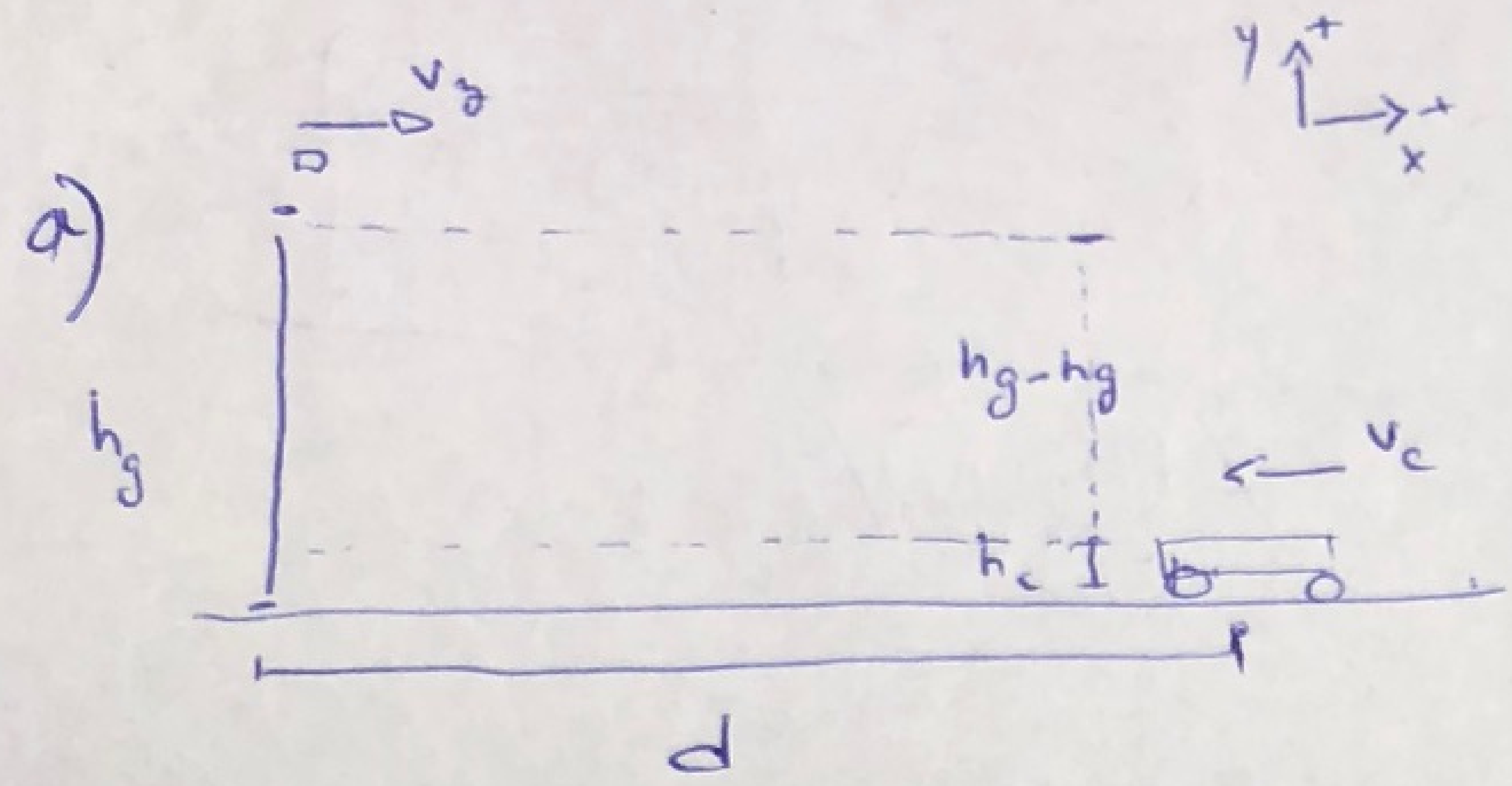
Problem 1 - MIDTERM 2018

a)



relative velocity

$(v_g - v_c)$

initial goose height $\quad y_0 = h_g$

height of windshield $\quad y = h_c$

$$y - y_0 = \cancel{v_0 t} - \frac{1}{2} g t^2$$

$$h_c - h_g = -\frac{1}{2} g t^2$$

express t

$$h_g - h_c = \frac{1}{2} g t^2 \quad \Rightarrow \boxed{t = \sqrt{\frac{2(h_g - h_c)}{g}}}$$

