Physics 7B Midterm 1 Solutions - Fall 2018
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Problem 1

(a) Let us first calculate the time of heating. From the definition of power, and using the formula relating heat and change in temperature , we have

$$Q = (\text{power})t = mC\Delta T \tag{1}$$

so that

$$t = \frac{mC\Delta T}{(\text{power})} = \frac{4 \cdot 1000 \cdot 100}{400}\ \text{s} \tag{2}$$
$$= 1000\ \text{s} \tag{3}$$

Now for the coefficient of linear expansion, we apply the linear expansion formula directly and see that

$$\alpha = \frac{\Delta L}{L_0 \Delta T} = \frac{0.08}{20 \cdot 100}\,(^\circ C)^{-1} \tag{4}$$
$$= 4 \cdot 10^{-5}\,(^\circ C)^{-1} \tag{5}$$

(b) (i) After a temperature change ΔT, we need the washer and screw radii to be equal:

$$a_0(1 + \alpha_w \Delta T) = r_0(1 + \alpha_s \Delta T) \tag{6}$$
$$\Delta T = \frac{r_0 - a_0}{a_0\alpha_w - r_0\alpha_s} \tag{7}$$

Then the amount of heat we need is

$$Q = (m_w c_w + m_s c_s)\Delta T \tag{8}$$
$$= (m_w c_w + m_s c_s)\left(\frac{r_0 - a_0}{a_0\alpha_w - r_0\alpha_s}\right) \tag{9}$$

Problem 2

(a) If we look at the motion of a gas particle moving only in the x-direction, the time between collisions is given by

$$\Delta t = \frac{2L}{v_x} \tag{10}$$

(b) The average force F_x on one of the walls considered in part (a) is given by

$$F_x = \frac{\Delta \bar{p}_x}{\Delta t} = \frac{2\bar{p}_x \bar{v}_x}{2L} = \frac{m\bar{v_x^2}}{L} \tag{11}$$

using the fact that the directions are all isotropic, we have $\bar{v^2} = \bar{v_x^2} + \bar{v_y^2} + \bar{v_z^2} = 3\bar{v_x^2}$, so

$$F = \Sigma F_x = \frac{mN\bar{v^2}}{3L} \tag{12}$$

(c) Using the equipartition theorem, we know that

$$K = \frac{3}{2} k_B T \tag{13}$$

since each gas particle has 3 degrees of freedom. Using $K = \frac{1}{2} m \bar{v^2}$,

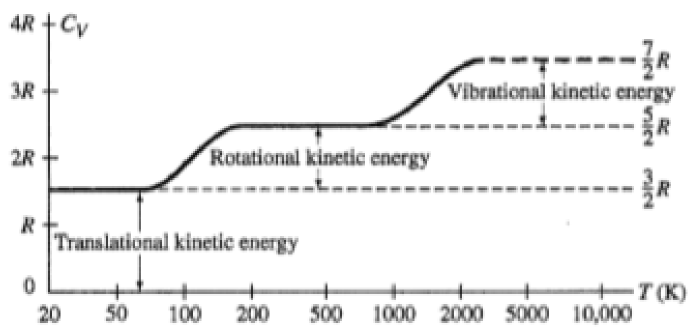
$$\bar{v^2} = 3 \frac{k_B T}{m} \tag{14}$$

so that

$$F = \frac{N k_B T}{L} \tag{15}$$

or, equivalently,

$$\frac{F}{L^2} L^3 = PV = N k_B T \tag{16}$$



(d) The plateaus in the figure above occur when various degrees of freedom of the diatomic molecule get unfrozen. Starting from the left, at low temperatures the molecule has only translational degrees of freedom, so $C_V = \frac{3}{2}R$. As the temperature increases, the rotational degrees of freedom are unfrozen so that $C_V = \frac{3+2}{2}R = \frac{5}{2}R$. Finally, as the temperature cantinues to increase, the vibrational degrees of freedom enter and we have $C_V = \frac{5+2}{2}R = \frac{7}{2}R$

Problem 3

(a) The energy released to bring the water from 25 °C to 0 °C is

$$m_w c_w (25°C - 0°C) = (3.0kg)(4.0kJ/kg \cdot° C)(25°C) = 300kJ \tag{17}$$

The energy needed to bring the ice from -10 °C to 0 °C is

$$m_{ice} c_{ice} (10°C) = (0.50kg)(2.0kJ/kg \cdot° C)(10°C) = 10kJ \tag{18}$$

Finally, the energy needed to change the ice to water at 0 °C is

$$m_{ice} L_f = (0.5kg)(300kJ/kg) = 150kJ \tag{19}$$

So the total energy to change all the initial ice to water is $10kJ + 150kJ = 160kJ < 300kJ$ To find the final temperature, we use conservation of energy:

$$160kJ + (0.5kg)(4.0kJ/kg \cdot^\circ C)(T - 0^\circ C) = (3.0kg)(4.0kJ/kg \cdot^\circ C)(25^\circ C - T) \quad (20)$$

$$T = 10^\circ C \quad (21)$$

(b) We calculate the entropy in each step of the process:

$$\text{Heating the Ice: } \Delta S_{heating} = m_{ice} c_{ice} \ln\left(\frac{T_0}{T_i}\right) = \ln\left(\frac{273}{263}\right) kJ/K \quad (22)$$

$$\text{Melting Ice: } \delta S_{melt} = \frac{m_{ice} L_f}{T_0} = \frac{150}{273} kJ/K \quad (23)$$

$$\text{Warming Water from Melted Ice: } \Delta S_{heating2} = m_{ice} c_{water} \ln\left(\frac{T_f}{T_0}\right) = 2\ln\left(\frac{283}{273}\right) kJ/K \quad (24)$$

$$\text{Cooling original water: } \Delta S_{cool} = m_w c_{water} \ln\left(\frac{T_f}{T_w}\right) = 12\ln\left(\frac{283}{298}\right) kJ/K \quad (25)$$

Problem 4 For the temperature of the joint to be fixed, we need

$$\frac{dQ}{dt}|_{Cu} = \frac{dQ}{dt}|_{Al} \quad (26)$$

$$k_{Cu} A \frac{T_H - T_M}{l} = k_{Al} A \frac{T_M - T_L}{l} \quad (27)$$

$$T_M = \frac{k_{Cu} T_H + k_A T_L}{k_{Cu} + k_{Al}} \quad (28)$$

$$(29)$$

Problem 5

(a) We have

(b) We find

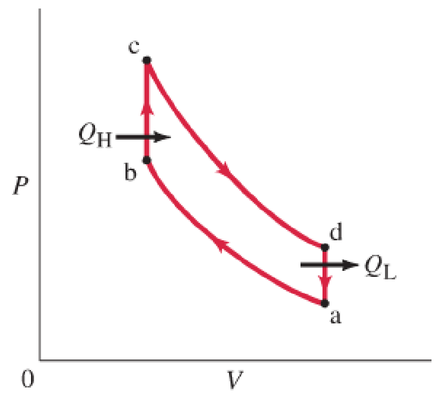
$$Q_{da} = nC_v(T_d - T_a) \quad (30)$$

$$Q_{bc} = nC_v(T_c - T_b) \quad (31)$$

(c) Using dS = dQ/T, we get

$$\Delta S_{da} = nC_v \ln(T_a/T_d) \quad (32)$$

$$\Delta S_{bc} = nC_v \ln(T_c/T_b) \quad (33)$$



(d) The effiency is given by

$$e = 1 - \left(\frac{V_b}{V_a}\right)^{\gamma-1} \tag{34}$$

$$= 1 - \left(\frac{1}{8}\right)^{2/5} \tag{35}$$