1. [40 points] **Short Questions**

(a) [10 points] Circle T or F for True or False

T   T F (i) The total electric charge of an isolated system remains constant regardless of changes within the system itself.

F   T F (ii) A plasma is a poor conductor. It's actually quite a good conductor

F   T F (iii) Any two opposite charges make a dipole. They need to be equal in magnitude.

T   T F (iv) The superposition principle for electric fields is well confirmed by experiment.

T   T F (v) A insulator's electrons are not free to move under electrical force.

T   T F (vi) Gauss's law is always valid.

T   T F (vii) The inverse of resistance and resistivity is conductance and conductivity.

T   T F (viii) The electric field is a vector field with SI units of Newtons per Coulomb (N/C) or, equivalently, volts per meter (V/m). The strength of the field at a given point is defined as the force that would be exerted on a positive test charge of +1 Coulomb placed at that point; the direction of the field is given by the direction of that force.

T   T F (ix) The capacitance of a collection of charges is determined solely by the geometry of the charge distribution.

T   T F (x) Electric charge is a characteristic of subatomic particles, and is quantized when expressed as a multiple of the so-called elementary charge $e = 1.6 \times 10^{-19}$ C.

T   T F (xi) An electric battery is composed of two dissimilar metals (or appropriate material such as carbon) and a solution called an electrolyte.

T   T F (xii) In order to have a steady-state electric current one needs a complete circuit.

F   T F (xiii) Conductivity plus Resistivity of a material is equal to one. $\sigma \cdot \rho = 1$

T   T F (xiv) In the microscopic model of electric current the validity of Ohm's law requires the electrons moving with a high velocity and drifting with a much lower one.

T   T F (xv) The lethal electric shock is around 0.1 Amperes (100 mA) for a period of a second or more.

F   T F (xvi) The electric flux is how much electric field passes through a surface per unit time. no time needed

F   T F (xvii) The electric potential is equal to the electric potential energy. $\Delta V = \frac{1}{q}\Delta U$

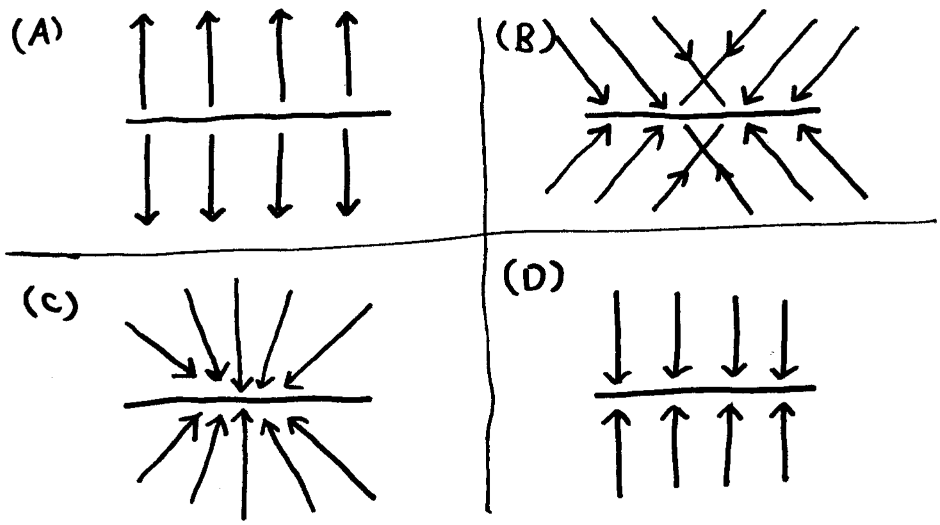
T   T F (xviii) A capacitor can store electric charge.

T   T F (xix) Electrical Resistivity depends upon Temperature .

T   T F (xx) Kirchhoff's Rules are really only conservation of charge and energy.

(b) [10 points] Circle correct answer Electrostatics

(i) Which of the following figures correctly depicts the field lines from an infinite uniformly negatively charged sheet? Note that the sheet is being viewed edge-on in all pictures.



(D)

(ii) In the diagram from part (i), what is wrong with figure B? (Pick only those statements that apply to figure B.) Circle all that apply.

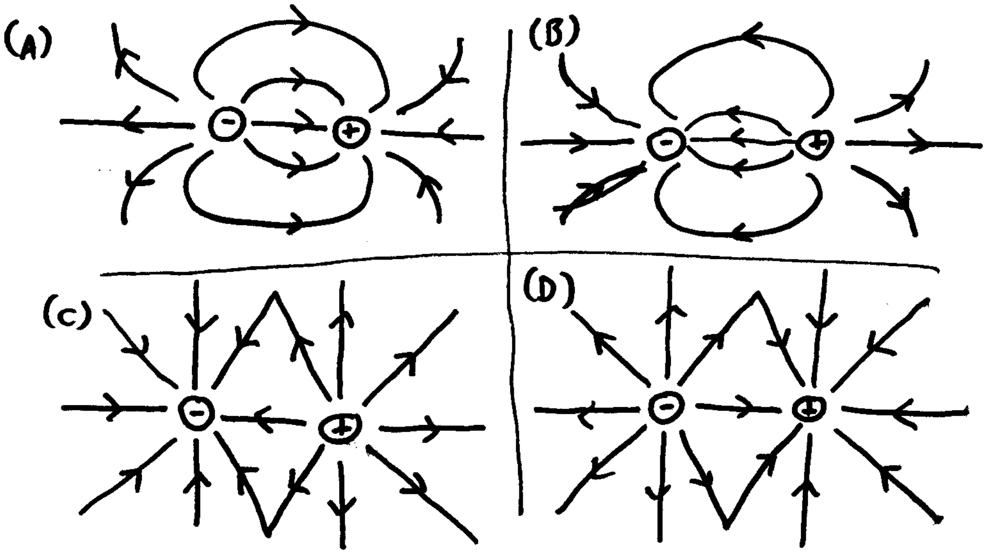
(A) Field lines cannot cross each other.

(B) The field lines should be parallel because of the sheet's symmetry.

(C) The field lines should spread apart as they leave the sheet to indicate the weakening of the field with distance.

(D) The field lines should always end on negative charges or at infinity.

(iii) Which of the following figures shows the correct electric field lines for an electric dipole?

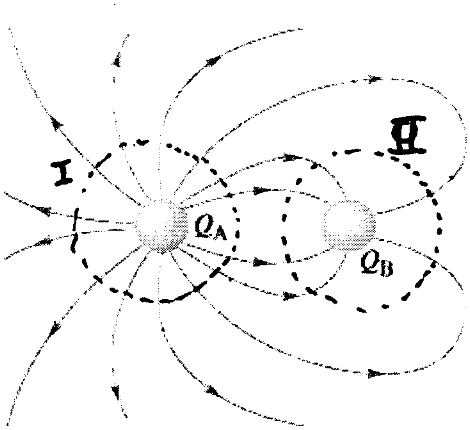


(B)

(iv) In the diagram from (iii) (Part D of figure), what is wrong with figure D? (Pick only those statements that apply to figure D.) Circle all that apply.

(A) Field lines cannot cross each other.
(B) The field lines should turn sharply as you move from one charge to the other.
(C) The field lines should be smooth curves in vacuum.
(D) The field lines should always end on negative charges or at infinity

(v) In the figure below, the electric field lines are shown for a system of two point charges, Q_A and Q_B. Which of the following could represent the magnitudes and signs of Q_A and Q_B? In the following, take q to be a positive quantity.



