MATH 16A MIDTERM 2 (001) 9.10AM - 10AM
PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.
CALCULATORS ARE NOT PERMITTED
YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK
SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE
REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT
THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM

Name and section: ____

GSI's name: ____

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. (25 points) Calculate the derivatives of the following functions: (You do not need to use the limit definition and you do not need to simplify your answers)

(a)

$$xe^{x^2}$$

Solution:

$f(x) = u(x)v(x)$, $u(x) = x$, $v(x) = e^{x^2}$ $\Rightarrow$ $u'(x) = 1$, $v'(x) = e^{x^2} \cdot 2x$

$\Rightarrow \frac{d}{dx}(x e^{x^2}) = 1 \cdot e^{x^2} + x e^{x^2} \cdot 2x$

(b)

$$\ln(\frac{e^x}{\sqrt{x^2-1}})$$

Solution:

$\ln(\frac{e^x}{\sqrt{x^2-1}}) = \ln(e^x) - \frac{1}{2}\ln(x^2-1)$

$= x - \frac{1}{2}\ln(x^2-1)$

$\Rightarrow \frac{d}{dx}(\ln(\frac{e^x}{\sqrt{x^2-1}})) = 1 - \frac{1}{2} \cdot \frac{2x}{x^2-1}$

2. (25 points) A company is selling a product. The demand equation for the product is

$$p = 169 - q^2$$

where p is the price per unit and q is the number of units sold.

(a) Determine the elasticity $E(p)$. At what price does demand have unit elasticity.

Solution:

$$p = 169 - q^2 \Rightarrow q = \sqrt{169 - p}$$

$$\Rightarrow \frac{dq}{dp} = \frac{1}{2} \cdot \frac{1}{\sqrt{169-p}} \cdot (-1)$$

$$\Rightarrow E(p) = \frac{-p}{q} \cdot \frac{dq}{dp} = \frac{-p}{\sqrt{169-p}} \cdot \frac{-1}{2\sqrt{169-p}}$$

$$= \frac{p}{2(169-p)}$$

$$E(p) = 1 \Rightarrow \frac{p}{2(169-p)} = 1 \Rightarrow p = 2(169-p) \Rightarrow p = \frac{338}{3}$$

(b) If they are selling 12 units, should they increase or decrease the price to raise revenue? Justify your answer.

Solution:

$$q = 12 \Rightarrow p = 169 - 12^2 = 169 - 144 = 25$$

$E(25) = \frac{25}{2 \cdot 144} < 1 \Rightarrow$ Demand is inelastic at $p = 25$

$\Rightarrow$ They should increase the price to raise revenue.

3. (25 points) Find and classify the relative extrema of the following function:

$$f(x) = x^{2/3} - x^{5/3}$$

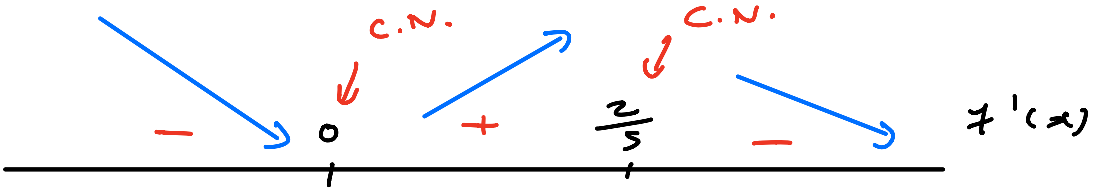
Be sure to carefully justify your answer.

Solution:

$$f'(x) = \frac{2}{3} \cdot \frac{1}{x^{1/3}} - \frac{5}{3} x^{2/3}$$

A/ $f'(x) = 0 \Rightarrow \frac{2}{3} \frac{1}{x^{1/3}} = \frac{5}{3} x^{2/3} \Rightarrow \frac{2}{5} = x$

B/ f' undefined $\Rightarrow x = 0$



$f'(1) = \frac{2}{3} - \frac{5}{3} < 0$

$f(\frac{1}{8}) = \frac{2}{3} \frac{1}{(\frac{1}{2})} - \frac{5}{3} \cdot (\frac{1}{2})^2$

$= \frac{4}{3} - \frac{5}{12} > 0$

$f'(-1) = \frac{-2}{3} - \frac{5}{3} < 0$

$\Rightarrow$ $f(0) = 0$ is a relative min

$f(\frac{2}{5}) = (\frac{2}{5})^{2/3} - (\frac{2}{5})^{5/3}$ is a relative max

4. What is the maximum possible value of $x+6y$ subject to the condition $x+y^2=4$, where x and y are non-negative numbers?

