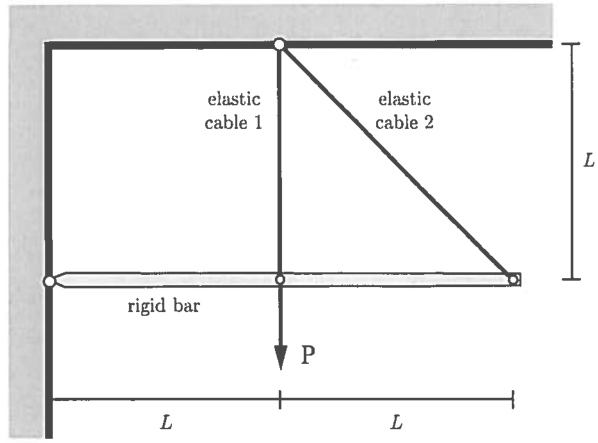
Problem #1 (40%)

A rigid bar hangs from two cables as shown in the figure. The cables can be considered linear elastic with an equal Young modulus E and have a cross section area A. All connections are pinned, and all members can be considered weightless.

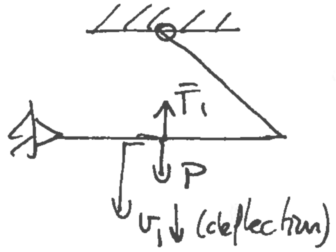
If the vertical load P shown in the figure is applied, determine: (1) the force in the cables, and (2) the deflection of the bar at its right tip.



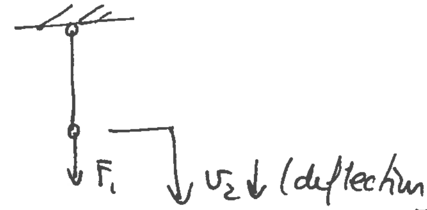
Statically indeterminate $\Rightarrow$ Force method (3 steps)
(degree of indeterminacy = 1)

$\boxed{\text{STEP 1}}$ Release the system, e.g. disconnect cable #1, leaving the force

Problem #1



Problem #2



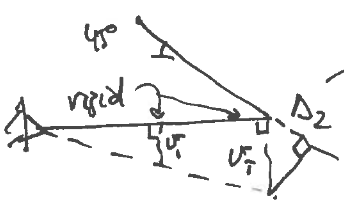
$\boxed{\text{STEP 2}}$ Solve for $v_1 \downarrow$ and $v_2 \downarrow$ in terms of P and F_1

Problem #2: $\downarrow v_2 = \Delta_1 = F_1 f_1$

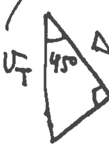
flexibility of cable #1: $f_1 = \frac{L}{EA}$

Problem #1

Kinematics



vertical deflection at the tip

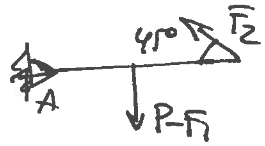


$\Delta_2 \rightarrow$ stretching of cable #2

$v_1 = \frac{1}{2} v_T$ (similar triangles)
v_T $\rightarrow$ deflection at the tip

$\downarrow v_T = \frac{\Delta_2}{\cos 45°} = \sqrt{2}\,\Delta_2$

Statics



$(\Sigma M_A = 0) \Rightarrow F_2 \sin 45° \cdot 2L = (P-F_1)\cdot L$

$\Rightarrow F_2 = \frac{1}{\sqrt{2}}(P-F_1)$

$\Rightarrow v_1 = \frac{1}{2} v_T = \frac{\sqrt{2}}{2}\Delta_2 = \frac{\sqrt{2}}{2} F_2 f_2$

$\downarrow v_1 = \frac{1}{2}(P-F_1) f_2$

Δ_2 = stretching cable #2 $= F_2 f_2$

f_2 = flexibility cable #2 $= \frac{\sqrt{2}L}{EA}$

Relax ...