5pts

displacement of car in x-direction

$$\Delta x_c = v_c t = v_c \sqrt{\frac{2(h_g - h_c)}{g}}$$

displacement of "poop" in x-direction

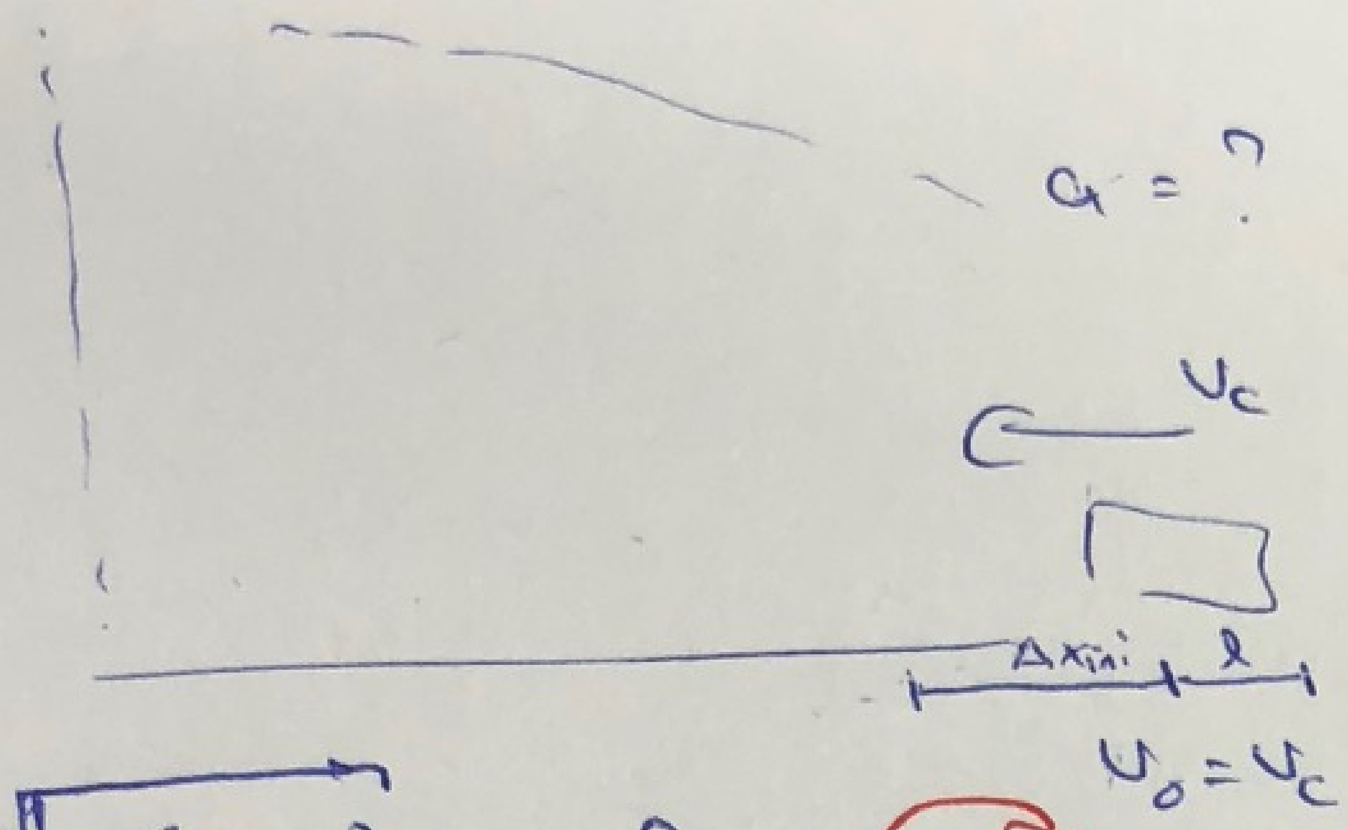
$$\Delta x_p = v_g t = v_g \sqrt{\frac{2(h_g - h_c)}{g}}$$

$$\boxed{d = \Delta_{x_c} + \Delta_{x_p} = (v_g + v_c)\sqrt{\frac{2(h_g - h_c)}{g}}}$$