$\Phi_I \sim +14$ lines

$\Phi_{II} \sim -6$ lines

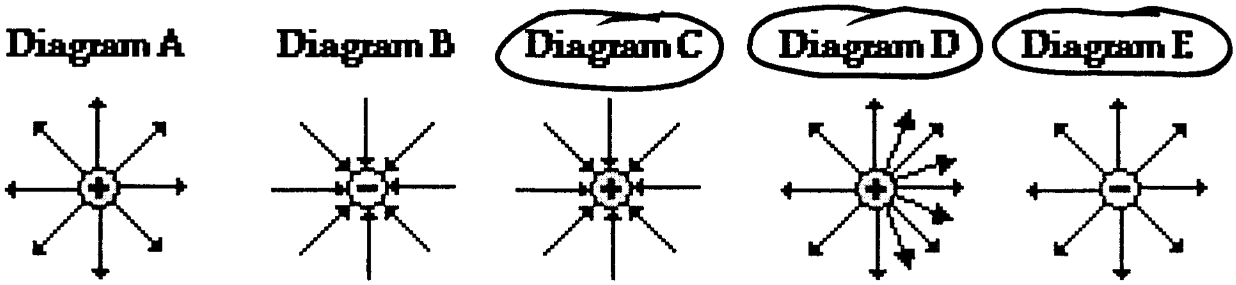
$\Rightarrow \Phi = \frac{q_{enc}}{\epsilon_0}$

$\Rightarrow \frac{Q_A}{Q_B} = -\frac{7}{3}$

& $Q_A > 0$
(lines go out)

(A) $Q_A = +q$ and $Q_B = -q$.
(B) $Q_A = +7q$ and $Q_B = -3q$.
(C) $Q_A = +3q$ and $Q_B = -7q$.
(D) $Q_A = -3q$ and $Q_B = +7q$.
(E) $Q_A = -7q$ and $Q_B = +3q$.
(F) All are correct.

(vi) Several electric field line patterns are shown in the diagrams below. Which of these patterns are incorrect? Circle the letter of the incorrect patterns and explain what is wrong with all incorrect diagrams.



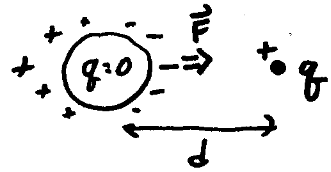
Direction   Symmetry   Direction

(vii) There are several ways to produce electrostatic charging. Which is **not** one?

(A) Rubbing two dissimilar materials together.
(B) Induction.
(C) Contact.
(D) Radio broadcast.
(E) Electric Sparking.
(F) None cause electrostatic charging.

(viii) A neutral conducting sphere of radius r is located a distance d from a dust particle of charge q. The conducting sphere exerts a force on the charged particle which is

(A) zero. (no net force)
(B) repulsive.
(C) attractive.
(D) not enough information to determine.



(ix) A fixed potential difference V exists between a pair of close parallel plates carrying opposite charges +Q and -Q. Which of the following would **not** increase the magnitude of charge that you could put on the plates?

$C = \frac{K\epsilon_0 A}{d}$ $\quad Q = CV$

(A) Increase the size of the plates. $A\uparrow \Rightarrow C\uparrow \Rightarrow Q\uparrow$
(B) Move the plates farther apart. $d\uparrow \Rightarrow C\downarrow \Rightarrow Q\downarrow$
(C) Fill the space between the plates with paper. $K\uparrow \Rightarrow C\uparrow \Rightarrow Q\uparrow$
(D) Increased the fixed potential difference V. $V\uparrow \Rightarrow Q\uparrow$
(E) None of the above.

(x) Sharper Image and other producers makes substantial profit from selling ion generators (electrode creates negative ions in the air) as air cleaner and antibacterial agent. How does generating negative ions clear the air of dust and bacteria?

(A) The ions are attracted to the dust and bacteria by polarizing them and make them charged. The dust is then attracted to any polarizable surface and sticks.

(B) The ions are attracted to the dust and bacteria and chemically react to disassemble them.

(C) The ions are attracted to the dust and bacteria and make them effectively charged. Since the Earth is positively charged by the large thunderstorms in the Amazon, the dust and bacteria are attracted to the floor.

(D) The coronel discharge that makes the negative ions is like a spark and actually burns up the dust and bacteria.

(E) It does not really work but the negative ions smell like fresh bleach so that people think the air is cleaner.

(c) [10 points] Concentric Cylinders

(c.1) [2 points] Without doing any calculation, draw eight electric field lines (solid) and four equipotential contours (dotted) in the electrical system shown in Figure 1 consisting of two concentric cylinders with charge $+q$ per unit length on the inner cylinder and q per unit length on the outer cylinder. Label the potential contours so that $V_1 > V_2 > V_3 > V_4$. Explicitly label any regions where the electric field is zero.

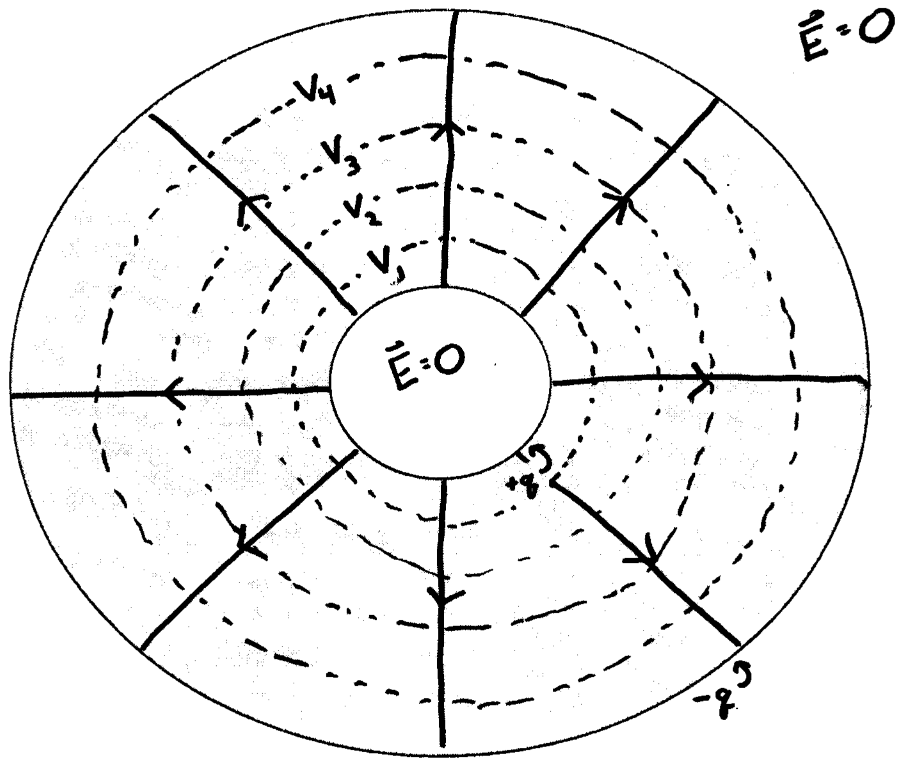


Figure 1: Figure for problem 1(c) showing two concentric cylinders with charge $+q$ per unit length on the inner cylinder and q per unit length on the outer cylinder.

(c.2) [2 points] What is an expression for the electric field in terms of the charge q and the radius given r_1 and r_2 are the inner and outer radii?

