

MATH 53 FINAL EXAM, PROF. SRIVASTAVA
MAY 11, 2018, 11:40PM–2:30PM, 155 DWINELLE HALL.

Name: Nikhil Srivastava

SID: _____

GSI: _____

NAME OF THE STUDENT TO YOUR LEFT: _____

NAME OF THE STUDENT TO YOUR RIGHT: _____

INSTRUCTIONS: Write all answers clearly in the provided space. This exam includes some space for scratch work at the bottom of pages 2 and 6 which will not be graded. Do not under any circumstances unstaple the exam. Write your name and SID on every page. Show your work — numerical answers without justification will be considered suspicious and will not be given full credit. You are allowed to bring one *single-sided handwritten letter size* cheat sheet. Calculators, phones, textbooks, and your own scratch paper are not allowed. **If you are seen writing after time is up, you will lose 20 points.**

When you are done, hand over your exam to your GSI *unless your GSI is Shiyu Li*, in which case hand it over to me.

UC BERKELEY HONOR CODE: *As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.*

Sign here: _____

Question:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
Points:	12	6	6	8	6	6	6	6	6	6	6	6	6	7	7	100

Do not turn over this page until your instructor tells you to do so.

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1. (12 points) Circle always true (T) or sometimes false (F) for each of the following. There is no need to provide an explanation. Two points each.

(a) Suppose $f(x, y)$ is differentiable and $f_x = 1$ and $f_y = -2$ at a point. Then there is a direction u such that $D_u f = 0$ at that point. (T) F

Choose $= \langle a, b \rangle$ so $a(1) + b(-2) = 0$.

(b) If the level curve of a differentiable function $g(x, y) = k$ intersects itself non-tangentially at a point P , then P must be a critical point of g . (T) F

Since ∇g is \perp to the level curve of g .

(c) If a and b are vectors in \mathbb{R}^3 then $a \times (a \times b)$ is always zero. T (F)

(d) The flux of $F = \langle x, 0, 0 \rangle$ across a sphere of radius 1 at the origin is strictly less than its flux across a sphere of radius 2 at the origin, where both are outwardly oriented. (T) F

Because $\text{flux} = \iiint_E \text{div}(F) dV = 2 \text{Vol}(E)$.

(e) If $\text{curl}(\nabla f) = \nabla f$ for a function f defined on \mathbb{R}^3 , then f must be a solution of the PDE

$$\partial^2 f / \partial x^2 + \partial^2 f / \partial y^2 + \partial^2 f / \partial z^2 = 0.$$

Take div of both sides;

$$0 = \text{div}(\text{curl}(\nabla f)) = \text{div}(\nabla f) = \Delta f$$

(f) If $S = \{(x, y, z) : f(x, y, z) = k\}$ is a level surface of a smooth function f with no critical points on S , then S must be orientable. (T) F

Take $\vec{n} = \frac{\nabla f}{\|\nabla f\|}$, which is a continuous normal vector since $\nabla f \perp S$.

[Scratch Space Below]

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2. Determine whether each of the following statements is true. If so, explain why, and if not, provide a counterexample.

(a) (3 points) If \mathbf{F} and \mathbf{G} are conservative vector fields defined on \mathbb{R}^3 then the sum $\mathbf{F} + \mathbf{G}$ is also conservative.

(b) (3 points) If $\mathbf{F} = \langle P, Q, R \rangle$ and $\mathbf{G} = \langle S, T, U \rangle$ are conservative vector fields defined on \mathbb{R}^3 , then the vector field

$$\mathbf{H} = \langle PS, QT, RU \rangle$$

with components equal to their entrywise products, is also conservative.

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3. (6 points) A particle moves along the intersection of the surfaces:

$$z = x^2 + \frac{y^2}{4}$$

and

$$x^2 + y^2 = 25.$$

Suppose its position vector at time t is $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ and we know that $x(0) = 3$, $y(0) = 4$, and $x'(0) = 4$. Calculate $y'(0)$ and $z'(0)$.

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4. Suppose $f(x, y) = xy$ and $x = r \cos \theta, y = r \sin \theta$.

(a) (4 points) Use the chain rule to find the partial derivatives $\partial f/\partial r$ and $\partial f/\partial \theta$.

(b) (4 points) Use this to approximate the value of f at the point $(r, \theta) = (1.001, -0.01)$.

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5. (6 points) Suppose $z = z(x, y)$ is a differentiable function satisfying $e^z = xyz$. Find $\partial z / \partial x$ and $\partial^2 z / \partial x^2$ as functions of x, y, z .

[Scratch Space Below]

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6. (6 points) Consider the function $f(x, y) = x^3/3 + y^3/3 + 5x - y$. Find and classify the critical points of the function $g(x, y) = |\nabla f(x, y)|^2$.

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7. (6 points) Find the extreme values of $f(x, y) = e^{-xy}$ in the region $D = \{(x, y) : x^2 + 4y^2 \leq 1\}$.

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8. (6 points) Compute the area of the simply connected “moustache” region enclosed by the parameterized curve

$$\mathbf{r}(t) = \langle 5 \cos(t), \sin(t) + \cos(4t) \rangle, \quad t \in [0, 2\pi].$$



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9. (6 points) Evaluate the integral

$$\iint_R e^{x+y} dA$$

where R is given by the inequality $|x| + |y| \leq 1$ by making an appropriate change of variables. (hint: sketch the region first)

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10. (6 points) Find the volume of the solid that lies between the paraboloid $z = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 2$.

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11. (6 points) Find the work done by the force field $\mathbf{F} = \langle z^2, x^2, y^2 \rangle$ on a particle moving along the line segment from $(1, 0, 0)$ to $(4, 1, 2)$.

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12. (6 points) A surface S is parameterized by

$$\mathbf{r}(u, v) = e^{-u^2} \langle 1, \sin(v), \cos(v) \rangle,$$

where

$$0 \leq u \leq \sqrt{\pi}, \quad u^2 \leq v \leq \pi.$$

Find its surface area.

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13. (6 points) Find the flux of the vector field

$$\mathbf{F}(x, y, z) = \langle y^3 z, x^3 z, 1 + e^{x^2+y^2} \rangle$$

through the paraboloid part S of the boundary of the solid region

$$z + x^2 + y^2 \leq 1; \quad z \geq 0,$$

where S is oriented upwards.

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14. (7 points) Let $\mathbf{F}(x, y, z) = \langle yz, -xz, 1 \rangle$. Let S be the portion of the paraboloid $z = 4 - x^2 - y^2$ which lies above the first octant $x \geq 0, y \geq 0, z \geq 0$; let C be the closed curve $C = C_1 + C_2 + C_3$ where the curves C_1, C_2, C_3 are formed by intersecting S with the xy, yz , and xz planes respectively, so that C is the boundary of S . Orient C so that it is traversed counterclockwise when seen from above in the first octant.

Use Stokes' theorem to compute the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ by reducing it to an appropriate surface integral over S .

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15. (7 points) Let S be the unit sphere centered at the origin, oriented outwards with normal vector \mathbf{n} , and let $f(x, y, z) = x + y^2 + z^3$. Calculate

$$\int \int_S D_{\mathbf{n}} f dS,$$

where $D_{\mathbf{n}}$ is the directional derivative along \mathbf{n} .