$\boxed{\text{STEP 3}}$ Impose back the compatibility constraint

$$\downarrow v_1 = \downarrow v_2$$

$$\Rightarrow \quad \frac{1}{2}(P - F_1)\underbrace{f_2}_{\frac{\sqrt{2}L}{EA}} = F_1 \underbrace{f_1}_{\frac{L}{EA}} \quad \Longrightarrow \quad F_1 = \frac{P}{1+\sqrt{2}}$$

From Problem #1 above

(statics) $F_2 = \frac{1}{\sqrt{2}}(P - F_1) = \frac{P}{1+\sqrt{2}} \quad (= F_1)$

and deflection at the tip

(kinematics) $v_T = \sqrt{2}\, F_2 \underbrace{f_2}_{\frac{\sqrt{2}L}{EA}} = 2F_2 \frac{L}{EA} = \frac{2}{1+\sqrt{2}} \frac{PL}{EA}$

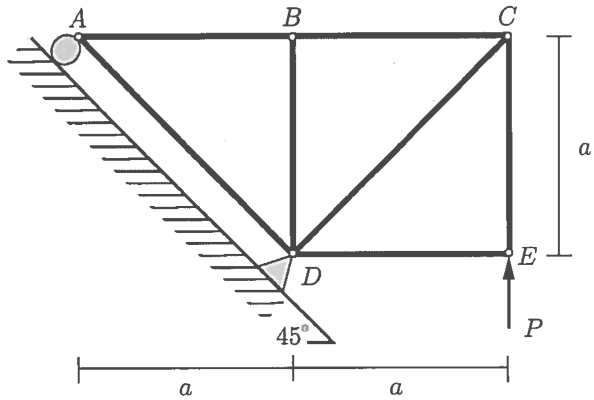
SUMMARY:

$$F_1 = \frac{P}{1+\sqrt{2}}$$

$$F_2 = \frac{P}{1+\sqrt{2}}$$

$$v_T = \frac{2}{1+\sqrt{2}} \frac{PL}{EA}$$

Problem #2 (25%)

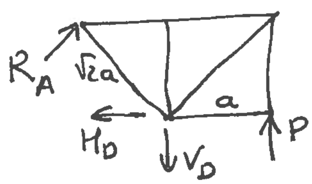


1. Determine the forces in all the members in the truss of the figure when the vertical load of value P shown in the figure is applied. Indicate clearly if the member is in tension or compression.

2. If all the members have the same $0.1 \times 0.1\ m^2$ square cross section, determine the maximum load value P that can be applied with a factor of safety of 1.5 if the material can only take $10MPa$ in tension or compression.

Remark: Express your results in terms of the length a if needed.

Reactions:



$(\sum M_D = 0) \Rightarrow R_A\sqrt{2}a = Pa \Rightarrow R_A = \frac{P}{\sqrt{2}}$

$(\sum F_{Hor} = 0) \Rightarrow H_D = R\cos 45 \Rightarrow H_D = \frac{P}{2}$

$(\sum F_{vert} = 0) \Rightarrow V_D = R\sin 45 + P \Rightarrow V_D = \frac{3P}{2}$

Part 1 Zero-force members: $F_{BD} = F_{DE} = 0$

① Joint E F_{EC}, 0, P $\Rightarrow F_{EC} = -P$

② Joint C: F_{BC}, F_{CD}, F_{EC}

$F_{CD}\cos 45 + F_{EC} = 0 \Rightarrow F_{CD} = -\sqrt{2}F_{EC}$

$\Rightarrow F_{CD} = P\sqrt{2}$

$F_{CD}\sin 45 + F_{BC} = 0 \rightarrow F_{BC} = -P$

③ Joint B F_{AB}, 0, F_{BC} $F_{AB} = F_{BC} = -P$

④ Joint A R_A, F_{AB}, F_{AD}

$F_{AD}\cos 45 - R_A\sin 45 = 0 \Rightarrow F_{AD} = R_A = \frac{P}{\sqrt{2}}$

$\underbrace{F_{AD}\sin 45}_{P/2} + \underbrace{R_A\cos 45}_{P/2} + \underbrace{F_{AB}}_{-P} = 0$ ✓

SUMMARY:

$F_{BD} = F_{DE} = 0$

$F_{AB} = -P$ (compression)

$F_{BC} = -P$ (compression)

$F_{AD} = P/\sqrt{2}$ (tension)

$F_{CD} = P\sqrt{2}$ (tension)