Cylindrical Symmetry $\Rightarrow$ $\vec{E} = \dfrac{Q_{enc}(r)/\ell}{2\pi r \epsilon_0}\hat{r}$ $\quad \dfrac{Q_{enc}(r)}{\ell} = \begin{cases} 0 & r<r_1 \\ +q & r_1<r<r_2 \\ 0 & r_2<r \end{cases}$

$$\Rightarrow \boxed{\vec{E} = \begin{cases} 0 & r<r_1 \\ \dfrac{+q}{2\pi\epsilon_0 r}\hat{r} & r_1<r<r_2 \\ 0 & r_2<r \end{cases}}$$

(c.3) [2 points] What is an expression for the potential V as a function of radius r? Set $V(r=0)=0$.

$$\Delta V = -\int_a^b \vec{E}\cdot d\vec{\ell}.$$ Take a radially outward path

$\Rightarrow d\vec{\ell} = \hat{r}\,dr$

$r<r_1 \Rightarrow V(r) = \int_0^r 0 = 0$

$r_1<r<r_2 \Rightarrow V(r) = -\int_0^{r_1} 0 + -\int_{r_1}^{r} \frac{q}{2\pi r\epsilon_0}\hat{r}\cdot\hat{r}\,dr = \frac{-q}{2\pi\epsilon_0}\ln\frac{r}{r_1}$

$r_2<r \Rightarrow V(r) = -\int_0^{r_1} 0 + -\int_{r_1}^{r_2}\frac{q}{2\pi r\epsilon_0}\hat{r}\cdot\hat{r}\,dr + -\int_{r_2}^{r} 0 = \frac{-q}{2\pi\epsilon_0}\ln\frac{r_2}{r_1}$

$$\Rightarrow \boxed{V(r) = \begin{cases} 0 & r<r_1 \\ \frac{-q}{2\pi\epsilon_0}\ln\frac{r}{r_1} & r_1<r<r_2 \\ \frac{-q}{2\pi\epsilon_0}\ln\frac{r_2}{r_1} & r_2<r \end{cases}}$$

(c.4) [2 points] What is the stored energy per unit length in the system at voltage V?

$$\mathcal{U} = \int \frac{1}{2}\epsilon_0 E^2\, dV = \int_{r_1}^{r_2} \underbrace{2\pi r\ell\, dr}_{dV}\, \frac{1}{2}\epsilon_0 \frac{q^2}{4\pi^2\epsilon_0^2 r^2}$$

$$\Rightarrow \frac{\mathcal{U}}{\ell} = \frac{q^2}{4\pi\epsilon_0}\int_{r_1}^{r_2}\frac{dr}{r}$$

$$\Rightarrow \boxed{\frac{\mathcal{U}}{\ell} = \frac{q^2}{4\pi\epsilon_0}\ln\frac{r_2}{r_1}}$$

$\leftarrow$ or $\rightarrow$

$$C = \frac{Q}{V} \Rightarrow \frac{C}{\ell} = \frac{q}{\frac{q}{2\pi\epsilon_0}\ln\frac{r_2}{r_1}} \Rightarrow \frac{C}{\ell} = \frac{2\pi\epsilon_0}{\ln\frac{r_2}{r_1}}$$

$$\mathcal{U} = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{q^2\ell^2}{2\pi\epsilon_0\ell/\ln\frac{r_2}{r_1}}$$

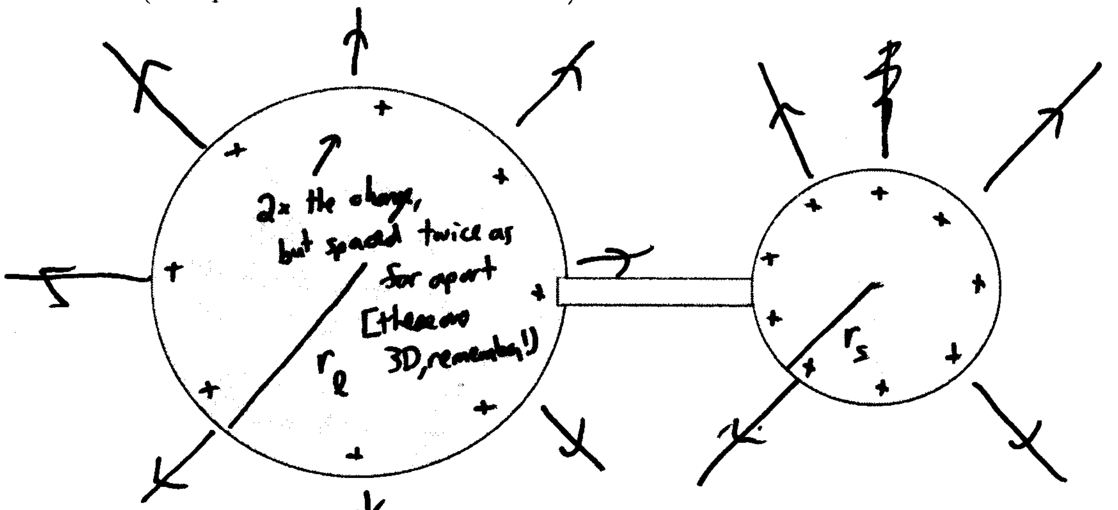
$$\Rightarrow \frac{\mathcal{U}}{\ell} = \frac{1}{2}\frac{q^2}{2\pi\epsilon_0}\ln\frac{r_2}{r_1}$$

$$\boxed{\frac{\mathcal{U}}{\ell} = \frac{q^2}{4\pi\epsilon_0}\ln\frac{r_2}{r_1}}$$

(c.5) [2 points] If the shaded region in between the cylinders is filled with a dielectric material with constant K while keeping the charge per unit length fixed, how do the quantities calculated in parts c.2, c.3, and c.4 change? Briefly describe your reasoning.

$$\epsilon_0 \mapsto K\epsilon_0 \Rightarrow \vec{E} \to \frac{\vec{E}}{K},\quad V \to \frac{V}{K},\quad \frac{\mathcal{U}}{\ell} \to \frac{\mathcal{U}/\ell}{K}$$

(d) [10 points] Two spheres placed far apart are positively charged to the same potential (as they are connected by a thin conducting wire) as seen in Figure 2. The radius r_s of the smaller spherical end is about half of the radius r_l of the larger end.
(d.1) [4 points] Sketch in the figure the distribution of the positive charges and the electric field lines. (See question sections below first.)



$q_l \sim 2q_s \Rightarrow$ twice as many field lines $\qquad \sigma_i \sim \frac{1}{r_i} \Rightarrow \sigma_l \sim \frac{\sigma_s}{2}$

Figure 2: Figure for problem 1(d) Two conducting spheres one with twice the diameter as the other and the two are connected by a conductor.

Explain your reasoning:
(d.2) [2 points] Give an expression for the charge q_i on each sphere for potential V.

$$V_i = \frac{q_i}{4\pi\epsilon_0 r_i}$$

• Connected by a wire ⇒ all @ same pot'l V

$$\Rightarrow \boxed{q_i = (4\pi\epsilon_0 V)\, r_i}$$

(d.3) [2 points] Give an expression for the electric field magnitude at the surface of each sphere for potential V.

$$\Delta E_{surface} = \frac{\sigma_{surface}}{\epsilon_0}$$

$$E_{inside} = 0 \Rightarrow \vec{E}_{surface} = \frac{\overbrace{q_i / 4\pi r_i^2}^{\sigma_i}\ \hat{r}}{\epsilon_0}$$

$\hat{r}$ normal to surface

$$= \frac{\cancel{4\pi\epsilon_0} r_i V}{4\pi r_i^2 \cancel{\epsilon_0}}$$

$$\boxed{E_{surface,i} = \frac{V}{r_i}}$$

Equivalently, you could have just started with $\vec{E}_i = \frac{q_i}{4\pi\epsilon_0 r_i^2}\hat{r}$!

