Pysics 7A - Midterm 1 Solution
Fall 2018 (Stahler)
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1. (a) The acceleration is $\mathbf{a}(t) = \dfrac{d\mathbf{v}(t)}{dt} = \boxed{\left(2\alpha t, \beta, \dfrac{\gamma}{t_0} e^{t/t_0}\right).}$

   (b) The position is $\mathbf{r}(t) = \mathbf{r}(0) + \int_0^t \mathbf{v}(t')\, dt' = \boxed{(1, 0, -3) + \left[\dfrac{\alpha t^3}{3}, \dfrac{\beta t^2}{2}, \gamma t_0 \left(e^{t/t_0} - 1\right)\right].}$

2. (a) In the vertical direction, the ball moves with an initial velocity v_{0y} and experiences a downward acceleration g. If the ball enters the pipe horizontally, it's vertical speed must be zero when it reaches the height H:

$$0^2 = v_{0y}^2 - 2gH \Rightarrow v_{0y} = \boxed{\sqrt{2gH}.}$$

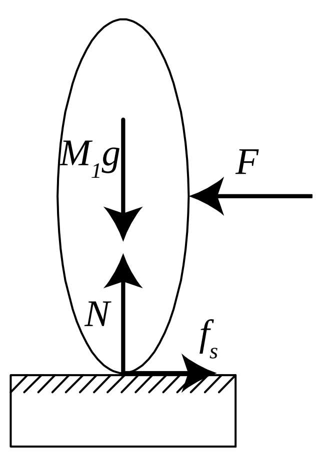
   (b) In the horizontal direction, the ball moves with a constant speed v_{0x}, so the time for the ball to reach the pipe is $t = D/v_{0x}$. During this time interval, the vertical velocity must decrease to zero:

$$0 = v_{0y} - gt = v_{0y} - \frac{gD}{v_{0x}} \Rightarrow v_{0y} = \boxed{\frac{gD}{v_{0x}}.}$$

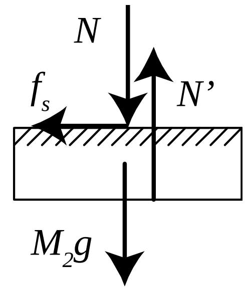
   (c) Equating the results of (a)(b), we obtain $v_{0x} = \boxed{\sqrt{\dfrac{gD^2}{2H}}.}$

   (d) From (a)(c), the angle θ_0 is given by $\theta_0 = \tan^{-1}(v_{0y}/v_{0x}) = \boxed{\tan^{-1}\left(\dfrac{2H}{D}\right).}$

3. (a) The free body diagram of the person is shown below. Here, F is the force exerted by the wall, $M_1 g$ the weight of the person, N the normal force exerted by the block, and f_s is the static friction force excerted by the surface of the block. Since the person has a tendency moving to the left, the friction force should point in the opposite (right) direction. Note that we only know $f_s \leq \mu_s N$, and its actual magnitude f_s is unknown.



(b) Similarly, the free body diagram of the block is shown below. The new force N' is the normal force exerted by the floor, and $M_2 g$ is the weight of the block.



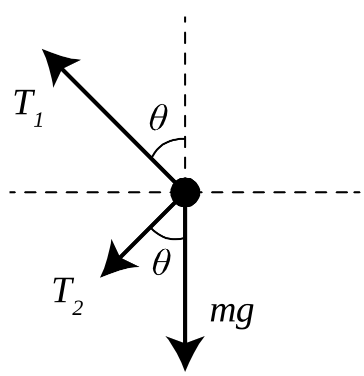
(c) Regard the person plus the block as a whole system with mass $M_1 + M_2$. The only horizontal external force acting on this system is $-F\hat{\mathbf{e}}_x$ from the wall, so the acceleration of the system is

$$\mathbf{a} = \boxed{-\frac{F}{M_1 + M_2}\hat{\mathbf{e}}_x.}$$

(d) The friction force f_s has to move the block M_2 with the acceleration a found in (c), so $f_s = M_2 F / (M_1 + M_2)$. In order for the man to not slip, this static friction force has to satisfy $f_s \leq \mu_s N$, where $N = M_1 g$ from the vertical force balance of the man. This implies

$$F \leq \boxed{\mu_s M_1 g \left(1 + \frac{M_1}{M_2}\right).}$$

4. (a) The free body diagram of the mass m is shown below.



(b) In the vertical direction, the mass is not moving, while in the horizontal direction the mass moves with an angular velocity ω and with a radius $L\sin\theta$, so

$$\begin{aligned} T_1\cos\theta - T_2\cos\theta &= mg, \\ T_1\sin\theta + T_2\sin\theta &= mL\sin\theta\omega^2. \end{aligned}$$

Solving the two equations for T_1 and T_2, we obtain

$$T_1 = \boxed{\frac{1}{2}m\left(L\omega^2 + g\sec\theta\right),}\; T_2 = \boxed{\frac{1}{2}m\left(L\omega^2 - g\sec\theta\right).}$$