MATH 1A MIDTERM 1 (001) 12.10PM - 1PM
PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

CALCULATORS ARE NOT PERMITTED

YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK

SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS FINISHED

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT

Name and section: ALEXANDER PAULIN

GSI's name: MASTER COPY (001)

Math 1A Midterm 1 (001) 12.10pm to 1pm

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. Determine the domains of the following functions:

(a) (15 points)

$$\sqrt[4]{\frac{x+1}{x+4}}$$

Solution:

$$\left.\begin{array}{l} \begin{matrix} x+1 \geq 0 \\ x+4 > 0 \end{matrix} \Rightarrow \begin{matrix} x \geq -1 \\ x > -4 \end{matrix} \Rightarrow x \geq -1 \\ \\ \begin{matrix} x+1 \leq 0 \\ x+4 < 0 \end{matrix} \Rightarrow \begin{matrix} x \leq -1 \\ x < -4 \end{matrix} \Rightarrow x < -4 \end{array}\right\} \Rightarrow \begin{matrix} \text{Domain is} \\ (-\infty, -4] \cup [-1, \infty) \end{matrix}$$

(b) (10 points)

$$\arcsin(3-2x)$$

Solution:

Domain of $\arcsin(x)$ is $[-1, 1]$

$-1 \leq 3 - 2x \leq 1 \iff -4 \leq -2x \leq -2 \iff 2 \geq x \geq 1$

$\Rightarrow$ Domain is $[1, 2]$

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2. (a) (15 points) Describe in words, how, starting with the graph $y = \cos(3x)$, one can draw the graph

$$y = 1 - \cos(x - \pi).$$

Solution:

$f(x) = \cos(3x) \Rightarrow f(\frac{1}{3}x) = \cos(x) \Rightarrow f(\frac{1}{3}(x-\pi)) = \cos(x-\pi)$

$\Rightarrow -f(\frac{1}{3}(x-\pi)) = -\cos(x-\pi) \Rightarrow 1 - f(\frac{1}{3}(x-\pi)) = 1 - \cos(x-\pi)$

1/ Stretch horizontally by factor 3.

2/ Shift to right by π.

3/ Reflect in x-axis.

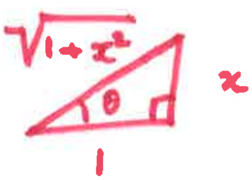
4/ Shift up by 1.

(b) (10 points) Express the following as an algebraic function:

$$\cos(\arctan(x))$$

Solution:

$\theta = \arctan(x) \Rightarrow x = \tan\theta \Rightarrow \tan\theta = \frac{x}{1}$



$\Rightarrow \cos\theta = \frac{1}{\sqrt{1+x^2}}$

$\Rightarrow \cos(\arctan(x)) = \frac{1}{\sqrt{1+x^2}}$

3. (25 points) Calculate (using the limit laws) the following limits. If a limit does not exist determine if it is ∞, $-\infty$ or neither.

(a)

$$\lim_{x\to 0}(e^{x^2+x}+x^2+1)$$

Solution:

$$\lim_{x\to 0}\left(e^{(x^2+x)}+x^2+1\right)$$

$$= e^{\lim_{x\to 0}(x^2+x)} + \lim_{x\to 0}(x^2+1)$$

$$= e^0 + 1 = 2$$

(b)

$$\lim_{x\to 1^+}\frac{x^2-9}{x^2+2x-3}$$

Solution:

$$\frac{x^2-9}{x^2+2x-3} = \frac{(x+3)(x-3)}{(x+3)(x-1)}$$

$$\lim_{x\to 1^+}(x+3) = L_1 \quad , \quad \lim_{x\to 1^+}(x-3) = -2$$

$$x-1 \to 0^+ \text{ as } x\to 1^+ \Rightarrow \lim_{x\to 1^+}\frac{x^2-9}{x^2+2x-3} = -\infty$$

