Name:____

Student ID Number:______________________________

Instructions:

1. You will have 80 minutes to complete the exam.
2. The exam is a total of 4 questions and each question is worth 10 points.
3. Unless stated otherwise, you may use results we proved in class and on the homework.
4. No books, notes, calculators, or electronic devices are permitted.
5. If you require additional space, please use the reverse side of the pages.
6. The exam has a total of 6 pages with the last page left blank for scratch work. Please verify that your copy has all 6 pages.

Question	**Score**	**Points**
1.		10
2.		10
3.		10
4.		10
Total		40

1. (a) For (E, d) a metric space, state what it means for a sequence $(x_n)_{n\in\mathbb{N}} \subset E$ to be **Cauchy**.
   (b) State what it means for a metric space (E, d) to be **complete**.
   (c) Let (E, d) be a compact metric space. Prove that (E, d) is complete.
   [Note: we proved this in class, but here I am asking you to write out a proof.]

2. (a) For metric spaces (E, d) and (E', d') and a function $f\colon E \to E'$, state what it means for f to be **uniformly continuous**.

   (b) Let (E, d) be a metric space and let $f, g\colon E \to \mathbb{R}$ be bounded, uniformly continuous functions. Recall that $f \cdot g\colon E \to \mathbb{R}$ is defined as

$$(f \cdot g)(x) = f(x)g(x) \qquad x \in E.$$

   Show that $f \cdot g$ is bounded and uniformly continuous.

3. Let (E, d) be a compact metric space and $S \subset E$.

   (a) Prove that S is dense in $\overline{S}$.

   (b) Suppose $f\colon S \to \mathbb{R}$ is uniformly continuous. Prove that f is bounded.

4. For $n \in \mathbb{N}$, consider the functions $f_n \colon \mathbb{R} \to \mathbb{R}$ defined as

$$f_n(x) = \frac{1}{1 + (n - x)^2}$$

(a) Prove $(f_n)_{n\in\mathbb{N}}$ converges pointwise on $\mathbb{R}$ and state the limit function.

(b) Prove $(f_n)_{n\in\mathbb{N}}$ does **not** converge uniformly on $\mathbb{R}$.

Scratch Work.