Solution:

Objective : Maximize $x+6y$

Constraint : $x+y^2=4$, $x,y \geq 0$

$x+y^2=4 \Rightarrow x = 4-y^2 \Rightarrow x+6y = (4-y^2)+6y = f(y)$

$x,y \geq 0$ and $x+y^2=4 \Rightarrow y \leq 2$

Domain $= [0,2]$

$f'(y) = -2y+6$

A/ $f'(y) = 0 \Rightarrow y = 3$

B/ f' continuous everywhere

$\Rightarrow$ 0, 2 are only critical numbers on $[0,2]$

$f(0) = 4$

$f(2) = 12$

$\Rightarrow$ The maximum possible value of $x+6y$ is 12 under the condition $x+y^2=4$ and $x,y \geq 0$

5. Sketch the following curve. If they exist, be sure to indicate, relative maxima and minima and inflection points. Show your working on this page and draw the graph on the next page.

$$y = \frac{x^2+1}{x}$$

Solution:

$$f(x) = \frac{x^2+1}{x} = x + \frac{1}{x}$$

<u>Domain</u> : $x \neq 0$

No x-intercept or y-intercept ($x^2+1 > 0$)

$\lim_{x \to \infty} f(x) = \infty$, $\lim_{x \to -\infty} f(x) = -\infty$ $\Rightarrow$ No horizontal asymptotes

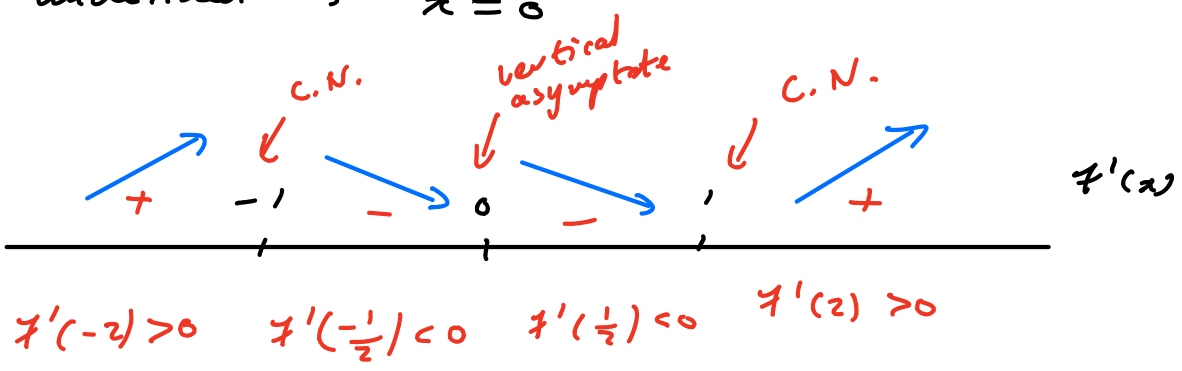
Vertical asymptote at $x = 0$

$f(x)$ <u>odd</u> .

$$f'(x) = 1 - \frac{1}{x^2}$$

A/ $f'(x) = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

B/ f' undefined $\Rightarrow x = 0$



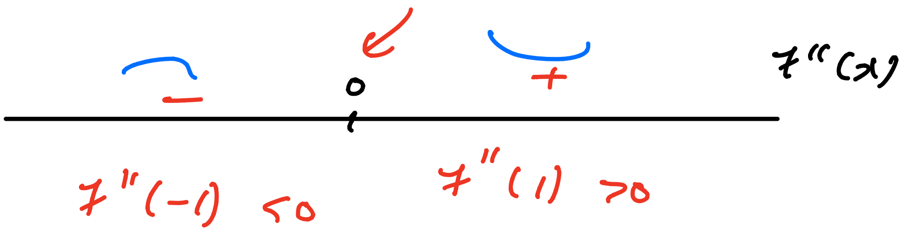
$$f''(x) = \frac{2}{x^3}$$

A/ $f''(x) = 0 \Rightarrow \frac{2}{x^3} = 0$ (No solutions)

