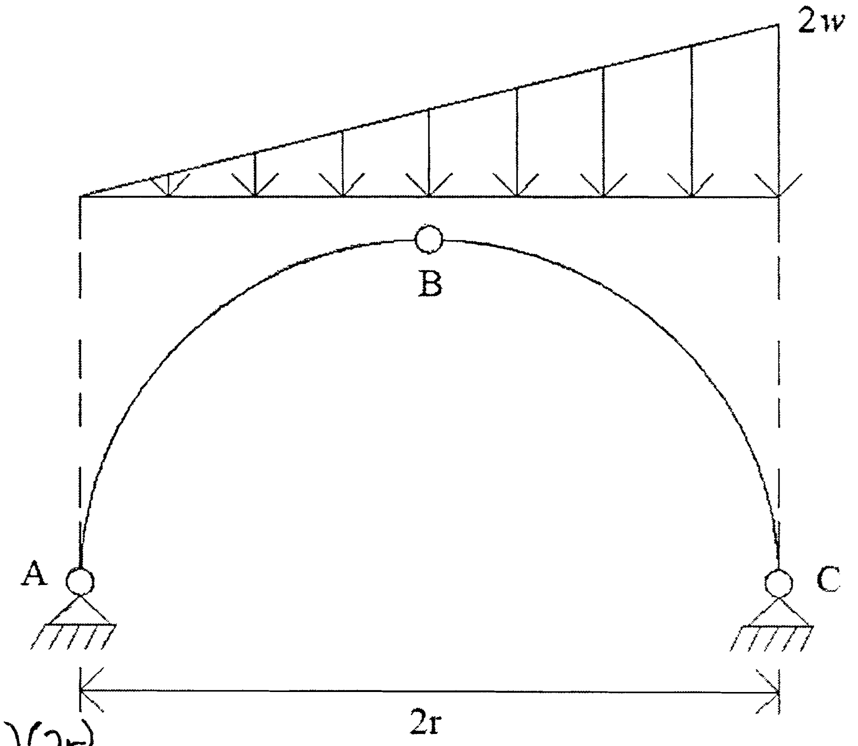
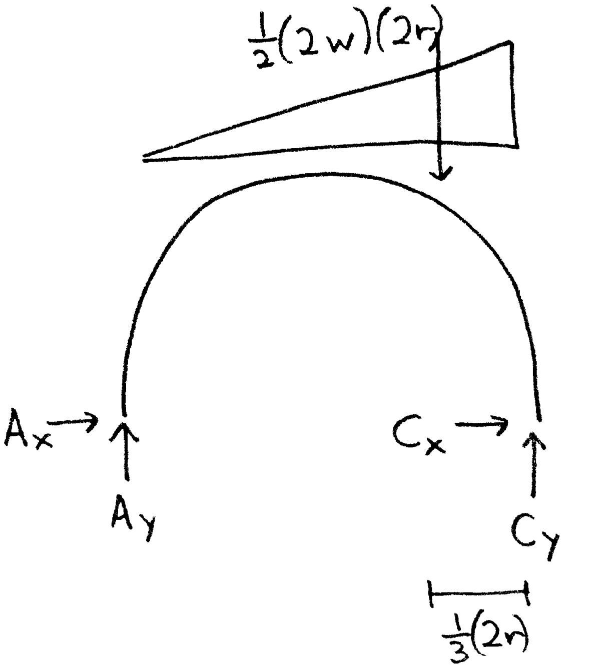
1. A weightless half-circle three-hinged arch of radius r is subjected to a vertical distributed load as shown. Hinge B is at the crest of the arch. **Calculate the force components at the three hinges. Put the results in the spaces below**. Indicate the directions of the reaction components at A and C and the force components at Hinge B acting on the left segment AB.

(Note switch)

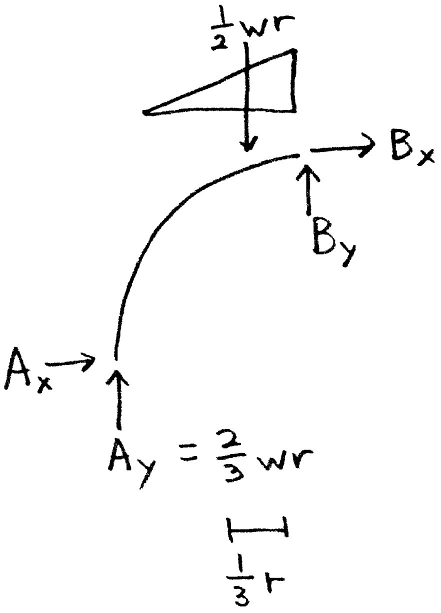
A_y $\frac{2}{3}wr \uparrow$	B_x $\frac{1}{2}wr \leftarrow$	C_x $\frac{1}{2}wr \leftarrow$
A_x $\frac{1}{2}wr \rightarrow$	B_y $\frac{1}{6}wr \downarrow$	C_y $\frac{4}{3}wr \uparrow$





