Name:____

Student ID Number:__________________________________

Instructions:

1. You will have 50 minutes to complete the exam.
2. The exam is a total of 4 questions and each question is worth 10 points.
3. Unless stated otherwise, you may use results we proved in class and on the homework.
4. No books, notes, calculators, or electronic devices are permitted.
5. If you require additional space, please use the reverse side of the pages.
6. The exam has a total of 6 pages with the last page left blank for scratch work. Please verify that your copy has all 6 pages.

Question	**Score**	**Points**
1.		10
2.		10
3.		10
4.		10
Total		40

1. (a) Given a bounded sequence $(x_n)_{n\in\mathbb{N}} \subset \mathbb{R}$, state the definition of the **limit supremum** $\limsup_{n\to\infty} x_n$.

   (b) Equip $\mathbb{R}$ with the usual metric, let $S \subset \mathbb{R}$ be a closed subset, and let $(x_n)_{n\in\mathbb{N}} \subset S$ be a bounded sequence. Show that $\limsup_{n\to\infty} x_n$ is contained in S.

2. (a) For (E, d) a metric space, state what it means for $(x_n)_{n\in\mathbb{N}} \subset E$ to be a **Cauchy sequence**.

   (b) State what it means for a metric space (E, d) to be **complete**.

   (c) Let E be an arbitrary set equipped with the trivial metric:

$$d(x,y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}.$$

   Show that E is complete.

3. (a) For (E, d) a metric space, state what it means for a subset $S \subset E$ to be **compact**.

   (b) Let (E, d) be as in the previous problem. Show that $S \subset E$ is compact if and only if S is finite.

4. (a) Let (E, d) and (E', d') be metric spaces, and let $f\colon E \to E'$ be a function. State what it means for f to be **continuous on** E.

   (b) Let $(\mathbb{R}^2, d_2)$ and $(\mathbb{R}, d)$ be the two-dimensional and one-dimensional Euclidean metric spaces, respectively. That is,

$$d_2\left((x_1, y_2), (x_2, y_2)\right) := \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$d(x_1, x_2) := |x_1 - x_2|.$$

   Show that the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x, y) = x^2 + y^2$$

   is continuous on $\mathbb{R}$.

Scratch Work.