Physics 7C Section 2
Spring 2005
Midterm II, April 5, 2005
Prof. Marco Battaglia

Choose four out of the five proposed problems. The test duration is 110 minutes.

1. The Fresnel equation gives the reflection coefficient $R_{||}$ for waves parallel to the plane of incidence as $R_{||} = \frac{tan(\theta_1 - \theta_2)}{tan(\theta_1 + \theta_2)}$, where the angles of incidence θ_1 and refraction θ_2 are defined by the Snell's law.
1) Derive the analytic expression of the Brewster's angle, at which the reflected light is fully polarised in the plane perpendicular to the plane of incidence.
2) Find the value of the Brewster's angle for light reflected from a water surface.

2. A B particle, with mass=5.2 GeV/c^2, is produced with a momentum p_x=40 GeV/c.
1) Compute the value of γ for the B particle.
2) The decay generates two identical pion particles, π, each with mass=0.14 GeV/c^2, such that the momentum of one of the pions, in the decaying B rest frame, projected along the line of flight of the B in the lab, is $p_{x'\ 1}$=2 GeV/c. Compute the angle between the two pions both in the B particle frame and in the laboratory frame. Do these angles depend on $p_{x'\ 1}$?

3. A cosmic ray experiment is conducted using two detectors: the first is located at an altitude of 2000 m and the second at 500 m. The first detector counts 550 muons/hour and the second 400 muons/hour. The difference in counts is entirely due to the decay of muons according to the exponential law $I(t) = I(0)e^{-t/\tau}$. The decay time τ for muons has been measured to be 2.25 μs, its mass is $M_\mu = 0.105$ GeV/c^2.
1) Determine the speed of the muons assuming Galilean relativity and comment whether the result is acceptable.
2) Determine the energy of the muons using the relativistic prescriptions.
(*Assume the muons to travel along a trajectory normal to the earth surface and, for point 2), approximate the muon velocity with c in computing the transit time*).

4. The relation $E = mc^2$ suggests that a photon γ of sufficient energy $E_\gamma = h\nu$ can generate an e^-e^+ electron-positron pair.
1) Can the pair creation process occur in vacuum (justify the answer) ?
2) Which is the minimum E_γ for it to occur with an electron e as spectator: $\gamma e \to e^+e^-e$?
3) Which is the minimum E_γ for it to occur with a nucleus N as spectator: $\gamma N \to e^+e^-N$?
(*Assume the spectator to be at rest in the laboratory*).

5. The light emitted by the SSA12 galaxy is measured in a spectrometer at the Keck II telescope. The λ=1216 Angstrom Ly-α H line is observed at a wavelength of 8185 Angstrom.
1) Determine the z value for the redshift of the galaxy and its recessional velocity in units of c.
2) Estimate the resolving distance for two stars in this Galaxy emitting light at λ=1216 Angstrom, knowing that the diameter of the telescope mirror is 10 m.
(*1 Angstrom* $= 10^{-10}$ *m, 1 ly = distance covered by light in one year. Take* $H_0 = 20$ *km* s^{-1} *Mly*$^{-1}$ *for the Hubble constant which relates the distance to the recessional velocity*)

7C S05 BATTAGLIA MT2 SOLUTIONS

1

1) AT BREWSTER'S ANGLE:

$$R_{\parallel} = \frac{\tan(\theta_B - \theta_2)}{\tan(\theta_B + \theta_2)} = 0$$