$$\Sigma M_{C\curvearrowleft +} = 0 = -A_y(2r) + \tfrac{1}{2}(2w)(2r)\left(\tfrac{1}{3}\right)(2r)$$

$$\Rightarrow A_y = \tfrac{2}{3}wr \uparrow$$



HALF ARCH

$$\Sigma M_B = 0 = -\underbrace{\tfrac{2}{3}wr}_{A_y}(r) + A_x(r) + \tfrac{1}{2}wr\left(\tfrac{1}{3}\right)r$$

$$\Rightarrow A_x = \tfrac{1}{2}wr \rightarrow$$

$$\Sigma F_y = 0 = A_y + B_y - \tfrac{1}{2}wr, \quad A_y = \tfrac{2}{3}wr$$

$$\Rightarrow B_y = \tfrac{1}{6}wr \downarrow$$

$$\Sigma F_x = 0 = A_x + B_x, \quad A_x = \tfrac{1}{2}wr$$

$$\Rightarrow B_x = \tfrac{1}{2}wr \leftarrow$$

WHOLE ARCH

$$\Sigma F_x = 0 = A_x + C_x, \quad A_x = \tfrac{1}{2}wr$$

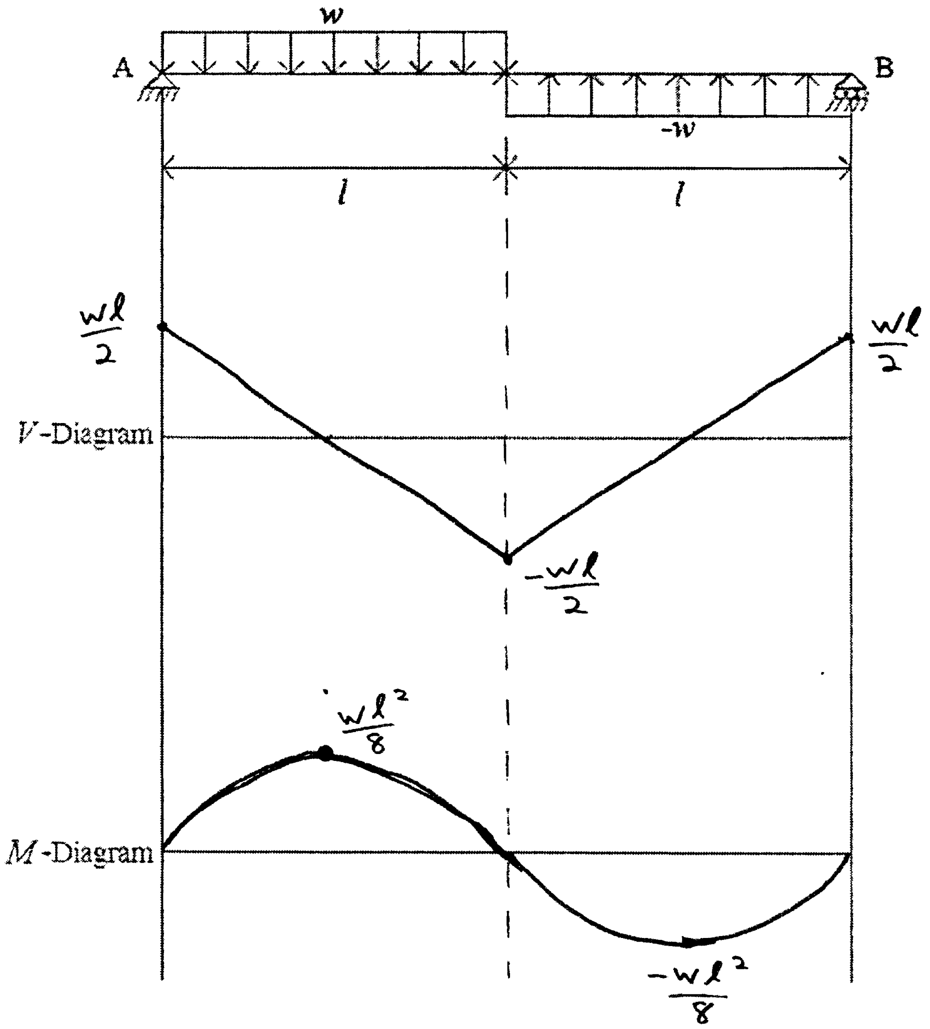
$$\Rightarrow C_x = \tfrac{1}{2}wr \leftarrow$$