5pts

b)

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$



$$\Delta x = \Delta x_{\text{initial}} + l = v_c\sqrt{\frac{2(h_g-h_c)}{g}} + l.$$ 3 pts

$$a = \frac{(\Delta x - v_0 t)(2)}{t^2} = \frac{\left[\cancel{v_c\sqrt{\frac{2(h_g-h_c)}{g}}} + l - \cancel{v_c\sqrt{\frac{2(h_g-h_c)}{g}}}\right](2)}{t^2}$$

7 pts

$$= \frac{2l}{\left(\sqrt{\frac{2(h_g-h_c)}{g}}\right)^2} = \frac{\cancel{2} g l}{\cancel{2}(h_g-h_c)} = \frac{g l}{(h_g-h_c)}$$

$\left(m/s^2 \text{ unit}\right)$

Ahmet Yildiz Midterm # 1, Problem 2 Solution

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(Dated: February 24, 2018)

I. SETUP

We want to know the velocity at which we must launch the ball, relative to the ground, in order for it to land back in our cart. This problem is going to involve two equations of motion: one for the ball going up and back down and another for the cart going up the ramp and back down. Therefore our first goal will be to find the two equations of motion.

II. FREE BODY DIAGRAMS

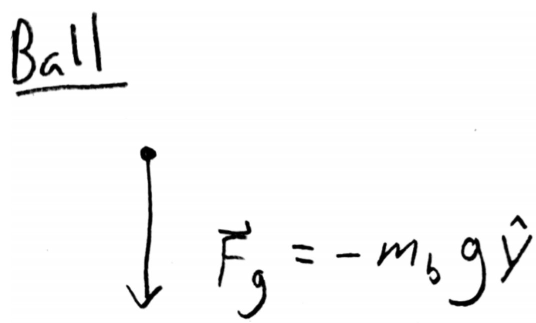


Figure 1. The free body diagram for the freely falling ball.

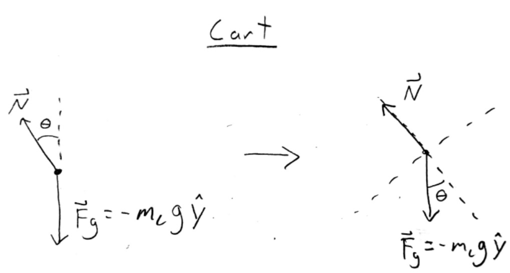


Figure 2. The free body diagram, and it's rotated counterpart, for the cart.

The free body diagrams for the ball and the cart can be seen in Figs. 1 and 2. The only force acting on the ball once it's released is gravity so the free body diagram is pretty simple. The free body diagram for the cart involves two forces: the normal force from the ramp and the force of gravity. We will find it beneficial to work in the rotated coordinate system in the next section.

III. NEWTON'S 2ND LAW

For the ball we have

$$m_b a_{b,y} = -m_b g \tag{1}$$

therefore

$$a_{b,y} = -g \tag{2}$$

where we have taken up to be the positive y direction.