DISCARD

$$\tan(\theta_B - \theta_2) = 0 \Rightarrow \theta_B - \theta_2 = n\pi$$

$$\Rightarrow n = 0 \text{ SINCE } \theta \in [0, \pi/2]$$

$$n_1 \sin\theta_B = n_2 \sin\theta_2 \; ; \; n_1 \neq n_2$$

$$\Rightarrow \theta_B = \theta_2 = 0$$

$$\Rightarrow R_{\parallel} \text{ UNDEFINED}$$

$$\tan(\theta_B + \theta_2) \rightarrow \infty \Rightarrow \theta_B + \theta_2 = \pi/2$$

$$n_1 \sin\theta_B = n_2 \sin\theta_2 = n_2(\pi/2 - \theta_2) = n_2 \cos\theta_B$$

$$\Rightarrow \theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

2) $n_2 = 1.33; \; n_1 = 1$

$$\Rightarrow \theta_B = \tan^{-1}(1.33) = 53.1^\circ = 0.926 \text{ rad}$$

2

1) $E = \gamma m c^2 = \sqrt{p_x^2 c^2 + m^2 c^4}$

5 pts.

$$\gamma = \sqrt{1 + \left(\frac{p_x}{mc}\right)^2}$$

$$= \sqrt{1 + \left(\frac{40\ \text{GeV}}{5.2\ \text{GeV}}\right)^2}$$

$\boxed{\gamma \simeq 7.8}$ $\left(\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.99 \simeq 1\right)$

2) B REST FRAME (S'): (5 pts)

20 pts.

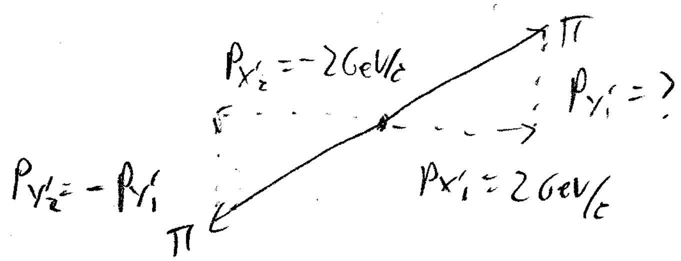
BEFORE

B

$p_{x'} = p_{y'} = 0$

$E = m_B c^2$

AFTER



$$E = 2\sqrt{(m_\pi c^2)^2 + (p_{x'_1}^2 + p_{y'_1}^2)c^2}$$

$\boxed{\text{MUTUAL ANGLE } \theta' = 180^\circ \text{ IN REST FRAME}}$

WE FIND $p_{y'_1}$ IN ORDER TO CALCULATE

θ IN LAB FRAME:

NEXT →

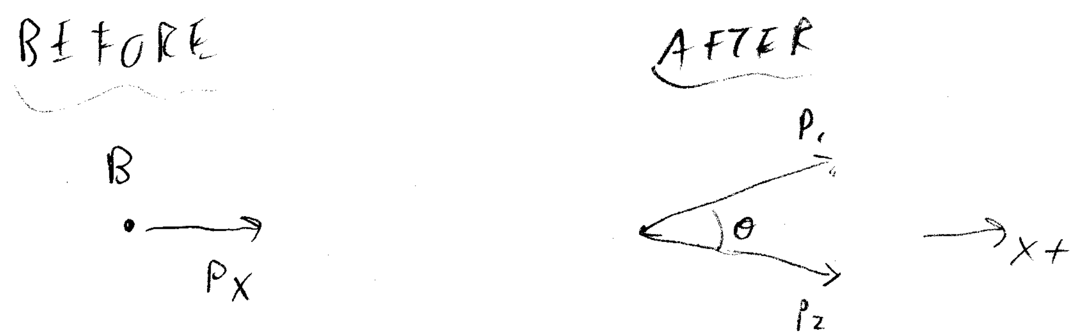
$$P_{y'_1} = \sqrt{\left(\frac{E}{2c}\right)^2 - (m_\pi c)^2 - (P_{x'_1})^2}$$

$$P_{y'_1}c = \sqrt{\frac{1}{4}(m_B c^2)^2 - (m_\pi c^2)^2 - (P_{x'_1}c)^2}$$

$$= \sqrt{0.25 \cdot (5.2\,\text{GeV})^2 - (0.14\,\text{GeV})^2 - (2\,\text{GeV})^2}$$

$$P_{y'_1} = 1.7\ \text{GeV}/c$$

IN LAB FRAME (S): (8 pts.)



