Problem 1 (20 pts.)

(a) We use the linear expansion formula

$$\Delta d = \alpha d \Delta T \tag{1}$$
$$4 \cdot 10^{-3}\ \text{cm} = (20 \cdot 10^{-6} (^\circ C)^{-1}) \Delta T \tag{2}$$
$$\Delta T = 50^\circ C \tag{3}$$

Taking room temperature to be about 25 $^\circ C$, we need a final temperature of $75^\circ C$

(b) We use the volume expansion formula

$$\frac{V'}{V} = 1 + 3\alpha \Delta T \tag{4}$$
$$= 1 + 3 \cdot 10^{-3} \tag{5}$$

Problem 2

(a) If we look at the motion of a gas particle moving only in the x-direction, the time between collisions is given by

$$\Delta t = \frac{2L}{v_x} \tag{6}$$

(b) The average force F_x on one of the walls considered in part (a) is given by

$$F_x = \frac{\Delta \bar{p}_x}{\Delta t} = \frac{2\bar{p}_x \bar{v}_x}{2L} = \frac{m\bar{v_x^2}}{L} \tag{7}$$

using the fact that the directions are all isotropic, we have $\bar{v^2} = \bar{v_x^2} + \bar{v_y^2} = 2\bar{v_x^2}$, so

$$F = \Sigma F_x = \frac{mN\bar{v^2}}{2L} \tag{8}$$

(c) Using the equipartition theorem, we know that

$$K = \frac{2}{2} k_B T \tag{9}$$

since each gas particle has 3 degrees of freedom. Using $K = \frac{1}{2} m\bar{v^2}$,

$$\bar{v^2} = 2\frac{k_B T}{m} \tag{10}$$

so that

$$F = \frac{N k_B T}{L} \tag{11}$$

or, equivalently,

$$\frac{F}{L} L^2 = P^* A = N k_B T \tag{12}$$

Problem 3

(a) First, we note that

$$P_a V_a = P_b V_b \tag{13}$$
$$P_c V_c^\gamma = P_b V_b^\gamma \tag{14}$$

so that

$$V_b = \left(\frac{T_L}{T_H}\right)^{\frac{1}{\gamma-1}} V_C \tag{15}$$
$$P_b = \frac{nRT_H}{V_b} \tag{16}$$

$$W_{ab} = nRT_H \ln\left(\frac{V_b}{V_a}\right) \tag{17}$$
$$W_{cd} = nRT_L \ln\left(\frac{V_d}{V_c}\right) \tag{18}$$
$$W_{bc} = \frac{3}{2}nR(T_H - T_L) \tag{19}$$
$$Q_{ab} = W_{ab} = nRT_H \ln\left(\frac{V_b}{V_a}\right) \tag{20}$$
$$Q_{cd} = W_{cd} = nRT_L \ln\left(\frac{V_d}{V_c}\right) \tag{21}$$
$$Q_{bc} = 0 \tag{22}$$
$$Q_{da} = \Delta E = \frac{3}{2}nR(T_H - T_L) \tag{23}$$

(b) The efficiency is

$$e = \frac{W}{Q_{in}} \tag{24}$$
$$= 1 - \frac{Q_{out}}{Q_{in}} \tag{25}$$
$$= 1 - \frac{T_L}{T_H} \frac{\ln\left(\frac{V_d}{V_c}\right)}{\frac{3}{2}\left(1 - \frac{T_L}{T_H}\right) + \ln\left(\frac{V_b}{V_a}\right)} \tag{26}$$

This efficiency must be less than the Carnot efficiency

(c)

$$\Delta S_{da} = \frac{3}{2} nR \ln\left(\frac{T_H}{T_L}\right) \tag{27}$$

$$\Delta S_{ab} = nR \ln\left(\frac{V_b}{V_a}\right) \tag{28}$$

$$\Delta S_{bc} = 0 \tag{29}$$

$$\Delta S_{cd} = nR \ln\left(\frac{V_d}{V_c}\right) \tag{30}$$

Problem 4

(a) For free expansion, we have

$$PV = P_f V_f \tag{31}$$

$$P_f = P\frac{V}{V_f} \tag{32}$$

For adiabatic expansion,

$$PV^\gamma = P_f V_f^\gamma \tag{33}$$

$$P_f = P\left(\frac{V}{V_f}\right)^\gamma \tag{34}$$

Since $\gamma > 1$, we see that free expansion leads to a larger volume

(b) Free expansion:

$$W = 0 \tag{35}$$

Adiabatic:

$$W = \int PdV = PV^\gamma \int \frac{dV'}{V'^\gamma} \tag{36}$$

$$= PV^\gamma \frac{V_f^{1-\gamma} - V^{1-\gamma}}{1-\gamma} \tag{37}$$

(c) Free expansion:

$$\Delta U = 0 \tag{38}$$

Adiabatic:

$$\Delta U = -\int dW = -PV^\gamma \frac{V_f^{1-\gamma} - V^{1-\gamma}}{1-\gamma} \tag{39}$$

(d) Free expansion:

$$\Delta S = nR \ln\left(\frac{V_f}{V_i}\right) \tag{40}$$

Adiabatic:

$$\Delta S = 0 \tag{41}$$

Problem 5

(a) Since the ice does not melt, the heat enetering the middle region must be equal to the heat leaving the mieddle region. Thus we need

$$\frac{Q_{wood}}{\Delta t} = \frac{Q_{glass}}{\Delta t} \tag{42}$$

$$\frac{k_w A}{l}(T_B - T_{water}) = \frac{k_g A}{l}(T_{water} - T_L) \tag{43}$$

$$T_B = -\frac{k_g}{k_w} T_L \tag{44}$$

where we have used $T_{water} = 0°\text{C}$.

(b) We need the total heat delivered to the ice to be equal to the energy required to melt the ice:

$$Q_{wood} + Q_{glass} = m_{ice} L_{ice} \tag{45}$$

or

$$\frac{m_{ice} L_{ice}}{\Delta t} = \frac{Q_{wood} + Q_{glass}}{\Delta t} \tag{46}$$

$$= \frac{k_w A}{l} T_B + \frac{k_g A}{l} T_L \tag{47}$$

$$= -\frac{k_g A}{l} T_L \tag{48}$$

where we have used $T_B = -2\frac{k_g}{k_w} T_L$ for this problem. Thus we have

$$\Delta t = \frac{m_{ice} L_{ice} l}{k_g A T_L} \tag{49}$$