For the cart we have the constraint that it stays on the ramp. If we define the coordinate on the ramp to be x', which is positive going up the ramp then Newton's Second Law says

$$m_c a_{c,x'} = -m_c g \sin(\theta) \tag{3}$$
$$m_c a_{c,y'} = N - m_c g \cos(\theta) = 0 \tag{4}$$

where the last equality in eq. (4) is the constraint that the cart stays on the ramp. Therefore

$$a_{c,x'} = -g \sin(\theta) \tag{5}$$

IV. 2D KINEMATICS

With our accelerations in hand we can now write down the equations of motion for the ball and the cart.

$$y_b(t) = vt - \frac{1}{2}gt^2 \tag{6}$$
$$x_c'(t) = v_0 t - \frac{1}{2}g\sin(\theta)t^2 \tag{7}$$

The condition that the ball and the cart meet at some later time, T, on the ramp is then

$$y_b(T) = 0 \tag{8}$$
$$x_c'(T) = 0 \tag{9}$$

which is two equations and we have two unknowns: v, T. Using eq. (9) we find

$$T = \frac{2v_0}{g\sin(\theta)} \tag{10}$$

plugging this into eq. (8) we find

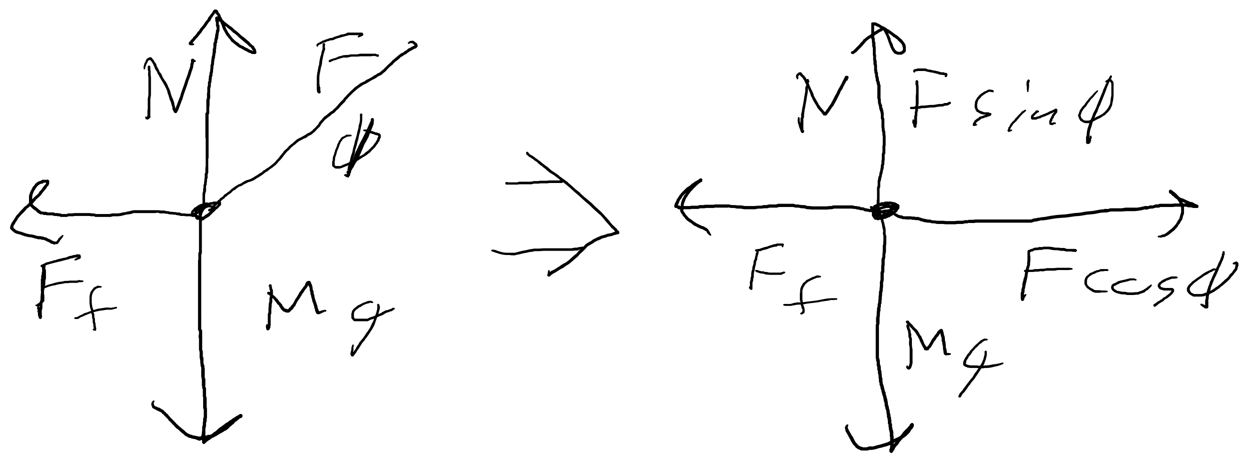
$$0 = v - \frac{1}{2}gT = v - \frac{1}{2}g\frac{2v_0}{g\sin(\theta)} = v - \frac{v_0}{\sin(\theta)} \tag{11}$$

and therefore

$$\boxed{v = \frac{v_0}{\sin(\theta)}} \tag{12}$$

Problem 3

(a) First, let's draw the FBD:



N2L in y-direction:

$$N + F\sin\phi - Mg = 0 \Rightarrow N = Mg - F\sin\phi$$

N2L in x-direction

$$F\cos\phi - F_f = F\cos\phi - \mu_k Mg + \mu_k F\sin\phi = 0$$

$$F = \frac{\mu_k Mg}{\cos\phi + \mu_k \sin\phi}$$

But we want the angle where this force is minimized

$$\frac{dF}{d\phi} = \frac{-\mu_k M g(-\sin\phi + \mu_k \cos\phi)}{(\cos\phi + \mu_k \sin\phi)^2}$$

$$= 0$$

So $\sin\phi = \mu_k \cos\phi$

$$\boxed{\phi = \arctan(\mu_k)}$$

Now lets find the force

$$F = \frac{\mu_k M g}{\cos\phi + \mu_k \sin\phi}$$

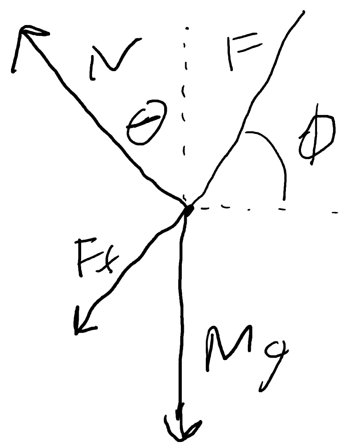
$$= \frac{\mu_k M g}{\cos\phi\,(1+\mu_k^2)}$$