JUST TRANSFORM FROM REST FRAME:

$$P_{x_1} = \gamma\left(P_{x'_1} + \frac{V_x}{c^2}E'_1\right)$$

$$P_{x_1}c = \gamma(P_{x'_1}c + \beta E'_1) \qquad = m_B c^2/2$$

$$= 7.8(2\,\text{GeV} + 1 \cdot 2.6\,\text{GeV})$$

$$P_{x_1} \simeq 36\ \text{GeV}/c \quad ; \quad P_{y_1} = P_{y'_1} \simeq 1.7\ \text{GeV}/c$$

NEXT →

$p_{x_2}c = 7.8\,(-2\,\text{GeV} + 1 \cdot 2.6\,\text{GeV})$

$p_{x_2} \simeq 4.7\,\text{GeV}/c \;,\; p_{y_2} \simeq -1.7\,\text{GeV}/c$

4 pts.

$$\theta = \cos^{-1}\left(\frac{\vec{p}_1 \cdot \vec{p}_2}{|\vec{p}_1|\,|\vec{p}_2|}\right) = \cos^{-1}\left(\frac{36 \cdot 4.7 - (1.7)^2}{\sqrt{36^2+1.7^2} \cdot \sqrt{4.7^2+1.7^2}}\right)$$

$$= \cos^{-1}\left(\frac{166}{36 \cdot 5}\right) = 23^\circ$$

MUTUAL ANGLE $\theta = 23^\circ$ IN LAB FRAME

(THIS ANGLE DEPENDS ON $p_{x_1}' \in [0, p_1']$) 3 pts.

3

t_1 @ 2000m ; t_2 @ 500m

$$I(t) = I(0)\, e^{-t/\tau}$$

$$\Rightarrow \frac{I(t_2)}{I(t_1)} = e^{-(t_2 - t_1)/\tau}$$

$$\Rightarrow \Delta t = -\tau \ln\left(\frac{400}{550}\right) \simeq (2.25\,\mu s)\cdot 0.318$$

$$\Delta t \simeq 0.717\ \mu s$$

1) GALILEAN: ALL TIMES AND DISTANCES SAME IN ALL FRAMES

$$v = \frac{1500\,m}{0.717 \times 10^{-6}\,s} = \boxed{20.93 \times 10^{8}\ m/s} \sim 7c!$$

NOT ACCEPTABLE

NEXT →

2) $\Delta t' = \gamma \underbrace{\Delta t}_{\text{proper time, in muon frame}}$

$$\gamma = \frac{\Delta t'}{\Delta t} = \frac{L/v}{\Delta t} \simeq \frac{L/c}{\Delta t}$$

$$\simeq \frac{1500\,m / 3e8\,m/s}{0.717 \times 10^{-6}\,s}$$

$$\simeq 6.98$$

$\Rightarrow E = \gamma m c^2 \simeq 6.98 \cdot 0.105\,GeV$

$\Rightarrow \boxed{E \simeq 0.733\,GeV}$

(4)

$\rightsquigarrow \qquad \longrightarrow x^+$

1) $\gamma \to e^+ e^-$

10 pts $E_\gamma = \sqrt{(mc^2)^2 + p_1^2 c^2} + \sqrt{(mc^2)^2 + p_2^2 c^2}$

$p_\gamma = \frac{E_\gamma}{c} = p_{x_1} + p_{x_2}$

$0 = p_{y_1} + p_{y_2} \Rightarrow p_{y_2} = -p_{y_1}$

$\Rightarrow p_{x_1} + p_{x_2} = \sqrt{[(mc)^2 + p_{y_1}^2] + p_{x_1}^2} + \sqrt{[(mc)^2 + p_{y_1}^2] + p_{x_2}^2}$

$\Rightarrow m = 0$ OR $p_{x_1}, p_{x_2} \to \infty$ + ($p_{y_1} = 0$ @ THRESH.)

