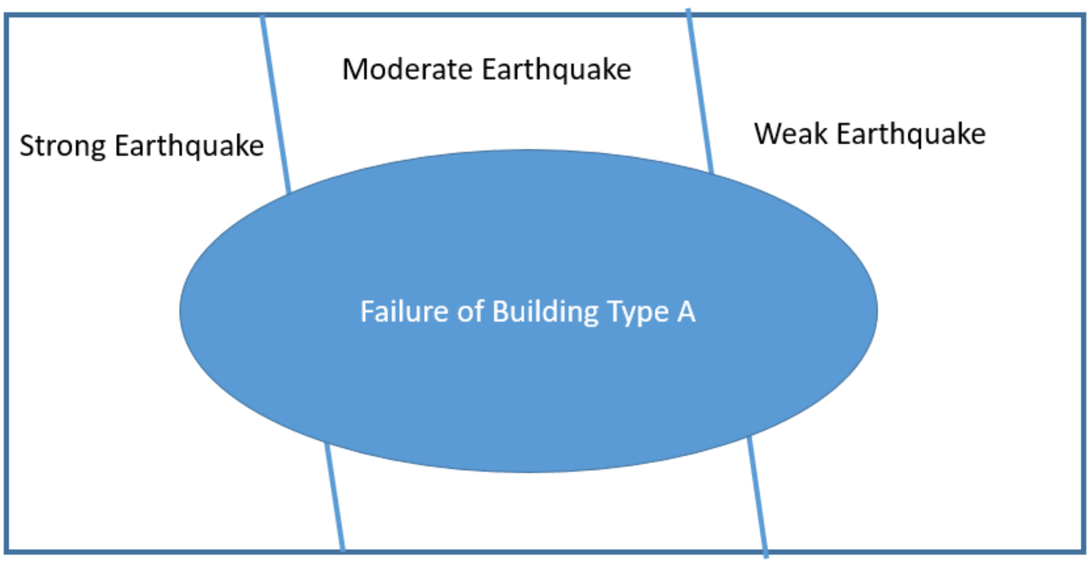
b. Determine the probability of failure of a Type A building if the impending hurricane indeed occured. [13 points]

c. If a Type A building failed, what is the probability that the hurricane was of moderate strength? [12 points]

d. An insurance company expects pay damages per failure of a Type A building caused by hurricane as follows: $1,000,000 for a strong hurricane; $100,000 for a moderate hurricane; and $10,000 for a weak hurricane. What is the mean and variance of damages expected to be paid by the insurance company for a Type A building failure? [10 points]

Solution:

a.



b. Let A denote the event of failure of Type Building A

$$P(A) = P(A \cap S) + P(A \cap M) + P(A \cap W)$$
$$= P(A|S)P(S) + P(A|M)P(M) + P(A|W)P(W)$$
$$= 0.3 * 0.1 + 0.1 * 0.3 + \ 0.01 * 0.6 = 0.066\ (if\ Version\ 1\ Exam)$$
$$= 0.25 * 0.1 + 0.15 * 0.3 + 0.01 * 0.6 = 0.076\ (if\ Version\ 2\ Exam)$$

c.

$$P(M|A) = \frac{P(M \cap A)}{P(A)} = \frac{P(A|M)P(M)}{P(A)}$$
$$= \frac{0.1 * 0.3}{0.066} = 0.454\ (V1)$$
$$= \frac{0.15 * 0.3}{0.076} = 0.592\ (V2)$$

d.

We basically have four different situations for the damage expense from the insurance company: Building Type A stands during the hurricane, which leads to $0 expense; $1,000,000 expense due to failure of Building Type A under a strong hurricane; $100,000 expense due to failure of Building Type A under a moderate hurricane; and $10,000 expense due to failure of Building Type A under a weak moderate hurricane. In order to derive the PMF of random variable “Damage Expense”, we have to calculate the following probabilities, and let DE denote the random variable:

$$P(DE = \$1{,}000{,}000) = P(Failure\ of\ A\ during\ strong\ hurricane)$$
$$= P(A \cap S) = P(A|S) * P(S) = 0.3 * 0.1 = 0.03$$
$$P(DE = \$100{,}000) = P(Failure\ of\ A\ during\ moderate\ hurricane)$$
$$= P(A \cap M) = P(A|M) * P(M) = 0.1 * 0.3 = 0.03$$
$$P(DE = \$10{,}000) = P(Failure\ of\ A\ during\ weak\ hurricane)$$
$$= P(A \cap W) = P(A|W) * P(W) = 0.6 * 0.01 = 0.006$$
$$P(DE = \$0) = P(standing\ of\ A\ during\ any\ hurricanes) = 1 - P(A) = 0.934$$

