MATH 16A MIDTERM 1(001)
PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

CALCULATORS ARE NOT PERMITTED

YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK

SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT

THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM

Name and section: ALEX PAULIN

GSI's name: KEVIN BUZZARD

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. (25 points) Determine the domains of the following functions:

(a)

$$\sqrt{1-2x}$$

Solution:

$1-2x \geqslant 0 \Rightarrow 1 \geqslant 2x \Rightarrow \frac{1}{2} \geqslant x$

$\Rightarrow$ Domain is $(-\infty, \frac{1}{2}]$.

(b)

$$\ln(\frac{x+1}{1-x})$$

Solution:

$\begin{matrix} x+1 > 0 \\ 1-x > 0 \end{matrix} \Rightarrow \begin{matrix} x > -1 \\ 1 > x \end{matrix} \Rightarrow -1 < x < 1$

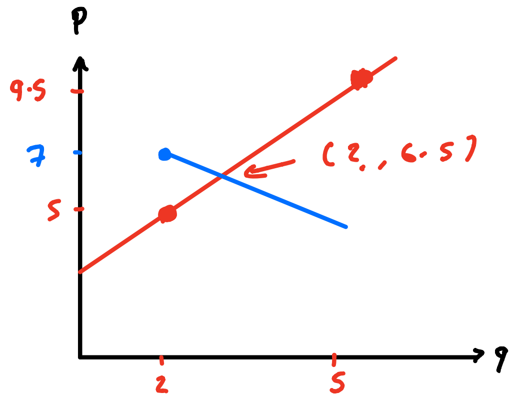
$\begin{matrix} x+1 < 0 \\ 1-x < 0 \end{matrix} \Rightarrow \begin{matrix} x < -1 \\ 1 < x \end{matrix} \Rightarrow$ No possible values of x

$\Rightarrow$ Domain is $(-1, 1)$

2. (25 points) A product is to be supplied and sold. If the price per unit is 5 dollars the supplier is willing to provide 2 unit. If the price per unit is 9.5 dollars the supplier is willing to provide 5 units. If the price per unit is 7 dollars the demand is 2 units. If the price per unit is less than 6.5 dollars there will be a shortage. If it is more than 6.5 dollars there will be a surplus.

   (a) Determine the supply and demand equations. Hint: Determine the supply equation first.

   Solution:



Supply : Slope $= \frac{9.5-5}{5-2} = \frac{4.5}{3} = \frac{3}{2}$

$\Rightarrow p - 5 = \frac{3}{2}(q-2)$

$\Rightarrow p = \frac{3}{2}q + 2 = S(q)$

$6.5 = \frac{3}{2}q + 2 \Rightarrow \frac{3}{2}q = 4.5$

$\Rightarrow q = 3$

Demand : Slope $= \frac{6.5-7}{3-2} = -\frac{1}{2}$

$p - 7 = -\frac{1}{2}(q-2)$

$\Rightarrow p = 8 - \frac{1}{2}q = D(q)$

   (b) At what price per unit will the demand be zero.

   Solution:

$D(0) = 8 \Rightarrow$ \$8 and above gives demand zero

3. Calculate the following limits. If they do not exist determine if they are ∞ or $-\infty$.

(a)

$$\lim_{x\to -1}\sqrt{\frac{x-3}{x-1}}$$

Solution:

$$\lim_{x\to -1}\sqrt{\frac{x-3}{x-1}} = \sqrt{\frac{-1-3}{-1-1}} = \sqrt{2}$$

(b)

$$\lim_{x\to -\infty}\frac{2x^7+6x^3-x^2+3}{-x^6+x^5+4x^3-x-1}$$

Solution:

$$\lim_{x\to -\infty}\frac{2x^7+6x^3-x^2+3}{-x^6+x^5+4x^3-x-1} = \lim_{x\to -\infty}\frac{2x^7}{-x^6} = \lim_{x\to -\infty} -2x = \infty \text{ (DNE)}$$

(c)

$$\lim_{x\to 1^-}\frac{2-x-x^2}{x^2-2x+1}$$

Solution:

$$\lim_{x\to 1^-}\frac{2-x-x^2}{x^2-2x+1} = \lim_{x\to 1^-}\frac{-(x+2)(x-1)}{(x-1)(x-1)} = \lim_{x\to 1^-}\frac{-(x+2)}{x-1}$$

$$\left.\begin{array}{l}\lim_{x\to 1^-} -(x+2) = -3 < 0 \\[2em] \text{1: } x-1<0 \quad x-1>0 \;\Rightarrow\; \lim_{x\to 1^-} x-1 = 0^-\end{array}\right\} \Rightarrow \lim_{x\to 1^-}\frac{-(x+2)}{x-1} = \infty \text{ (DNE)}$$

4. Let $f(x) = \begin{cases} \frac{\sqrt{x}-a}{x-4} & \text{if } x \neq 4 \\ b & \text{if } x = 4 \end{cases}$ for some real numbers a and b.

Find values of a and b such that $f(x)$ continuous at $x = 4$? Carefully justify why $f(x)$ is continuous at $x = 4$ for these values.

Solution:

$$\lim_{x \to 4} \sqrt{x} - a = \sqrt{4} - a = 2 - a$$

$$\lim_{x \to 4} x - 4 = 4 - 4 = 0$$

$$2 - a \neq 0 \Rightarrow \lim_{x \to 4} \frac{\sqrt{x} - a}{x - 4} \text{ DNE}$$

$$2 - a = 0 \Rightarrow \text{UNCLEAR QUOTIENT}$$

$$\Updownarrow$$

$$a = 2$$

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} = \lim_{x \to 4} \frac{\sqrt{x} - 2}{(\sqrt{x} - 2)(\sqrt{x} + 2)} = \lim_{x \to 4} \frac{1}{\sqrt{x} + 2}$$

$$= \frac{1}{\sqrt{4} + 2} = \frac{1}{4}$$

$\Rightarrow$ $a = 2$, $b = \frac{1}{4}$ gives a continuous function at $x = 4$.

You may assum $x \neq 3$

5. Using limits, calculate the derivative of $f(x) = \frac{x^2-9}{x^2-x-6}$. Are there any points in the graph $y = f(x)$ with horizontal tangent line?

Solution:

$$\frac{x^2-9}{x^2-x-6} = \frac{(x+3)(x-3)}{(x-3)(x+2)} = \frac{x+3}{x+2}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\frac{x+h+3}{x+h+2} - \frac{x+3}{x+2}}{h}$$

$$= \lim_{h \to 0} \frac{(x+2)(x+h+3) - (x+3)(x+h+2)}{h(x+2)(x+h+2)}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} \cancel{+hx} \cancel{+3}x + \cancel{2x} + 2h \cancel{+6} - \cancel{x^2} - \cancel{xh} - \cancel{2x} - \cancel{3}x - 3h - \cancel{6}}{h(x+2)(x+h+2)}$$

$$= \lim_{h \to 0} \frac{-1}{(x+2)(x+h+2)} = \frac{-1}{(x+2)^2}$$

$$\Rightarrow f'(x) = \frac{-1}{(x+2)^2}$$

$f'(x) = 0 \Rightarrow \frac{-1}{(x+2)^2} = 0$ which has no solutions.

$\Rightarrow$ There are no horizontal tangent lines

END OF EXAM

Blank

B Lauh

Blank