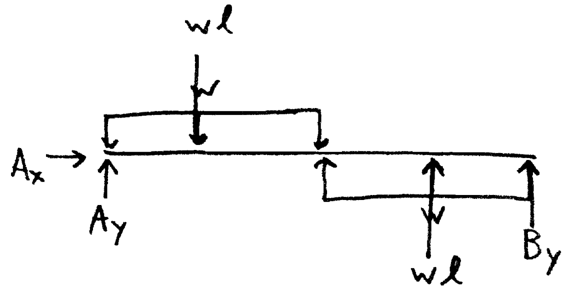
$$\Sigma F_y = 0 = A_y + C_y - \tfrac{1}{2}(2w)(2r), \quad A_y = \tfrac{2}{3}wr$$

$$\Rightarrow C_y = \tfrac{4}{3}wr \uparrow$$

2. A weightless beam of length $2l$ is subjected to a distributed vertical load. The loading on the left half of the beam is a uniform downward load of w per unit length, while the right half is subjected to a uniform upward load of w per unit length as shown in the figure. **Calculate the reaction forces and draw the shear and moment diagrams** and **fill in the values** of the reactions and the maximum moment and shear in the space below:

A_y $\frac{1}{2}wl \uparrow$ B_y $\frac{1}{2}wl \downarrow$ M_{max} $\frac{1}{8}wl^2$ V_{max} $\frac{1}{2}wl$





$$\Sigma F_x = 0 = A_x$$

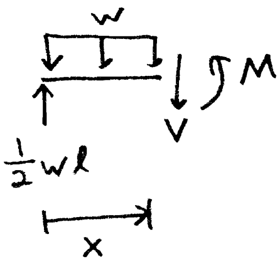
$$\Sigma M_{A\circlearrowleft +} = 0 = -wl\left(\frac{l}{2}\right) + wl\left(\frac{3}{2}l\right) + B_y(2l)$$

$$\Rightarrow B_y = \frac{1}{2}wl \downarrow$$

$$\Sigma F_y = 0 = A_y - \underbrace{\frac{1}{2}wl}_{B_y} - wl + wl$$

$$\Rightarrow A_y = \frac{1}{2}wl \uparrow$$

From A →



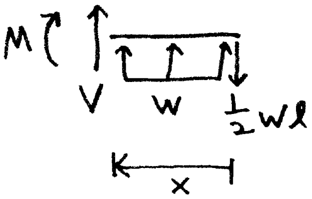
$$\Sigma F_y = 0 = A_y - wx - V \qquad A_y = \tfrac{1}{2}wl$$

$$\Rightarrow V = \tfrac{1}{2}wl - wx$$

$$\Sigma M_{x} = 0 = M + wx\left(\tfrac{x}{2}\right) - A_y x \qquad A_y = \tfrac{1}{2}wl$$

$$\Rightarrow M = \tfrac{1}{2}wlx - \tfrac{1}{2}wx^2$$

From B ←



$$\Sigma F_y = 0 = B_y + wx + V \qquad B_y = -\tfrac{1}{2}wl$$

$$\Rightarrow V = \tfrac{1}{2}wl - wx$$

$$\Sigma M_{x} = 0 = -M + wx\left(\tfrac{x}{2}\right) + B_y x \qquad B_y = -\tfrac{1}{2}wl$$

$$\Rightarrow M = \tfrac{1}{2}wx^2 - \tfrac{1}{2}wlx$$

→ M_{max} when $V = 0$

$$0 = \tfrac{1}{2}wl - wx$$

$$\Rightarrow x = l/2$$

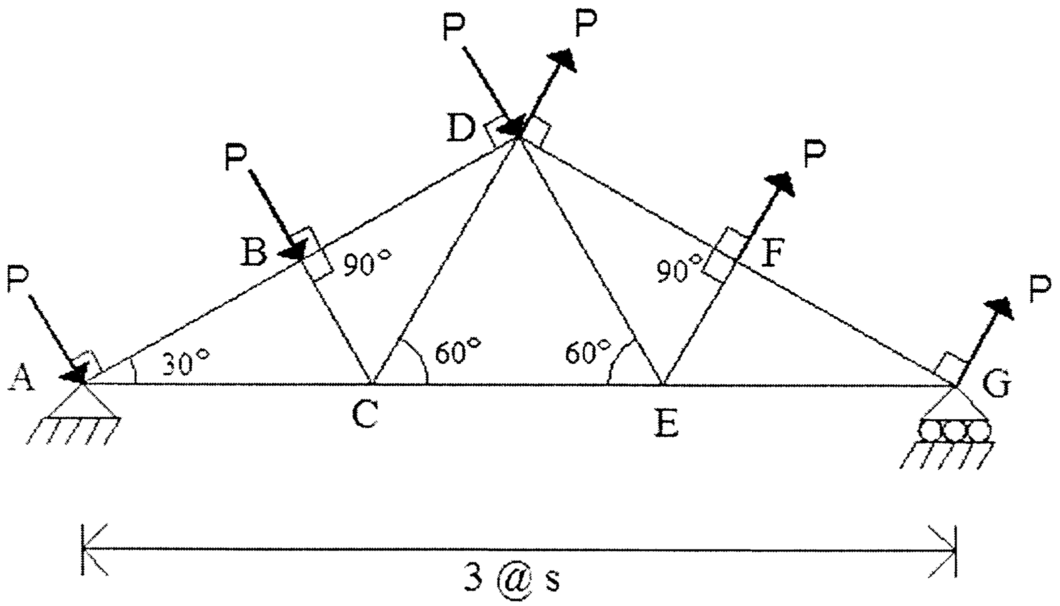
$$\Rightarrow M_{max} = \tfrac{1}{8}wl^2$$