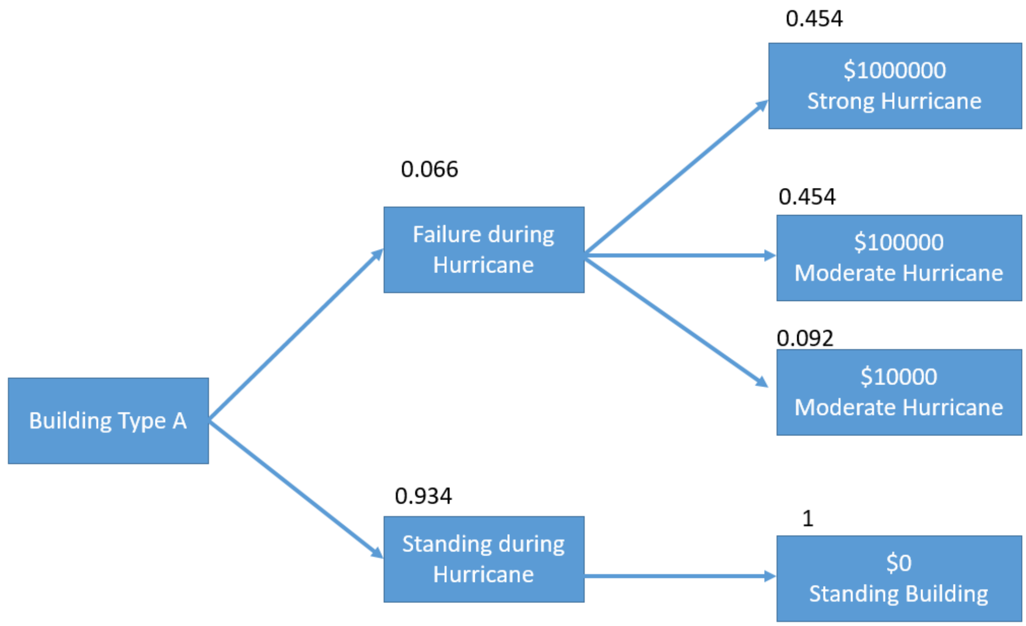
Damage Expense (V1)	$1,000,000	$100,000	$10,000	$0
P	0.03	0.03	0.006	0.934

$$\mu = 1000000 * 0.03 + 100000 * 0.03 + 10000 * 0.006 + 0 * 0.934 = 33060$$
$$\sigma^2 = 0.03 * (1000000 - 33060)^2 + 0.03 * (100000 - 33060)^2 + 0.006 * (10000 - 33060)^2 + 0.934 * (0 - 33060)^2 = 2.92 * 10^{10}$$

Damage Expense (V2)	$1,000,000	$100,000	$10,000	$0
P	0.025	0.045	0.006	0.924

$$\mu = 1000000 * 0.025 + 100000 * 0.045 + 10000 * 0.006 + 0 * 0.924 = 29560$$
$$\sigma^2 = 0.025 * (1000000 - 29560)^2 + 0.045 * (100000 - 29560)^2 + 0.006 * (10000 - 29560)^2 + 0.924 * (0 - 29560)^2 = 2.46 * 10^{10}$$

An example tree diagram for deriving PMF for damage expenses of insurance company can be presented as follows.



2. The joint PMF of precipitation, X(in) and runoff, Y(cfs) (discretized here for simplicity) due to storms at a given location is as follow:

	Y			
X	0	1	2	3
0	0.13	0.10	0.07	0.03
1	0.12	0.16	0.08	0.04
2	0.02	0.06	0.08	0.04
3	0.01	0.02	0.02	0.02

a. What is the probability of $P(X > 2)$ [4 points]
b. What is the probability of $P(X > 2 \mid Y < 3)$ [6 points]
c. What is the marginal PMF of X? [6 points]
d. What is the conditional PMF of X|Y = 3? [6 points]
e. What is the conditional mean and variance of Y|X=3? [8 points]

Solution (V1):
a.

$$P(X > 2) = P(X = 3) = 0.01 + 0.02 + 0.02 + 0.02 = 0.07$$

b.

$$P(X > 2|Y < 3) = \frac{P(X > 2, Y < 3)}{P(Y < 3)}$$

$$= \frac{0.01 + 0.02 + 0.02}{1 - (0.03 + 0.04 + 0.03 + 0.02)} = 0.0568$$

c.