Note

$$\cos\phi = \cos(\arctan \mu_k) = (\mu_k^2+1)^{-1/2}$$

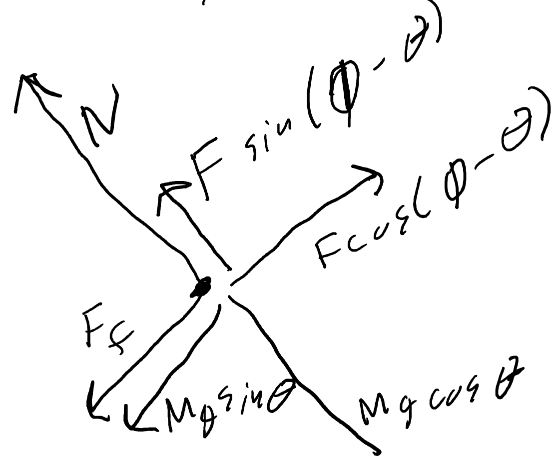
So $$\boxed{F = \frac{\mu_k M g}{(1+\mu_k^2)^{1/2}}}$$

(b) Now the box is on an incline:



Let's choose x to lie along the incline and y to be $\perp$ to it.

$\Rightarrow$



Let's use N2L for y:

$$N + F\sin(\phi-\theta) - Mg\cos\theta = 0$$

$$N = Mg\cos\theta - F\sin(\phi-\theta)$$

And for x:

$$F\cos(\phi-\theta) - Mg\sin\theta - F_f$$

$$= F\cos(\phi-\theta) - Mg\sin\theta - \mu_k Mg\cos\theta$$

$$+ \mu_k F\sin(\phi-\theta)$$

$$= 0$$

Let's solve this for F:

$$F = \frac{Mg(\sin\theta + \mu_k \cos\theta)}{\cos(\phi-\theta) + \mu_k \sin(\phi-\theta)}$$

And we want to minimize this:

$$\frac{dF}{d\phi} = \frac{-Mg(\sin\theta + \mu_k\cos\theta)(-\sin(\phi-\theta) + \mu_k\cos(\phi-\theta))}{(\cos(\phi-\theta) + \mu_k\sin(\phi-\theta))^2}$$

$$= 0$$

so $\sin(\phi-\theta) = \mu_k\cos(\phi-\theta)$

$$\boxed{\phi = \theta + \arctan\mu_k}$$

And now the Force:

$$F = \frac{Mg(\sin\theta + \mu_k\cos\theta)}{\cos(\phi-\theta) + \mu_k\sin(\phi-\theta)}$$

$$= \frac{Mg(\sin\theta + \mu_k\cos\theta)}{\cos(\arctan\mu_k)(1+\mu_k^2)}$$

$$\boxed{F = \frac{Mg(\sin\theta + \mu_k\cos\theta)}{(1+\mu_k^2)^{1/2}}}$$

Physics 7A Spring 2018 Yildiz Midterm 1 Problem 4 Solution

GSI: James Reed Watson

February 28, 2018

There are three degrees of freedom in the problem, the tension in the upper rod, T_1, the tension in the lower rod, T_2, and the angle the rods make with the vertical, θ. Therefore, three equations are needed. The sum of forces in the y-directions for both masses:

$$(T_1 - T_2)\cos\theta = m_1 g \tag{1}$$
$$2T_2 \cos\theta = m_2 g \tag{2}$$

The centripetal acceleration provides the following constraint:

$$(T_1 + T_2)\sin\theta = m_1 \Omega^2 R = m_1 \Omega^2 L \sin\theta \tag{3}$$

Equation (2) can be solved immediately to find that $T_2 = m_2 g / 2\cos\theta$. This is then plugged into equation (1) to find:

