Name:____

Student ID Number:__________________________________

Instructions:

1. You will have 50 minutes to complete the exam.
2. The exam is a total of 4 questions and each question is worth 10 points.
3. There are two survey questions on Page 6 worth 1 bonus point each.
4. Unless stated otherwise, you may use results we proved in class and on the homework.
5. No books, notes, calculators, or electronic devices are permitted.
6. If you require additional space, please use the reverse side of the pages.
7. The exam has a total of 7 pages with the last page reserved for scratch work. Please verify that your copy has all 7 pages.

Question	**Score**	**Points**
1.		10
2.		10
3.		10
4.		10
Survey		+2
Total		40

1. (a) Let V and W be normed spaces. For a linear transformation $T\colon V \to W$, define the **operator norm** of T.

   (b) Let $C([0,1])$ denote the normed space of continuous functions $f\colon [0,1] \to \mathbb{R}$ with norm

$$|f|_\infty = \sup\{|f(t)|\colon 0 \leq t \leq 1\}.$$

   Fix $x, y \in [0,1]$ and define a linear transformation $\delta_{(x,y)}\colon C([0,1]) \to \mathbb{R}^2$ by

$$\delta_{(x,y)}(f) = (f(x), f(y)).$$

   Letting $\mathbb{R}^2$ have the usual norm, determine (with proof) the operator norm of $\delta_{(x,y)}$.

2. (a) For a map $R\colon \mathbb{R}^n \to \mathbb{R}^m$, state what is means for R to be **sublinear**.

   (b) Consider $f\colon \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$$f(x_1, x_2) = \left(x_1 x_2,\ x_1 + x_2^2,\ x_1 + x_2\right).$$

   For $p = (p_1, p_2) \in \mathbb{R}^2$, compute $(Df)_p$ and prove that the corresponding Taylor remainder is sublinear.

3. (a) Let $k \in \mathbb{N}$. State what it means for a map

$$T\colon \underbrace{\mathbb{R}^n \times \cdots \times \mathbb{R}^n}_{k \text{ times}} \to \mathbb{R}^m$$

to be k-**linear**.

(b) Let $f\colon \mathbb{R}^2 \to \mathbb{R}^3$ be the function from Question 2.(b). Show that for $p = (p_1, p_2) \in \mathbb{R}^2$, f is twice-differentiable at p.

4. (a) Let $U \subset \mathbb{R}^n$ be open and let $f_k\colon U \to \mathbb{R}^m$, $k \in \mathbb{N}$, be functions of class C^r. State what means for the sequence $(f_k)_{k\in\mathbb{N}}$ to be **uniformly** C^r **convergent**.

   (b) Let $B = \{p \in \mathbb{R}^2 \colon |p| < 1\}$ be the open unit ball in $\mathbb{R}^2$. Consider the functions $f_k\colon B \to \mathbb{R}^2$ defined by

$$f_k(x_1, x_2) = \left(x_1^2 + \frac{x_1}{k},\ x_2 + \frac{1}{k}\right).$$

   Show that $(f_k)_{k\in\mathbb{N}}$ is uniformly C^1 convergent.

The following (optional) questions are worth 1 bonus point each:

- Rate the difficulty of homework assignments:
  (a) They are too easy.
  (b) They are reasonable.
  (c) They are too hard.
  (d) I can't even
  (e) Other:

- In what way(s) can the class be improved?

Scratch Work: