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Fall 2008, Math H110
Second Midterm

22 Oct., 2008
1:10-2:00 PM

This is a closed book closed notes exam.

Attempt all problems. Write solutions on these sheets. Ask for scratch paper if the fronts and backs of these pages are not sufficient; put your name on any such extra sheets and hand them in with your exam.

Credit for an answer may be reduced if a large amount of irrelevant or incoherent material is included along with the correct answer.

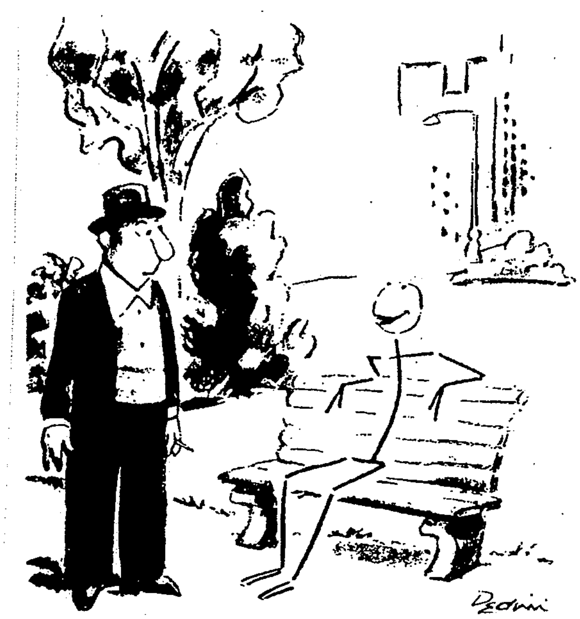
Questions begin on the next sheet. Fill in your name on this sheet now, but do not turn the page until the signal is given. At the end of the exam, stop writing and close your exam as soon as the ending signal is given, or you will lose points.

Think clearly, stay calm.

Your name ______________________________

Leave blank for grading

1	/ 30
2	/ 30
3(a)	/ 20
3(b)	/ 20
Σ	/100



"I got rid of all the excess in my life."

1. (30 points, 10 points each.) Complete the following definitions. You may use, without defining them, any terms or symbols that our text defines before defining the word or symbol asked for. Your definitions do not have to have exactly the same wording as those in the text, but for full credit they should be clear, and mean the same thing as those definitions. More space is provided for the answers than a concise answer is likely to need.

(a) Suppose V is an n-dimensional vector space and W an m-dimensional vector space over a field F, with bases $\beta = \{v_1, \ldots, v_n\}$ and $\gamma = \{w_1, \ldots, w_m\}$ respectively, and $\mathsf{T}\colon \mathsf{V} \to \mathsf{W}$ is a linear transformation. Then the matrix $[\mathsf{T}]_\beta^\gamma$ representing T with respect to this pair of bases means the $m \times n$ matrix A whose entries A_{ij} satisfy

____

____ .

(b) If $\mathsf{T}\colon \mathsf{V} \to \mathsf{W}$ is a linear transformation between two vector spaces over a field F, then T^t: ________ → ________ (called the "transpose" of T) is the linear transformation defined by ____

____ .

(c) If V is a vector space and W a subspace, and v an element of V, then the *coset* of W containing v, denoted ________________________, means

____

____ ,

and the set of all cosets of W in V is denoted ______________________ .

2. (30 points, 10 points each.) For each of the items listed below, either *give an example* with the property stated, or give a brief reason why *no such example exists.*

If you give an example, you do *not* have to prove that it has the property stated; however, your examples should be specific; i.e., even if there are many objects of a given sort, you should name a particular one. If you give a reason why no example exists, don't worry about giving reasons for your reasons; a simple statement will suffice.

(a) A square matrix A over C which is not diagonalizable.

(b) A polynomial $f(t) \in \mathsf{P}(C)$ which does not split over C.

(c) A transition matrix A such that $\lim_{n \to \infty} A^n$ does not exist.

3. (40 points, 20 points each) Prove the following statements. You may assume all results proved in our text in the readings so far.

(a) Suppose V is an n-dimensional vector space, and we have elements $v_1, \ldots, v_n \in V$ and $f_1, \ldots, f_n \in V^*$ such that

$$f_i(v_j) = \delta_{ij} \qquad \text{for all } i, j \in \{1, \ldots, n\}$$

(where δ_{ij} denotes the Kronecker delta function, equal to 1 if $i = j$ and to 0 otherwise).

Show that $\{v_1, \ldots, v_n\}$ is a basis of V.

(3, continued)

(b) Suppose V is a vector space and W_1 a subspace. Show that there exists a subspace $W_2 \subseteq V$ such that $V = W_1 \oplus W_2$; i.e., such that $W_1 + W_2 = V$ and $W_1 \cap W_2 = \{0\}$. (You may assume V finite-dimensional if you wish.)