University of California, Berkeley
Physics 110B Spring 2004 (*Strovink*)

EXAMINATION 3

Directions: Do all three problems, which have unequal weight. This is a closed-book closed-note exam except for Griffiths, Pedrotti, a copy of anything posted on the course web site, and anything in your own original handwriting (not Xeroxed). Calculators are not needed, but you may use one if you wish. Laptops and palmtops should be turned off. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Show all your work. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Express your answer in terms of the quantities specified in the problem. Box or circle your answer.

Problem 1. (30 points)
In a medium with (fixed) conductivity σ, the ratio of complex fields $\tilde{H}/\tilde{E}$ is given by the complex admittance

$$\tilde{Z}^{-1} = \frac{\tilde{H}}{\tilde{E}}$$
$$= \sqrt{\frac{\epsilon}{\mu}}\Big\{\Big[\frac{\sqrt{1+\beta^2}+1}{2}\Big]^{\frac{1}{2}} + i\Big[\frac{\sqrt{1+\beta^2}-1}{2}\Big]^{\frac{1}{2}}\Big\} ,$$

where $\beta \equiv \frac{\sigma}{\epsilon\omega}$, and the (fixed) dielectric constant ϵ and the (fixed) magnetic permeability μ are due to the effects of bound electrons.

At normal incidence at the interface between two dissimilar materials 1 and 2, the (complex) electric field amplitude reflected back into material 1 is expressed as a (complex) ratio $\tilde{\mathcal{R}}$ to the (complex) incident amplitude. By matching boundary conditions for the electric and magnetic fields, $\tilde{\mathcal{R}}$ is routinely found to be given by the standard result

$$\tilde{\mathcal{R}} = \frac{\tilde{Z}_1^{-1} - \tilde{Z}_2^{-1}}{\tilde{Z}_1^{-1} + \tilde{Z}_2^{-1}} .$$

Consider the case in which material 1 is an insulator ($\beta = 0$) and material 2 is a poor conductor ($\beta \ll 1$), with $\sqrt{\epsilon_1/\mu_1} = \sqrt{\epsilon_2/\mu_2}$. Calculate the reflection amplitude ratio $\tilde{\mathcal{R}}$ to lowest nonvanishing order in β.

Problem 2. (35 points)
An ideal wave plate of thickness D with phase retardation difference

$$\delta \equiv (n_s - n_f)\frac{\omega D}{c} ,$$

having its slow (n_s) axis along the direction $(\hat{x}\cos\phi + \hat{y}\sin\phi)$, is represented by the Jones matrix

$$M_{\rm W}(\phi) =$$
$$\begin{pmatrix} \cos\frac{\delta}{2} + i\sin\frac{\delta}{2}\cos 2\phi & i\sin\frac{\delta}{2}\sin 2\phi \\ i\sin\frac{\delta}{2}\sin 2\phi & \cos\frac{\delta}{2} - i\sin\frac{\delta}{2}\cos 2\phi \end{pmatrix}$$

A beam of light traveling in the $\hat{z}$ direction passes through a device that consists of the following ideal components:

- First, a quarter-wave-plate (QWP: $\delta = \frac{\pi}{2}$) with slow axis at $\phi = +45°$;
- Next, an x polarizer;
- Finally, a QWP with slow axis at $\phi = -45°$.

(**a.**) (10 points)
Calculate the Jones matrix that represents the effect of this device.

(**b.**) (15 points)
Given that a beam is observed to emerge from this device, calculate its state of polarization.

(**c.**) (10 points)
The Mueller matrix for this device is

$$\frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} .$$

Assume that the incident beam is *natural light* (100% unpolarized, $\langle|E_x|^2\rangle = \langle|E_y|^2\rangle$). Calculate the fraction I'/I_0 of the incident irradiance that this device transmits.

Problem 3. (30 points)
At $t = 0$, charge $+e$ lies at $(x, y, z) = (0, 0, b/2)$ and charge $-e$ lies at $(x, y, z) = (0, 0, -b/2)$.

(**a.**) (10 points)
Identify the lowest-l nonvanishing electrostatic multipole moment(s) of the charge distribution (you don't need to calculate its (their) magnitude).

(**b.**) (5 points)
The static charge distribution in (a.) now is set into oscillation: as time advances, the position vector of each charge is multiplied by the same factor $1 + \epsilon \cos \omega t$, where ω and $0 < \epsilon \ll 1$ are real constants. Using the fact that a static electric multipole corresponding to a given l and m, when caused to oscillate, yields E-type (TM) multipole radiation of the *same* l and m, what type(s) of lowest-l radiation (*e.g.* $\mathrm{E}2_1$) is (are) emitted?

(**c.**) (15 points)
For E-type (TM) radiation of type $\mathrm{E}l_m$, the magnetic field $\tilde{\vec{B}}$ ($\perp \hat{r}$) is proportional to the vector spherical harmonic $\vec{X}_{lm}$:

$$\tilde{\vec{B}} \propto \vec{X}_{lm}(\theta, \phi) \equiv \vec{L}\, Y_{lm}(\theta, \phi) .$$

Also, in the far zone,

$$\tilde{\vec{E}} \approx c\tilde{\vec{B}} \times \hat{r} .$$

Finally, in spherical polar coordinates,

$$i\vec{L} \equiv \vec{r} \times \nabla = \hat{\phi}\frac{\partial}{\partial\theta} - \frac{\hat{\theta}}{\sin\theta}\frac{\partial}{\partial\phi} .$$

Write down a function $f(\theta, \phi)$ such that the angular distribution of the radiated power P in the far zone $b \ll \frac{2\pi c}{\omega} \ll r$ is proportional to it:

$$\frac{dP}{d\Omega} \propto f(\theta, \phi) .$$