X	0	1	2	3
P	0.33	0.40	0.20	0.07

$$P(X = 0) = 0.13 + 0.10 + 0.07 + 0.03 = 0.33$$
$$P(X = 1) = 0.12 + 0.16 + 0.08 + 0.04 = 0.40$$
$$P(X = 2) = 0.02 + 0.06 + 0.08 + 0.04 = 0.20$$
$$P(X = 3) = 0.01 + 0.02 + 0.02 + 0.02 = 0.07$$

d.

X|Y=3	0|Y=3	1|Y=3	2|Y=3	3|Y=3
P	0.23	0.308	0.308	0.154

$$P(Y = 3) = 0.03 + 0.04 + 0.04 + 0.02 = 0.13$$

e.

Y|X=3	0|X=3	1|X=3	2|X=3	3|X=3
P	1/7	2/7	2/7	2/7

$$\mu = 0 * \frac{1}{7} + 1 * \frac{2}{7} + 2 * \frac{2}{7} + 3 * \frac{2}{7} = 1.714$$

$$\sigma^2 = \frac{1}{7} * (0 - 1.714)^2 + \frac{2}{7} * (1 - 1.714)^2 + \frac{2}{7} * (2 - 1.714)^2 + \frac{2}{7} * (3 - 1.714)^2 = 1.061$$

Solution (V2):

a.

$$P(X > 2) = P(X = 3) = 0.01 + 0.03 + 0.01 + 0.02 = 0.07$$

b.

$$P(X > 2|Y < 3) = \frac{P(X > 2, Y < 3)}{P(Y < 3)}$$

$$= \frac{0.01 + 0.03 + 0.01}{1 - (0.03 + 0.04 + 0.02 + 0.02)} = 0.0562$$

c.

X	0	1	2	3
P	0.33	0.40	0.20	0.07

$$P(X = 0) = 0.11 + 0.10 + 0.09 + 0.03 = 0.33$$
$$P(X = 1) = 0.18 + 0.10 + 0.08 + 0.04 = 0.40$$
$$P(X = 2) = 0.02 + 0.06 + 0.10 + 0.02 = 0.20$$
$$P(X = 3) = 0.01 + 0.03 + 0.01 + 0.02 = 0.07$$

d.

X|Y=3	0|Y=3	1|Y=3	2|Y=3	3|Y=3
P	0.273	0.363	0.182	0.182

$$P(Y = 3) = 0.03 + 0.04 + 0.02 + 0.02 = 0.11$$

e.

Y|X=3	0|X=3	1|X=3	2|X=3	3|X=3
P	1/7	3/7	1/7	2/7

$$\mu = 0 * \frac{1}{7} + 1 * \frac{3}{7} + 2 * \frac{1}{7} + 3 * \frac{2}{7} = 1.571$$

$$\sigma^2 = \frac{1}{7} * (0 - 1.571)^2 + \frac{3}{7} * (1 - 1.571)^2 + \frac{1}{7} * (2 - 1.571)^2 + \frac{2}{7} * (3 - 1.571)^2 = 1.102$$

3. In Lab3, we looked at rainfall data in San Francisco over 1960 – 2002 with two variables: the number of rainy days per year and the cumulative yearly rainfall in inches. Let:

E_1 = the number of rainy days in SF in a future year is > 60 days

E_2 = the amount of yearly cumulative annual rainfall in SF in a future year is > 20 in

The following code has been used to calculate the coefficient of variation of yearly cumulative rainfall in SF, $P(E_1 \cap E_2)$ *and* $P(E_1 \cup E_2)$.

```
clear; close all; clc
load('SFrainfall.dat');
days=SFrainfall(:,1);  % # of rainy days in a season
rain=SFrainfall(:,2);  % seasonal rainfall

cov_rain = cov(rain); % Caclulate coefficient of variation of
rainy days

nTotal = length(days); % number of samples
nE1 = sum(days>60);    % number of days > 60
```

```
Pr_E1 = nE1/nTotal;     % Pr(E1)

nE2 = sum(rain>20);     % number of rainfall > 20
Pr_E2 = nE2/nTotal;     % Pr(E2)

Pr_E1E2 = Pr_E1 * Pr_E2  % intersection Pr(E1∩E2)

Pr_E1_E2 = Pr_E1 + Pr_E2 % union Pr(E1UE2)
```

Question: The above codes contain multiple errors. Circle the lines of codes that contain errors, and write the corrected line of codes here [30 points]

Solution:
For both versions, errors are:
1. cov_rain
2. Pr_E1E2
3. Pr_E1_E2

Correction:
1.
mean_rain = mean(rain);
std_rain = std(rain);
cov_rain = std_rain ./ mean_rain

2.
E1E2 = sum(rain > 20 & days > 60)
Pr_E1E2 = E1E2/nTotal;

3.
Pr_E1_E2 = Pr_E1 + Pr_E2 – Pr_E1E2