REQUIRE A SPECTATOR PARTICLE:

OPTIONAL

$\gamma A \to e^+ e^- A'$

$E_\gamma + \sqrt{(m_A c^2)^2 + p_A^2 c^2} =$

$\sqrt{(mc^2)^2 + p_1^2 c^2} + \sqrt{(mc^2)^2 + p_2^2 c^2}$

$+ \sqrt{(m_A c^2)^2 + p_{A'}^2 c^2}$

$p_\gamma + p_{xA} = \frac{E_\gamma}{c} + p_{xA} = p_{x_1} + p_{x_2} + p_{x_{A'}}$

$p_{yA} = p_{y_1} + p_{y_2} + p_{yA'}$

NEXT $\rightarrow$

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$$\Rightarrow \quad p_{x_1} + p_{x_2} + p_{xA'} - p_{xA} = \sqrt{(mc)^2 + p_{x_1}^2 + p_{y_1}^2} + \sqrt{(mc)^2 + p_{x_2}^2 + p_{y_2}^2} + \sqrt{(m_A c)^2 + p_{xA'}^2 + p_{yA'}^2} - \sqrt{(m_A c)^2 + p_{xA}^2 + p_{yA}^2}$$

$$\sqrt{(m_A c)^2 + p_{xA}^2 + p_{yA}^2} - p_{xA} = \sqrt{(m_A c)^2 + p_{xA'}^2 + p_{yA'}^2} - p_{xA'} + \sqrt{(mc)^2 + p_{x_1}^2 + p_{y_1}^2} - p_{x_1} + \sqrt{(mc)^2 + p_{x_2}^2 + p_{y_2}^2} - p_{x_2}$$

$\Rightarrow$ $p_{xA} < p_{xA'}$ REQUIRED FOR SOLUTION

- TAKE $p_{xA} = p_{yA} = 0$ AS INSTRUCTED
- AT THRESHOLD, $p_{y_1} = p_{y_2} = 0$; $p_{x_1} = p_{x_2}$

$$m_A c = \sqrt{(m_A c)^2 + p_{yA'}^2} - p_{xA'} + 2\sqrt{(mc)^2 + p_{x_1}^2} - 2p_{x_1}$$

NEXT →

2) AT THRESHOLD:

6 pts.

$$mc = 3\sqrt{(mc)^2 + p^2} - 3p$$

$$(mc + 3p)^2 = 9\left[(mc)^2 + p^2\right]$$

$$(mc)^2 + 6(mc)p + \cancel{9p^2} = 9(mc)^2 + \cancel{9p^2}$$

$$p = \frac{4}{3} mc$$

$$\rightarrow E_\gamma = 3\sqrt{(mc)^2 + \frac{16}{9}(mc)^2} - mc^2$$

$$\boxed{E_\gamma = 4mc^2 \simeq 2.04 \text{ MeV}}$$

3) $m_N >> m \Rightarrow p \sim \frac{m^2 c}{m_N}$, $P_N = ?$

9 pts.

$$m_N c = \sqrt{(m_N c)^2 + p_N^2} - p_N + 2\sqrt{(mc)^2 + \left(\frac{m^2 c}{m_N}\right)^2} - \frac{2m^2 c}{m_N}$$

$$0 \simeq \frac{p_N^2}{2m_N c} - p_N + 2mc + \left(\frac{m}{m_N}\right)^2 - \frac{2m^2 c}{m_N}$$

$$p_N\left(1 - \frac{p_N}{2m_N c}\right) \simeq 2mc\left(1 - \frac{m}{m_N}\right) + O\left(\frac{m}{m_N}\right)^2$$