(c)

$$\lim_{x\to\infty} (\ln(3+x) - \ln(1+x))$$

Solution:

$$\lim_{x\to\infty} (\ln(3+x) - \ln(1+x)) = \lim_{x\to\infty} \ln\left(\frac{3+x}{1+x}\right)$$

$$\lim_{x\to\infty} \frac{3+x}{1+x} = \lim_{x\to\infty} \frac{\frac{3}{x}+1}{\frac{1}{x}+1} = \frac{1}{1} = 1$$

$$\Rightarrow \lim_{x\to\infty} (\ln(3+x) - \ln(1+x)) = \ln(1) = 0$$

(d)

$$\lim_{x\to-\infty} \frac{x^4-9}{x^3+7}$$

Solution:

$$\lim_{x\to-\infty} \frac{x^4-9}{x^3+7} = \lim_{x\to-\infty} \frac{1-\frac{9}{x^4}}{\frac{1}{x}+\frac{7}{x^4}}$$

$$1-\frac{9}{x^4} \to 1 \text{ as } x\to-\infty$$

$$\frac{1}{x}+\frac{7}{x^4} \to 0^- \text{ as } x\to-\infty$$

$$\Rightarrow \lim_{-\infty} \frac{x^4-9}{x^3+7} = -\infty.$$

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4. (a) (20 points) Prove, using ϵ, δ methods, that

$$\lim_{x\to 1} \frac{2x^2 - x - 1}{x - 1} = 3.$$

Hint: First simplify the function when $x \neq 1$.

Solution:

$x \neq 1 \Rightarrow \dfrac{2x^2 - x - 1}{x-1} = \dfrac{(2x+1)(x-1)}{(x-1)} = 2x+1$

Must prove $\lim\limits_{x \to -1} (3x+1) = -2$

Let $\varepsilon > 0$. $|(2x+1) - 3| < \varepsilon \Leftrightarrow |2x - 2| < \varepsilon$

$\Leftrightarrow |x-1| < \dfrac{\varepsilon}{2}$.

If $\delta = \dfrac{\varepsilon}{2}$ then $0 < |x-1| < \delta \Rightarrow$

$\left| \dfrac{2x^2 - x - 1}{x-1} - 3 \right| < \varepsilon$

$\Rightarrow \lim\limits_{x \to 1} \dfrac{2x^2 - x - 1}{x-1} = 3$

(b) (5 points) Is $\frac{2x^2-x-1}{x-1}$ differentiable at $x = 1$?

Solution:

No, it is <u>not</u> defined at $x = 1$.

PLEASE TURN OVER

5. (a) (15 points) Using the direct definition of the derivative to calculate the derivative of the function

$$f(x) = x^{3/2}.$$

What is the domain of the $f'(x)$?

Solution:

$(x > 0)$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^{3/2} - x^{3/2}}{h} = \lim_{h\to 0} \frac{\left((x+h)^{3/2} - x^{3/2}\right)\left((x+h)^{3/2} + x^{3/2}\right)}{h\left((x+h)^{3/2} + x^{3/2}\right)}$$

$$= \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h\left((x+h)^{3/2} + x^{3/2}\right)} = \lim_{h\to 0} \frac{3x^2 + 3xh + h^2}{(x+h)^{3/2} + x^{3/2}}$$

$$= \frac{3x^2}{2x^{3/2}} = \frac{3}{2}\sqrt{x}$$

Domain is $(0, \infty)$ ($x = 0$ is an end point of domain of $f(x)$ so $f'(0)$ DNE)

(b) (10 points) Show that the line $y = 3x + 2$ is **not** a tangent line to some point on the graph $y = f(x)$.

Solution:

$f'(x) = 3 \iff \frac{3}{2}\sqrt{x} = 3 \Rightarrow \sqrt{x} = 2 \iff x = 4$

$(4, 8)$ is <u>not</u> on $y = 3x+2 \Rightarrow$

$y = 3x + 2$ is <u>not</u> a tangent line.

END OF EXAM

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