(d.4) [2 points] Give an expression for the electric field magnitude at the surface of any convex conductor (radius of curvature r) at potential V. Explain why forks are more dangerous in microwave ovens than spoons.

Based on above, $E_i = \frac{V}{r_i}$

The tines on a fork have a very small radius of curvature, so the Electric field is much higher, potentially approaching the breakdown field of air, causing sparks.

MT2 - Prof. Smoot (lecture 3)

Prob 2:

a (7 pts) (1 pt each)

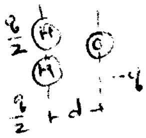
(i) H $\left[F = -\frac{ke^2}{r^2}\hat{r} = -\frac{(9\times10^9)(1.6\times10^{-19})^2}{(0.53\times10^{-10})^2}\hat{r} = -8\times10^{-8}\,\hat{r}\ N \right]$

(ii) F $\left[E = +\frac{ke}{r^2}\hat{r} = \frac{(9\times10^9)(1.6\times10^{-19})}{(0.53\times10^{-10})^2}\hat{r} = 5\times10^{11}\,\hat{r}\ \frac{N}{C} \right]$

(iii) C $\left[V = \frac{ke}{r} = \frac{(9\times10^9)(1.6\times10^{-19})}{0.53\times10^{-10}} = 27.2\ V \right]$

(iv) A $\left[|E| = \frac{\sigma}{2\epsilon_0} = \frac{10^{-4}}{2\times8.85\times10^{-12}} = 5.6\times10^{6}\ \frac{N}{C} \right]$

numbering glitch

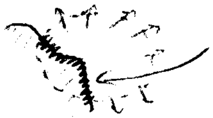
(iv) E $\left[p = qd \right.$ 

$\rightarrow q = \frac{p}{d} = \frac{6.1\times10^{-30}}{10^{-10}} = 6.1\times10^{-20}C \quad \frac{e}{1.6\times10^{-19}C} \sim 0.4e \left. \right]$ in units of 'e'

So, $q/2$ on each H-atom $= \boxed{0.2e}$

(v) B $\left[|\vec{\tau}| = |\vec{p}\times\vec{E}| = pE\sin\theta^{\,90^\circ} = (6.1\times10^{-30})(5\times10^{4}) \approx 3\times10^{-25}\ Nm \right]$

(vi) H $\left[\text{Gauss' law} \rightarrow \text{Flux} = Q_{enclosed}/\epsilon_0 = \frac{-1.6\times10^{-19}}{8.85\times10^{-12}} = -1.81\times10^{-8}\ \frac{Nm^2}{C} \right]$

(vii) A $\left[\text{Gauss' law} \rightarrow \text{flux} = Q_{enc}/\epsilon_0 \right.$ 
$Q_{enc} = \text{flux}\times\epsilon_0 = (2\times10^5)(8.85e\ldots) = 1.77e{-6}\ C$

Grading scheme: (1 pt each problem)
Max. score = 7 pts.
So, 7 or 8 probs correct → 7 pts.
1,2,3,4,5, or 6 probs. correct → that's your score.

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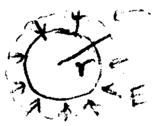
b (3 pts)

(i) F (lower case) $\left[\right.$ 

Gaussian surface very close to Earth's surface.

$-EA = Q/\epsilon_0 \Rightarrow Q = -\epsilon_0 E A = -(8.85e{-12})(150)4\pi\times(6.4e6)^2$, $A = 4\pi r^2$

$\boxed{Q = -6.8\times10^{5}\,C}$.

(ii) G $\left[V = \frac{kQ}{r} = \frac{(9\times10^9)(-6.8\times10^5)}{6.4\times10^6} = -9.56\times10^{5} \approx \boxed{-10^{9}\,V} \right.$

(iii) B $\left[\right.$ Superconductors & semiconductors have several other properties that are irrelevant here.

Grading scheme: 1pt each problem.

Prob 2.

$\boxed{c}$ (5 pts) (i) D $\left[1A = \frac{1C}{s} = 1\frac{C}{s} \times \frac{1e^-}{1.6\times10^{-19}C} \approx 6\times10^{18} \frac{e}{s} \right]$

(ii) D $\left[V = 4\times1.5V = 6V \quad I = 0.3A \Rightarrow R = \frac{V}{I} = \frac{6V}{0.3A} = 20\Omega \right]$

(iii) C $\left[P = V_{rms} I_{rms} \rightarrow I_{rms} = \frac{P}{V_{rms}} = \frac{1100W}{120V} \approx 9\,A\ (rms) \right]$

(iv) B $\left[R = \frac{\rho l}{A} = \frac{(1.6\times10^{-8})(305)}{\pi\left(\frac{2.58826\times10^{-3}}{2}\right)^2} \approx 1\Omega \right]$

(v) C $\left[\rho_T = \rho_0(1+\alpha\Delta T) \right.$

$R_0 = \frac{\rho_0 l}{A}$ & $R_T = \frac{\rho_T l}{A} \Rightarrow \frac{R_T}{R_0} = \frac{\rho_T}{\rho_0} \Rightarrow \boxed{R_T = R_0\frac{\rho_T}{\rho_0}}$

$R_T = 0.3(1+0.0068\times15) = \underline{0.33\Omega}$.

1 pt. each prob.

$\boxed{d}$ (5pts) - 1 pt each.

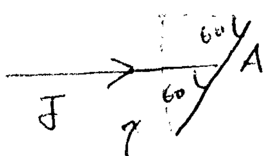
(i) C [pg. 692 (text)] $\rightarrow$ 100mA

(ii) B $\left[R = \rho l/A \rightarrow \rho = \frac{AR}{l} = \frac{(0.1)^2\, 200}{0.2} = 10\ \Omega\cdot m \right]$

(iii) C $\left[J = \underbrace{\sigma}_{\text{conductivity}} E = \frac{1}{\underbrace{\rho}_{\text{resistivity}}} E \Rightarrow E = \rho J = 1\times10 = 10\frac{V}{m} \right]$

iv) $\boxed{\text{Invalid answer}}$. (everyone gets credit)

As stated, the current density $\vec{J}$ arrived at the area at 60°



$I = \vec{J}\cdot\vec{A} = JA\cos30 = JA\sin60$
$= \underline{0.087\,A}$.

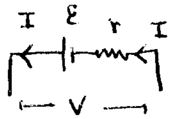
Effective area is the vertical $= A\sin60° \approx 0.866\,A$

v) B $\left[I_{rms} = \frac{I_{peak}}{\sqrt{2}} = \frac{15A}{\sqrt{2}} = 10.6\,A \right]$

Prob 2

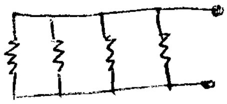
a 5pts.

(i) D

[

$V_{net} = \mathcal{E} - Ir = 0.15 - (20\times10^{-3})(0.25)$
$= 0.145\ V$]

(ii) C

[

$R_{eq} = \left(\frac{1}{R}+\frac{1}{R}+\frac{1}{R}+\frac{1}{R}\right)^{-1} = \frac{R}{4} = \frac{10}{4} = 2.5\,\Omega$]

(iii) A

[$\underset{(\text{series})}{C_{eq}} = \left(\frac{1}{C}+\frac{1}{C}+\frac{1}{C}+\frac{1}{C}\right)^{-1} = \frac{C}{4} = \frac{10}{4} = 2.5\,pF$]

(iv) C

[V is equal in parallel]

(v) D

[$U = \frac{1}{2}\frac{Q^2}{C}$ $\rightarrow$ If $Q \rightarrow 2Q$ $\quad U \rightarrow 4U$]

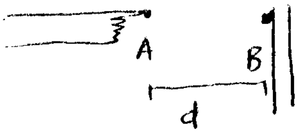
1pt each part

MT2 - Prof. Smoot (lecture 3).

Prob 3:

(also see comm[illegible] after solutions)

(a) (5 pts) There is a potential difference $\Delta V = |V_B - V_A|$ between the student and the cable. The (approximate) E field is then $\boxed{E \approx \frac{\Delta V}{d}}$ where d is the distance between the two.



$V_A = -300\,000\ V = -3\times10^5\ V$

$V_B = 0\ V$

$$\boxed{\Delta V = 3\times10^5\ V}$$

If this E field is strong enough to cause the electric breakdown of the air in the vicinity – a current will flow through the (now conducting) air and into the student. The power here is sufficient to electrocute the hapless academic.

The breakdown field for air (text · pg 633 · Table 24-1) is

$$\boxed{E_{breakdown} = 3\times10^6\ \frac{V}{m}}$$

4 – calculation
1 – explanation

$$= \frac{\Delta V}{d} \Rightarrow \boxed{d = \frac{\Delta V}{E_{breakdown}} = \frac{3\times10^5\ V}{3\times10^6\ \frac{V}{m}} = 0.1\ m}$$

(b) Power transmitted $= P_{trans} = VI \Rightarrow \boxed{I = \frac{P_{trans}}{V}}$

(b.1) (2 pts) Power loss along cable $= P_{loss} = I^2 R_{cable}$

$\boxed{R_{cable} = 0.31\ \frac{\Omega}{km} \times L}$ where L is the cable length in km.

$$\therefore P_{loss} = \left(\frac{P_{trans}}{V}\right)^2 (0.31\,L) \Rightarrow \boxed{P_{loss} = \frac{P_{trans}^2}{V^2}\,0.31\,L}$$

3b)

(b.2) Given that P_{loss} is 20% of P_{trans} ($\therefore P_{delivered} = 0.8\, P_{trans}$)

(2 pts)

$\Rightarrow \dfrac{P_{loss}}{P_{trans}} = 0.2$ (use P_{loss} formula from (b.1))

$$\frac{1}{P_{trans}}\left(\frac{P_{trans}^2}{V^2}\, 0.31\, L\right) = 0.2$$

$P_{delivered}$ is 80% of

$$\Rightarrow \boxed{P_{trans} = \frac{0.2\, V^2}{0.31\, L}} = \frac{0.2\,(6\times10^5)^2}{0.31\times 7000} = 3.3\times10^7\ \text{W}$$

$$\boxed{P_{delivered} = 26\ \text{MW}}$$

Note: The line is bipolar with the ends at $+3\times10^5$ V and -3×10^5 V (the load in between). The pot. diff. (V) is $3\times10^5 - -3\times10^5 = \underline{6\times10^5}$ V.

note applies to (b.3) also.

Grading: 3 common mistakes:
(i) voltage = 300 kV (ii) Using L = 7000 or cm
(iii) only finding P_{trans}

1 to 2 mistakes	→ score = 1/2
3 mistakes	→ score = 0/2

(b.3)

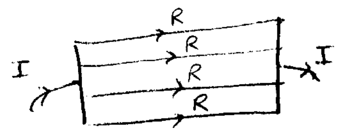
(i) Current: $P_{trans} = VI \Rightarrow I = \dfrac{P_{trans}}{V} = \dfrac{5\times10^9\,\text{W}}{2\times(800\times10^3\,\text{V})} = \boxed{3125\ \text{A}}$

(1 pt) → (voltage error not penalized again)

$\therefore \boxed{I \simeq 3.13\ \text{kA}}$ (3 sig figs).

(ii) P_{loss} 4 wires in parallel

(1 pt) Could calculate 2 ways:



R → resis. of each line

(1) I splits 4 ways → $\frac{I}{4}$ in each line.

$$P_{loss} = 4\times P_{loss}^{(1\,line)} = 4\times\left[\left(\frac{I}{4}\right)^2 R\right] = \frac{I^2 R}{4}$$

(2) OR; equivalent Resis. of 4 wires is (parallel) → $\frac{R}{4}$.

$$\therefore \boxed{P_{loss} = I^2\left(\frac{R}{4}\right)} = (3.13\times10^3)^2\left(\frac{0.31\times1400}{4}\right)$$

(I → found above to be 3.13 kA
R = 0.31 L = $0.31\ \frac{\Omega}{\text{km}}\times 1400$ km)

$$\therefore \boxed{\begin{aligned}P_{loss} &= 1.1\times10^9\ \text{W}\\ &= 1.1\ \text{GW}\end{aligned}}$$

(iii) $R = \dfrac{\rho l}{A} \Rightarrow$ If $A \to \frac{1}{2}A$, resistance doubles

(1 pt) $\therefore P_{loss}^{(new)} = 2\, P_{loss}^{(old)} = \boxed{2.2\ \text{GW}}$

3c)

C.1
(2 pts) $P_{trans} = VI = (20\text{ kV})(2200\text{ A}) = \boxed{44\text{ MW}}$ (at 80 K)

C.2

Normal Conductor . $P^N_{loss} = I^2R = I^2\, 0.31\, L$

(1 pt) → $\left(\dfrac{P^N_{loss}}{L}\right) = I^2\, 0.31 = (2200)^2\, 0.31 = \boxed{1.5\,\dfrac{\text{MW}}{\text{km}}}$

Power cost of refrigeration $= \boxed{P_{ref} = \frac{1}{2} P^N_{loss}}$ ← stated in problem

(1 pt) → $\boxed{\dfrac{P_{ref}}{\text{length}} = 0.75\,\dfrac{\text{MW}}{\text{km}}}$

C.3

(i) $P_{max} = V I_{new} = V(1.2\, I_{old}) = 1.2\, P_{trans}$ ← from C.1

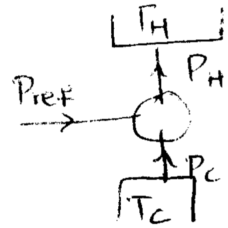
(2 pts) (20% more current)

$\boxed{P_{max} = 53\text{ MW}}$ (at 40 K)

(ii) While the transmitted power goes up, the refrigeration system has to do more work to keep the temperature at 40K (than it did at 80K).
(2 pts)

The refrigerator is described in the problem.

* Heat leak is the same ⇒ P_c is same (this is heat removed from cable)



* $\dfrac{\text{efficiency}}{\text{ideal efficiency}} = \text{constant}$

$\Rightarrow \dfrac{e}{e_{ideal}} = \text{constant} \Rightarrow \dfrac{(P_c/P_{ref})}{(T_L/(T_H-T_L))} = \dfrac{(P_c/P_{ref}')}{(T_L'/(T_H-T_L'))}$

$\left(e = \dfrac{Q_c}{W} = \dfrac{P_c}{P_{ref}}\right)$
↑ energy ↑ power

$T_L = 80K$ $T_H = 300K$
P_{ref} = old power used

$T_L' = 40K$
P_{ref}' = new power used

3c)

$\boxed{C.3}$ continued

(ii)

part $\boxed{C.2}$

$$\therefore \quad P_{ref}' = P_{ref}\left(\frac{T_L}{T_L'}\right) \cdot \frac{T_H - T_L'}{T_H - T_L} = (0.75)\left(\frac{80}{40}\right)\left(\frac{300-40}{300-80}\right)$$

(approx. as 1 is OK)

$$P_{ref}' \simeq 2.4\, P_{ref} = 1.8\ \frac{MW}{km}$$

(per length)

(*) $\therefore$ increase in refr. power $= \Delta P_{ref} = 1.4\, P_{ref} \simeq 1\ \frac{MW}{km}$.