NEXT →

$$\rightarrow p_N \simeq 2mc\left(1-\frac{m}{M_N}\right)\cdot\left(1+\frac{p_N}{2M_Nc}\right)$$

$$\simeq 2mc - 2mc\,\frac{m}{M_N} + 2mc\,\frac{p_N}{2M_Nc} + O\left(\frac{m}{M_N}\right)^2$$

$$p_N \simeq \frac{2mc\left(1-\frac{m}{M_N}\right)}{1-\frac{m}{M_N}} = 2mc + O\left(\frac{m}{M_N}\right)^2$$

$$\Rightarrow E_\gamma = \sqrt{(M_Nc^2)^2 + p_N^2c^2} - M_Nc^2 + 2\sqrt{(mc^2)^2 + p^2c^2}$$

$$\simeq \frac{1}{2}\frac{p_N^2 c^2}{M_N c^2} + 2mc^2 + \frac{p^2 c^2}{mc^2}$$

$$\simeq 2mc^2 + \frac{1}{2}\frac{4m^2c^2}{M_N} + \frac{m^4c^2}{mM_N^2}$$

$$\simeq 2mc^2 + \frac{m}{M_N}2mc^2 + mc^2\cdot\left(\frac{m}{M_N}\right)^2$$

$$\boxed{E_\gamma \simeq 2mc^2\left(1+\frac{m}{M_N}\right) + O\left(\frac{m}{M_N}\right)^2 \simeq 1.02\ \text{MeV}}$$

STARTING w/ $p \simeq \emptyset$ TO GIVE THIS IS ACCEPTABLE;

-2 FOR $E_\gamma = 2mc^2 = 1.02$ MeV

$\boxed{5}$

$$\left.\begin{aligned}\lambda &= 8185 \text{ Å}\\ \lambda_0 &= 1216 \text{ Å}\end{aligned}\right\}$$

1) $\quad \nu = \sqrt{\dfrac{1-\beta}{1+\beta}}\,\nu_0$

10 pts

$$\frac{c}{\lambda} = \sqrt{\frac{1-\beta}{1+\beta}}\,\frac{c}{\lambda_0} \Rightarrow \lambda = \sqrt{\frac{1+\beta}{1-\beta}}\,\lambda_0$$

$$\frac{1+\beta}{1-\beta} = \left(\frac{\lambda}{\lambda_0}\right)^2 \Rightarrow 1+\beta = (1-\beta)\left(\frac{\lambda}{\lambda_0}\right)^2$$

$$\beta\left[1+\left(\frac{\lambda}{\lambda_0}\right)^2\right] = \left(\frac{\lambda}{\lambda_0}\right)^2 - 1$$

$$\beta = \frac{(\lambda/\lambda_0)^2 - 1}{(\lambda/\lambda_0)^2 + 1}$$

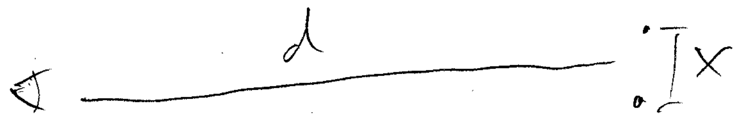
$$\boxed{\beta \simeq 0.9568} \quad 8 \text{ pts}$$

$$\boxed{z = \frac{\lambda - \lambda_0}{\lambda_0} = 5.731} \quad 2 \text{ pts}$$

next →

2)

15 pts.



$x \simeq d\,\Delta\theta$ $\qquad$ −1 FOR USING 1216 Å

5 pts. $$\Delta\theta = \frac{1.22\lambda}{D} \simeq \frac{1.22 \cdot 8185\,\text{Å}}{10\ m} \simeq 9.96e\text{-}8\ rad$$

5 pts. $$H_0 d = v \Rightarrow d = \frac{\beta z}{H_0} \simeq \frac{0.9568 \cdot 3\times10^{8}\,m/s}{20\times10^{3}\,\frac{m}{s}\cdot(Mly)^{-1}}$$

$$= 14350\ Mly$$

$$\Rightarrow x \simeq 1430\,ly \simeq 1.36\times10^{19}\ m$$ 5 pts.