Physics 7C Section 2
Spring ~~2004~~ **Final**, May 19, 2005
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Choose one out of the two problems in both section A and B and two out of the three problem in section C. The test duration is 170 minutes.

A 1. Consider the interface between vacuum and a gas of refractive index n. i) Use the Huygens principle to derive the Snell's law for light reaching the interface at an angle θ_0 and emerging at an angle θ_1. ii) Determine the value of the refractive index n, such that light propagates inside the gas at the same speed as a muon ($M_\mu = 105$ MeV/c^2) of total energy $E_\mu = 150$ MeV and suggest a material to be used.

A 2. Consider a model for the photoelectric effect based on Maxwell classical theory of light. An experiment is conducted with Na, for which the maximum wavelength necessary to extract an electron using single photons is 683 nm. When the light of a 830 nm IR laser with 15 mW of power on a 1 mm^2 spot is used which would be the time delay before electrons are emitted ?

B 1. An experiment is aiming at the production of a new particle Z' with mass = 1000 GeV/c^2 using electron and positrons collisions through the reaction $e^-e^+ \to Z' \to e^+e^-$. Find the positron energy necessary for producing the Z' particle i) if the positrons collide on electrons at rest in the lab frame, ii) if the positrons collide head-on with electrons of the same energy. iii) In both cases, find the minimum positron energy necessary for the outgoing electron and positron momenta to exceed 100 GeV/c. iv) Determine the radius of curvature of the 100 GeV/c e^- and e^+ trajectories, when a $B = 3$ T solenoidal magnetic field is applied.

B 2. The energy spectrum of cosmic rays has an upper bound, due to their scattering against the Cosmic Microwave Backgrond photons, known as Greisen-Zatsepin-Kuzmin (GZK) cut-off. Using a simple model, where the scattering is due to head-on collision and the cutoff is defined by the reaction $p\gamma \to p\pi$, ($M_p = 938$ MeV/c^2, $M_\pi = 139$ MeV/c^2, $E_\gamma = kT = 3 \times 10^{-10}$ MeV) becoming kinematically allowed, estimate the value of the GZK cut-off energy.

C 1. Consider an electron, e^-, in a 1D, infinite potential well of length L. Due to the uncertainty principle its velocity v is non-zero, if L is finite. Find the value of L for which the electron would become relativistic, taking this to correspond to the β value at which its relativistic mass exceeds the rest mass, $M_e = 0.511$ MeV/c^2, by more than 5 %.

C 2. An electron, e^-, rotating around an anti-electron, or positron e^+, forms a bound system, called Positronium. i) Write the potential $U(r)$ felt by the electron and its radial Schrodinger equation, ii) write the positronium 1S wave function and iii) find the most probable value for the electron radius and compare it to the case of H. (Hints: since the mass of the "nucleus" here is equal to that of the electron $M_e = 0.511$ MeV/c^2, its energy must be taken into account. In the case of hydrogen $C_{100} = \frac{1}{\sqrt{\pi}} a_0^{-3/2}$, where $a_0 = \frac{\hbar^2}{m_e k e^2}$=0.053 nm is the Bohr radius. How is this modified in positronium ? Remember that the solution of the radial Schrodinger equation for ℓ=0 is a function of the kind Ce^{-ar}, where a is a coefficient to be determined.)

C 3. A radioactive isotope decays by α (= ${}^4_2He$) emission due to quantum tunneling of the binding potential barrier. i) Assuming that the potential felt by the α particle is due to the Coulomb potential induced by its interaction with the reduced nucleus (Z-2) and that their distance is $r = 9$ fm, compute the height of the barrier. ii) Knowing that the α particle is emitted with an energy E_α=9 MeV, find its tunnelling probability by approximating the potential with a step potential of the same height as found at i) and 10 fm width. iii) Estimate the time needed to the isotope to decay, assuming that the α hits the potential step with a frequency depending on its (non-relativistic) velocity. (Z=100)