For 1 km cable ; $\boxed{\Delta P_{ref} = 1 MW}$

(*) increase in transmitted power $= \Delta P_{trans} = 0.2\, P_{trans}$
$= \boxed{8.8\ MW}$

$\therefore$ Net power gain $= 8.8\ MW - 1\ MW$
$= \boxed{7.8\ MW}$.

Partial credit (variable) for $\boxed{C.3}$ for some (RELEVANT) meaningful physi[cs]

If you didn't do the problem right, partial credit is a reward for some meaningful response → regrade requests in such cases are futile).

Breakdown : usually : (1) Understanding of net power gain → 1 pt
(2) Fridge analysis → 1 pt

Prob 3: Common errors.

3(a) Anything other than dielectric breakdown - no credit

3(b) (i) Using $\frac{V^2}{R}$ for power loss → fundamental mistake!

R_{cable} is in series with R_{load}; so V across R_{cable} is NOT the full V (600kV).

(ii) Unit mistakes: $R = 0.31 L$ (Units are $0.31 \frac{\Omega}{km} \times L$ ← in km)

Lots of students entered L in meters.

(iii) For (b.2) → $P_{delivered} = P_{trans} - P_{loss}$
$= 0.8 P_{trans}$.

Lots of students found just P_{trans}.

(NOTE: ~~(i)~~, (ii), (iii), (iv) all present → no credit left !!)

(iv) $\Delta V = 600 kV$ (bipolar line).

Lots of students used 300kV. (same for b.3)

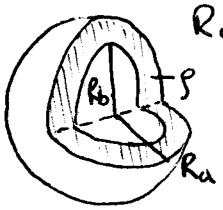
$\Delta V = 800 \times 2 = 1600 kV$

(NOTE: (ii) & (iv) → I tried not to penalize more than once → as long as other things made sense)

(v) Forgetting that resistance for 4 cables in parallel is $\frac{R}{4} = \frac{0.31L}{4}$.

3(c) Most students got (c.1) & (c.2) and P_{max} in (c.3). Very few got the refrigerator analysis. Approximating $e_{ideal} = \frac{T_L}{T_H - T_L}$ as $\frac{T_L}{T_H}$ was given full credit.

4) A nonconducting sphere has a uniform charge density ρ and outer radius R_a. It is hollowed out to an inner radius R_b. Find $\vec{E}(\vec{r})$.



we must find the electric field in three regions

For $r<R_b$:

$$\Phi_E = \oint\!\!\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}, \quad q_{enc}=0 \text{ for } r<R_b, \text{ so } \boxed{\vec{E}(\vec{r})=0, \text{ for } r<R_b}$$

$R_b\le r\le R_a$: In this case, $q_{enc} = \rho\frac{4}{3}\pi(r^3-R_b^3)$, so from Gauss's Law

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \frac{4\pi\rho}{3\epsilon_0}(r^3-R_b^3)$$

The Gaussian surface we make is a sphere with radius r such that $R_b<r<R_a$. The electric field is spherically symmetric, and so it is constant in magnitude along the surface of the Gaussian surface, and it is perpendicular to the surface everywhere as well. The vector $d\vec{A}$ is always perpendicular to the surface, so $\vec{E}(r)\cdot d\vec{A} = E(r)dA$ on the Gaussian surface. So we find

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = E(r)\oint\!\!\oint_{\text{sphere}} dA = E(r)\int_0^\pi d\theta\sin\theta\int_0^{2\pi}d\phi\, r^2 = E(r)r^2(2)(2\pi)$$

So putting this all together, we find

$$E(r)4\pi r^2 = \frac{4\pi\rho}{3\epsilon_0}(r^3-R_b^3) \rightarrow \boxed{\vec{E}(r) = \frac{\rho(r^3-R_b^3)}{3\epsilon_0 r^2}\hat{r}, \text{ for } R_b<r<R_a}$$

$r>R_a$: Now $q_{enc} = \rho\frac{4}{3}\pi(R_a^3-R_b^3)$, so using the same technique as above, we find

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \frac{\rho 4\pi}{3\epsilon_0}(R_a^3-R_b^3) \rightarrow E(r)4\pi r^2 = \frac{\rho 4\pi}{3\epsilon_0}(R_a^3-R_b^3)$$

$$\text{so } \boxed{\vec{E}(r) = \frac{\rho(R_a^3-R_b^3)}{3\epsilon_0 r^2}\hat{r}, \text{ for } r>R_a}$$

b. Find $V(r)$ from the results in (a). Set $V(\infty)=0$. Plot $E(r)$ and $V(r)$ for $0\le r\le 2R_a$

we will use the equation $V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{l}$

For $r>R_a$: $V(r)-V(\infty) = -\int_\infty^r \vec{E}(r)\cdot d\vec{l}$ Now since $\vec{E}(r)$ points radially outward from the origin and $d\vec{l}$ is radially inward from infinity, then $\vec{E}(r)\cdot d\vec{l} = -E(r)dl$, but since dl comes inward from infinity then $dl=-dr$, so we find that $\vec{E}(r)\cdot d\vec{l} = E(r)dr$

$$\text{so } V(r)-V(\infty) = -\int_\infty^r \frac{\rho\, dr'}{3\epsilon_0 r'^2}(R_a^3-R_b^3) = -\frac{\rho(R_a^3-R_b^3)}{3\epsilon_0}\left[-\frac{1}{r'}\right]_\infty^r = \frac{\rho(R_a^3-R_b^3)}{3\epsilon_0 r}$$

Since $V(\infty)=0$, we have $\boxed{V(r) = \frac{\rho(R_a^3-R_b^3)}{3\epsilon_0 r}, \text{ for } r>R_a}$

For $R_b\le r\le R_a$: $V(r)-V(R_a) = -\int_{R_a}^r \frac{\rho}{3\epsilon_0}\frac{(r'^3-R_b^3)}{r'^2}dr'$

$$= \frac{-\rho}{3\epsilon_0}\left[\int_{R_a}^r r'dr' - R_b^3\int_{R_a}^r\frac{dr'}{r'^2}\right] = \frac{-\rho}{3\epsilon_0}\left[\frac{1}{2}(r^2-R_a^2) - R_b^3\left(-\frac{1}{r}+\frac{1}{R_a}\right)\right]$$

$$= \frac{\rho}{3\epsilon_0}\left[\frac{1}{2}(R_a^2-r^2) + R_b^3\left(\frac{1}{R_a}-\frac{1}{r}\right)\right]$$, so by evaluating $V(R_a)$ using the above expression

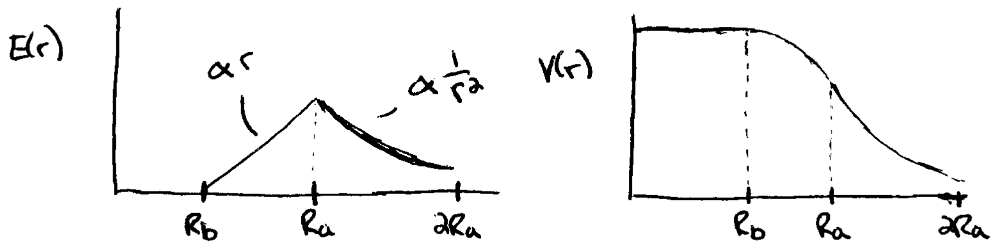
$$\boxed{V(r) = \frac{\rho(R_a^3-R_b^3)}{3\epsilon_0 R_a} + \frac{\rho}{3\epsilon_0}\left[\frac{1}{2}(R_a^2-r^2)+R_b^3\left(\frac{1}{R_a}-\frac{1}{r}\right)\right] = \frac{\rho}{3\epsilon_0}\left[\frac{1}{2}(3R_a^2-r^2)-\frac{R_b^3}{r}\right] \text{ for } R_b\le r\le R_a}$$

4) b. continued...