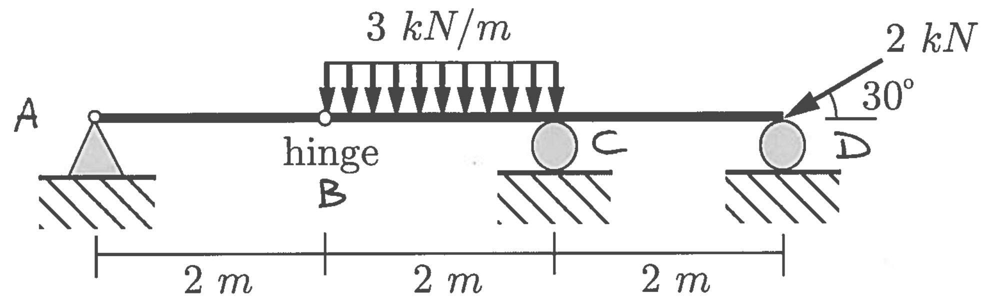
$F_{EC} = -P$ (compression)

Part 2 Maximum force in all members $= \sqrt{2}P$

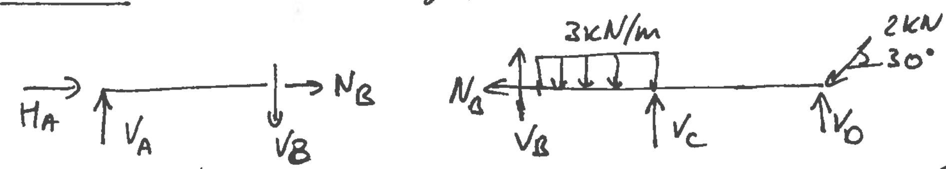
$$\Rightarrow \frac{\sqrt{2}P}{\underbrace{A}_{0.1\cdot 0.1 = 10^{-2}m^2}} \le \frac{\overbrace{\sigma_{max}}^{=10MPa}}{\underbrace{FS}_{1.5}} \Longrightarrow \boxed{P_{max} = 47.14\,kN}$$

Problem #3 (35%)

Draw the axial force, transversal shear force and bending moment diagrams for the beam shown in the figure. Indicate the characteristic values (min/max values, values at the ends and supports, slopes, linear/parabolic/cubic distributions,...).



Reactions: (cut at the hinge, no moment)



$V_A = V_B = 0$

$H_A = -N_B$

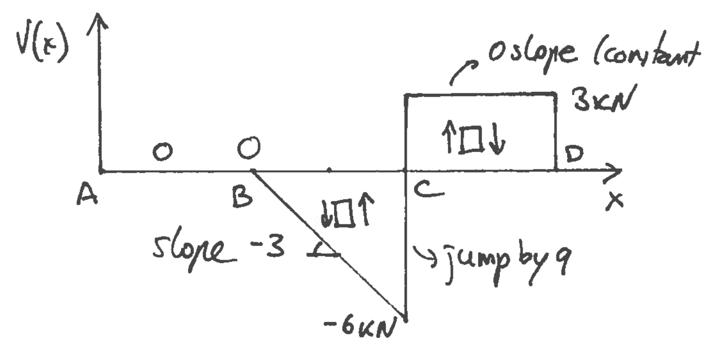
$(\Sigma F_{horiz} = 0)$ $N_B + 2\cos 30 = 0 \Rightarrow N_B = -\sqrt{3}\ kN$ <0 opposite than assumed

$(\Sigma M_C = 0)$ $\underset{0}{V_B} \cdot 2 - V_D \cdot 2 - 3 \cdot 2 \cdot 1 + 2\sin 30 \cdot 2 = 0 \Rightarrow V_D = -2\ kN$

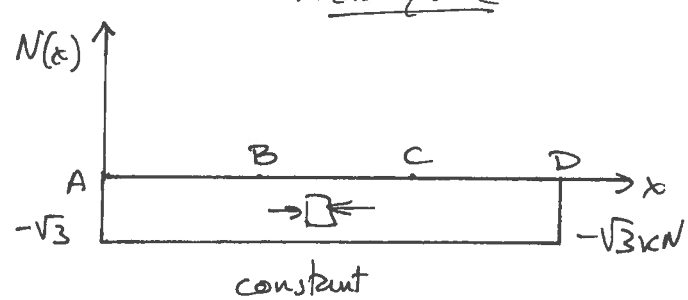
$(\Sigma M_D = 0)$ $V_B \cdot 4 + 2 \cdot V_C - 3 \cdot 2 \cdot 3 = 0 \Rightarrow V_C = 9\ kN$

Diagrams

Transverse shear force



Axial force



Bending moment