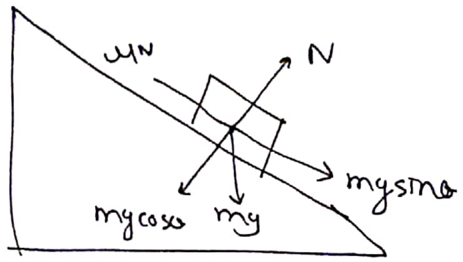
$$T_1 = \frac{m_2 g}{2\cos\theta} + \frac{m_1 g}{\cos\theta} \tag{4}$$

Plugging it into equation (3) one obtains:

$$\frac{m_2 g}{2\cos\theta} + \frac{m_1 g}{\cos\theta} + \frac{m_2 g}{2\cos\theta} = m_1 \Omega^2 L \tag{5}$$
$$\cos\theta = (\frac{m_1 + m_2}{m_1})\frac{g}{\Omega^2 L} \tag{6}$$

Q5

(a)



$N = mg\cos\theta \quad — (1)$

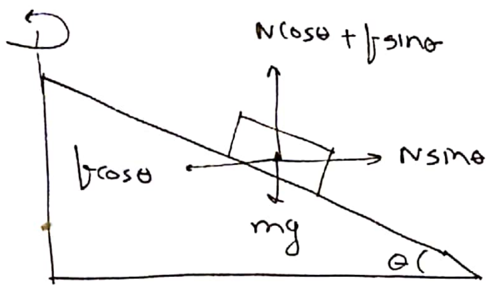
$\mu N = mg\sin\theta \quad — (2)$

from (1) and (2)

$\mu\, mg\cos\theta = mg\sin\theta$

$\boxed{\mu = \tan\theta}$

(b)



$\Sigma F_x \Rightarrow f\cos\theta - N\sin\theta = mr\omega^2$

$\Rightarrow \mu N\cos\theta - N\sin\theta = mr\omega^2 \quad — (1)$

$\Sigma F_y = 0 \quad N\cos\theta + f\sin\theta = mg$

$N\cos\theta + \mu N\sin\theta = mg$

$N[\cos\theta + \mu\sin\theta] = mg$

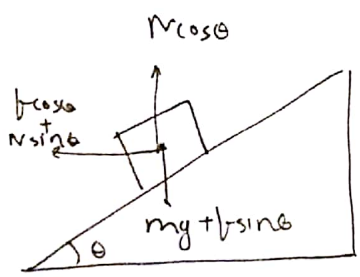
$N = \dfrac{mg}{(\cos\theta + \mu\sin\theta)} \quad — (2)$

from (1) and (2)

$[\mu\cos\theta - \sin\theta] \times \dfrac{\cancel{m}g}{[\cos\theta + \mu\sin\theta]} = \cancel{m}(d\cos\theta)\omega^2$

$$\boxed{\sqrt{\frac{g}{d\cos\theta}\,\frac{(\mu\cos\theta - \sin\theta)}{(\cos\theta + \mu\sin\theta)}} = \omega}$$

(c)



$\Sigma F_y = 0 \Rightarrow N\cos\theta = mg + f\sin\theta$

$N\cos\theta = mg + \mu N\sin\theta$

$N\cos\theta - \mu N\sin\theta = mg$

$N(\cos\theta - \mu\sin\theta) = mg$

$\boxed{N = \dfrac{mg}{\cos\theta - \mu\sin\theta}} \quad — (1)$

$\Sigma F_x : f\cos\theta + N\sin\theta = mr\omega^2$

$\mu N\cos\theta + N\sin\theta = m\, d\cos\theta\, \omega^2$

$N(\mu\cos\theta + \sin\theta) = md\cos\theta\, \omega^2 \quad — (2)$

from (1) and (2)

$$\frac{\cancel{m}g}{\cos\theta - \mu\sin\theta} \times (\mu\cos\theta + \sin\theta) = \cancel{m}d\cos\theta\,\omega^2 \Rightarrow \boxed{\omega = \sqrt{\frac{g}{d\cos\theta}\,\frac{\mu\cos\theta + \sin\theta}{\cos\theta - \mu\sin\theta}}}$$

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