For $r < R_b$: Since $\vec{E}(r) = 0$, then $V(r) - V(R_b) = -\int_{R_b}^{r} 0 \cdot d\vec{l} = 0$, so $V(r) = V(R_b)$

and we just evaluate the previous expression at $r = R_b$

$$\boxed{V(r) = \frac{\rho}{3\epsilon_0}\left[\frac{1}{2}(3R_a^2 - R_b^3) - R_b^2\right] = \frac{\rho}{2\epsilon_0}(R_a^2 - R_b^2) \quad , \text{for } r < R_b}$$



c. Now we replace the nonconducting sphere with two conducting ~~spheres~~ shells with the same two radii R_a and R_b and one is given charge $+q$ and the other $-q$. Calculate the stored energy and capacitance.

The electric field between the plates is found again from Gauss's law, where I will put $+q$ on the inner shell.

$$\oiint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0} \rightarrow E(r) 4\pi r^2 = \frac{q}{\epsilon_0} \rightarrow \vec{E}(r) = \frac{q\,\hat{r}}{4\pi\epsilon_0 r^2} \quad \text{for } R_b < r < R_a$$

Next I will calculate the potential difference between the shells

$V_{ab} = -\int_{R_a}^{R_b} \vec{E}(r) \cdot d\vec{l}$, we use the same argument to convert $\vec{E}(r) \cdot d\vec{l} = E(r)dr$

$$= -\int_{R_a}^{R_b} \frac{q\,dr}{4\pi\epsilon_0 r^2} = \frac{-q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_{R_a}^{R_b} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{R_b} - \frac{1}{R_a}\right)$$

Now the capacitance is found by $C = \frac{q}{V_{ab}}$

$$\boxed{C = \frac{4\pi\epsilon_0}{\left(\frac{1}{R_b} - \frac{1}{R_a}\right)} = 4\pi\epsilon_0\left(\frac{R_a R_b}{R_a - R_b}\right)}$$

The stored energy in a capacitor is found by $U = \frac{1}{2}\frac{Q^2}{C}$, so

$$\boxed{U = \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{R_b} - \frac{1}{R_a}\right) = \frac{q^2}{8\pi\epsilon_0}\left(\frac{R_a - R_b}{R_a R_b}\right)}$$

d. Now when a dielectric completely fills the region between the spheres, we find the induced electric field in the dielectric reduces the overall field in the dielectric, and the capacitance is increased by a factor of K.

$$C_D = KC_0 = \boxed{4\pi\epsilon_0 K\left(\frac{R_a R_b}{R_a - R_b}\right) = C_D}$$

Now if the fluid only fills up to a radius R_c where $R_b < R_c < R_a$ then we can find the capacitance of the system by treating it as two capacitors in series, where

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

In this case $C_1 = 4\pi\epsilon_0 K\left(\frac{R_c R_b}{R_c - R_b}\right)$

$C_2 = 4\pi\epsilon_0 K\left(\frac{R_a R_c}{R_a - R_c}\right)$

So we find $\boxed{C_{eq} = \left[\frac{1}{4\pi\epsilon_0 K}\frac{R_c - R_b}{R_c R_b} + \frac{1}{4\pi\epsilon_0}\frac{R_a - R_c}{R_a R_c}\right]^{-1}}$

4) e. How much work was done to move the fluid in from far away to its final position between the spheres?

To calculate how much work is done by a person to move the fluid, we use

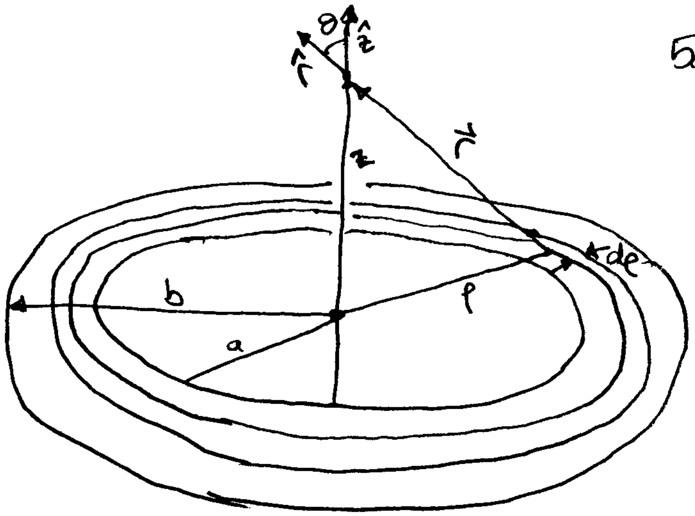
$W = U_f - U_i$, where $U_f = \frac{1}{2}\frac{q^2}{C_D}$, $U_i = \frac{1}{2}\frac{q^2}{C_o}$

$$W = \frac{q^2}{2}\left(\frac{1}{C_D} - \frac{1}{C_o}\right) = \frac{q^2}{2}\left(\frac{R_a - R_b}{4\pi\epsilon_0 K R_a R_b} - \frac{R_a - R_b}{4\pi\epsilon_0 R_a R_b}\right) = \boxed{-\frac{q^2}{2}\left[\frac{(K-1)(R_a - R_b)}{4\pi\epsilon_0 K R_a R_b}\right] = W}$$

So positive work is done by the electric field

F. Now the dielectric is removed and a conducting wire joins the two shells together. A charge +Q is introduced into the region between the spheres. What is the net charge on the facing surfaces of the two spheres?

$q_{net} = -Q$ on the facing surfaces because $E=0$ inside conductors, so we muts have $-Q$ to cancel $+Q$ between the plates, and we must have the same potential everywhere.