Solution (continued) :

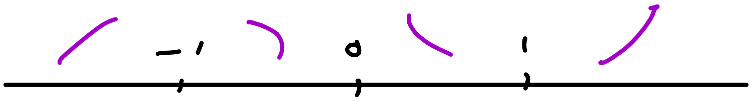
3) f'' undefined when $x = 0$

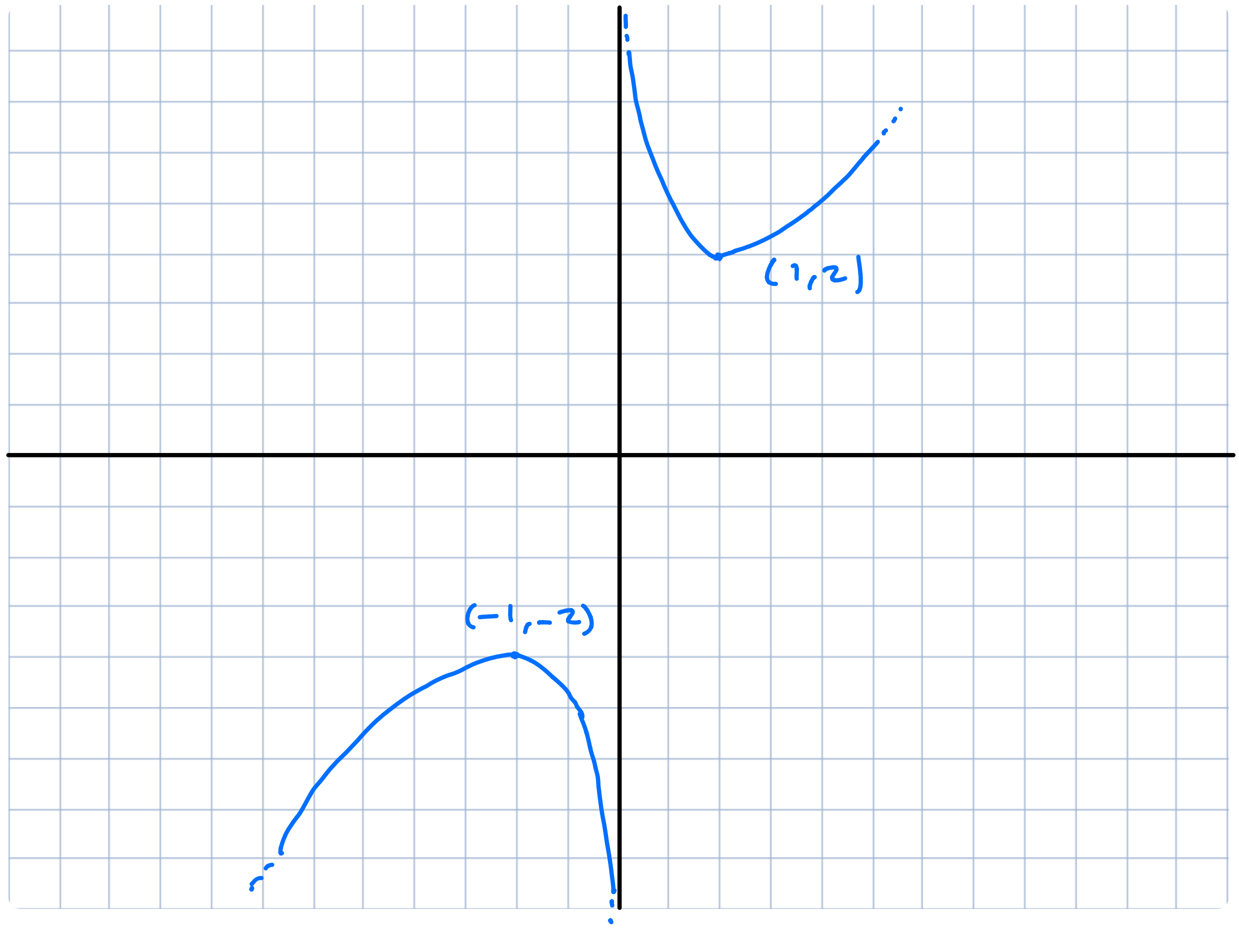
Not inflection (vertical asymptote)



$f(1) = 2$

$f(-1) = -2$





END OF EXAM