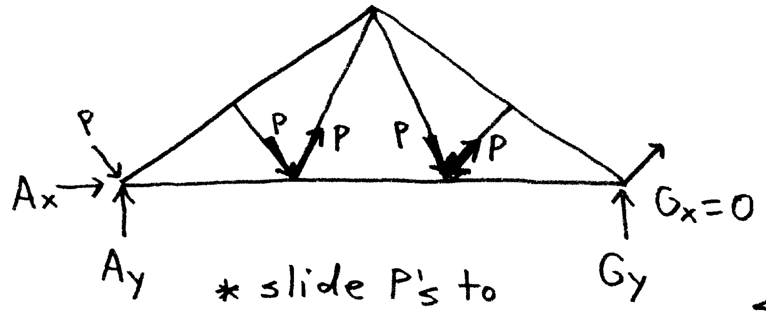
→ V_{max} at supports

$$\Rightarrow V_{max} = \tfrac{1}{2}wl$$

3. A roof truss is subjected to a loading system as indicated in the drawing. **Calculate reaction force components at Supports A and G and the forces in the 6 left truss members listed below.** Put the results in the table. Indicate the directions of the reaction components and also indicate tension (+) or compression (-) for the truss forces.

A_x	$3P \leftarrow$	AB	0	CD	$+P$
A_y	$\sqrt{3}/2\, P \uparrow$	AC	$+2.5P$	CE	$+1.5P$
G_x	0	BD	0	---	[illegible]
G_Y	$\sqrt{3}/2\, P \downarrow$	BC	$-P$	---	





* slide P's to bottom truss elements

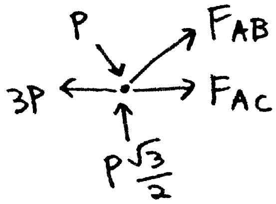
$$\Sigma M_{A} = 0 = G_y(3s) + P\left(\frac{\sqrt{3}}{2}\right)(3s) \quad (\sin 60)$$

$$\rightarrow G_y = P\frac{\sqrt{3}}{2} \downarrow$$

$$\Sigma F_y = 0 = A_y - P\frac{\sqrt{3}}{2} \quad (= G_y) \Rightarrow A_y = P\frac{\sqrt{3}}{2} \uparrow$$

$$\Sigma F_x = 0 = A_x + 6P\left(\tfrac{1}{2}\right) \quad (\cos 60) \Rightarrow A_x = 3P \leftarrow$$

A



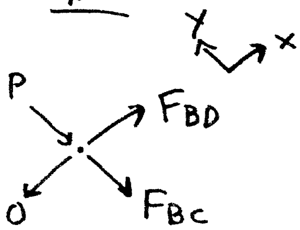
$$\Sigma F_y = 0 = \tfrac{1}{2} F_{AB} + P\tfrac{\sqrt{3}}{2} - P\tfrac{\sqrt{3}}{2}$$

$$\Rightarrow F_{AB} = 0$$

$$\Sigma F_x = 0 = F_{AC} + \tfrac{1}{2}P - 3P$$

$$\Rightarrow F_{AC} = +2.5P$$

B



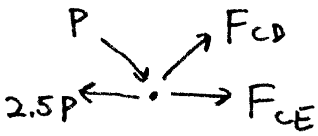
$$\Sigma F_x = 0 = F_{BD}$$

$$\Rightarrow F_{BD} = 0$$

$$\Sigma F_y = 0 = -P - F_{BC}$$

$$\Rightarrow F_{BC} = -P$$

C



$$\Sigma F_y = 0 = F_{CD}\left(\tfrac{\sqrt{3}}{2}\right) - P\tfrac{\sqrt{3}}{2}$$

$$\Rightarrow F_{CD} = +P$$

$$\Sigma F_x = 0 = F_{CE} - 2.5P + P\left(\tfrac{1}{2}\right) + P\left(\tfrac{1}{2}\right)$$

$$\Rightarrow F_{CE} = +1.5P$$