5a)

The reference voltage at ∞ is 0

We may therefore exploit the trick

$$-\int_{\infty} \vec{E}\cdot d\vec{l} = -\int_{\infty}\int_{Q} \frac{k\,dQ}{r^2}\,\hat{r}\cdot d\vec{l}$$

$$= \int_{Q} k\,dQ \int_{\infty} \frac{-\hat{r}\cdot\hat{r}\,dr}{r^2}$$

$$= \int \frac{k\,dQ}{r} \quad (+2)$$

The distance r from the point of observation to the charge element sourcing the field is constant over the rings of radius ρ and thickness $d\rho$. So we take these to be our charge elements.

$$dQ = \sigma\, 2\pi\rho\, d\rho$$

$$r = \sqrt{\rho^2+z^2} \quad (+2)$$

Then $$V(z) = \int_a^b \frac{2k\sigma\pi\rho\,d\rho}{\sqrt{\rho^2+z^2}}$$

$$= \int_{a^2+z^2}^{b^2+z^2} \sigma k\pi\, u^{-\frac{1}{2}}\,du$$

$$= 2\sigma k\pi\, u^{\frac{1}{2}}\Big|_{a^2+z^2}^{b^2+z^2}$$

$$V(z) = 2\sigma k\pi\left(\sqrt{b^2+z^2} - \sqrt{a^2+z^2}\right)$$
(+2)

b) Recognize that, by conservation of energy, $\frac{1}{2}mv^2 = qV(0)$ (+2)

$$\frac{1}{2}mv^2 = 2\sigma qk\pi(b-a)$$

$$v = \sqrt{\frac{4\sigma qk\pi}{m}(b-a)} \quad (+2)$$

c) $\vec{E} = -\vec{\nabla}V$

as $V = V(z)$, $-\vec{\nabla}V = -\hat{z}\frac{d}{dz}V(z)$ (+2)

$$= k\sigma\pi z\hat{z}\left(\frac{1}{\sqrt{b^2+z^2}} - \frac{1}{\sqrt{a^2+z^2}}\right)$$
(+3)

d) Use the same rings of charge (+1)

Then only $\hat{z}$ component survives

$d\vec{E}(z) = dE(z)\,\hat{z}$ (+1)

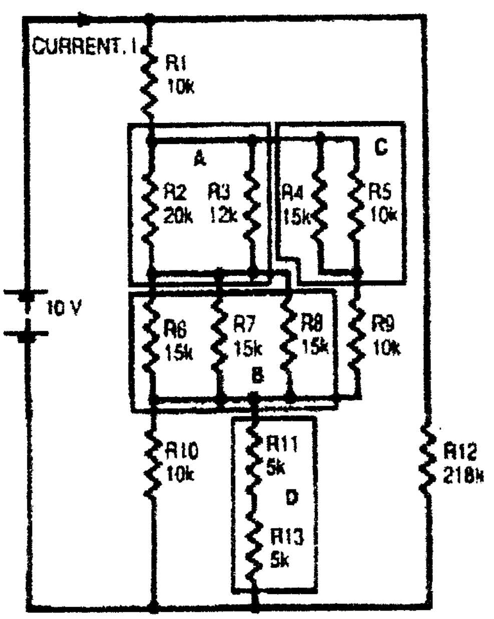
Take then the projection onto $\hat{z}$, $\hat{r}\cdot\hat{z} = \cos\theta = \frac{z}{\sqrt{\rho^2+z^2}}$ (+1)

$$\vec{E}(z) = \hat{z}\int k\,dq\,\frac{\hat{r}\cdot\hat{z}}{r^2} = \hat{z}\int_a^b \frac{2\pi k\sigma\rho\,d\rho\, z}{(\rho^2+z^2)^{\frac{3}{2}}} = \pi k\sigma z\hat{z}\int_{a^2+z^2}^{b^2+z^2} du\, u^{-\frac{3}{2}}$$

$$= 2\pi k\sigma z\hat{z}\left(\frac{1}{\sqrt{b^2+z^2}} - \frac{1}{\sqrt{a^2+z^2}}\right) \quad (+2)$$

6)

a. What is the current through resistor R12?



To answer this question we can use Kirchoff's loop rule, noting that the sum in potential drops around any closed loop is zero in a circuit, or by realizing that resistors in parallel have the same voltage across them. Either way, we find the current through R12 is

$$I_{12} = \frac{10V}{218K\Omega} \approx \boxed{4.6\times10^{-5}\ A = .046 mA = I_{12}}$$

b. what are the equivalent resistors R_A, R_B, R_C, and R_D?

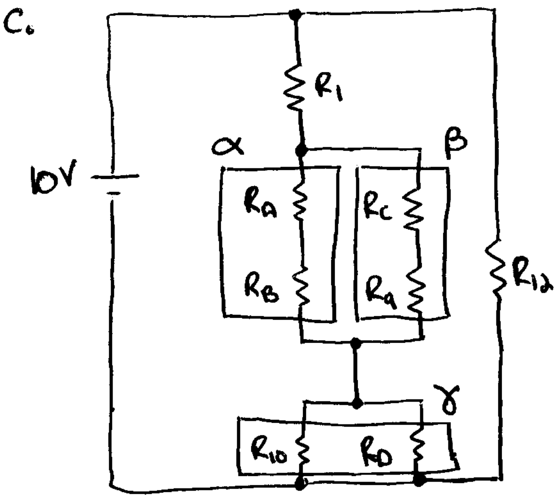
$$\frac{1}{R_A} = \frac{1}{R_2} + \frac{1}{R_3} \rightarrow \boxed{R_A = \frac{R_2R_3}{R_2+R_3} = 7.5K\Omega}$$

$$\frac{1}{R_B} = \frac{1}{R_7} + \frac{1}{R_6} + \frac{1}{R_8} \rightarrow \frac{R_6R_7R_8}{R_6R_7+R_6R_8+R_7R_8} = \boxed{R_B = 5K\Omega}$$

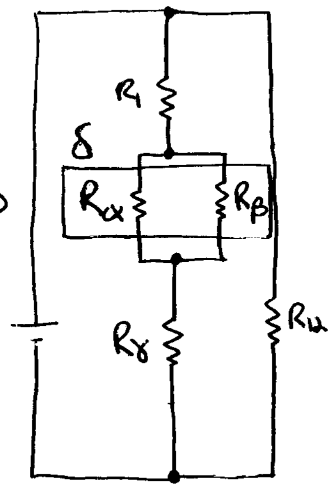
$$\frac{1}{R_C} = \frac{1}{R_4} + \frac{1}{R_5} \rightarrow \boxed{R_C = \frac{R_4R_5}{R_4+R_5} = 6K\Omega}$$

$$\boxed{R_D = R_{11}+R_{13} = 10K\Omega}$$

c.



⇒



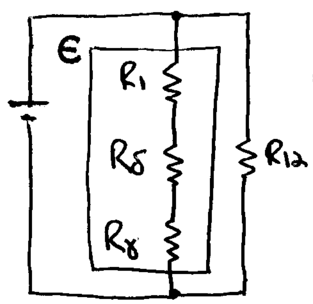
where

$R_\alpha = R_A + R_B = 12.5K\Omega$

$R_\beta = R_C + R_D = 16K\Omega$

$R_\gamma = \left(\frac{1}{R_D} + \frac{1}{R_{10}}\right)^{-1} = \frac{R_DR_{10}}{R_D+R_{10}} = 5K\Omega$

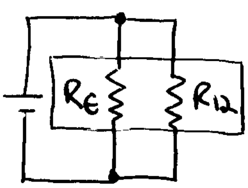
d.



where

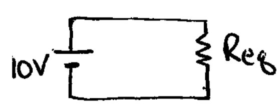
$$R_\delta = \left(\frac{1}{R_\alpha} + \frac{1}{R_\beta}\right)^{-1} = \frac{R_\alpha R_\beta}{R_\alpha + R_\beta} \approx 7.02K\Omega$$

e.



where $R_\epsilon = R_1 + R_\delta + R_\gamma \approx 22.02K\Omega$

F.



where $R_{eq} = \left(\frac{1}{R_\epsilon} + \frac{1}{R_{12}}\right)^{-1} \approx \boxed{20K\Omega = R_{eq}}$

$$I_{tot} = \frac{10V}{R_{eq}} \approx \boxed{5\times10^{-4} A = 0.5 mA = I_{tot}}$$