UNIVERSITY OF CALIFORNIA AT BERKELEY

Physics 105

Spring 2014

SAMPLE FINAL EXAMINATION 1

BASED ON PROBLEMS FROM THE COURSE TEXT BY TAYLOR

Please keep this exam booklet closed until the beginning of the exam is announced.

Please try to do all your work on the front and back pages of this exam. If that is not enough space, you may attach additional paper. Please write your name on all pages.

No calculators or other electronic devices are allowed.

Please circle your final answer.

NAME:

STUDENT ID NUMBER:

SIGNATURE:

Problem 1

(a) Write down the Lagrangian $\mathcal{L}(x_1, x_2, \dot{x}_1, \dot{x}_2)$ for two particles of equal masses, $m_1 = m_2 = m$, confined to the x axis and connected by a spring with potential energy $U = \frac{1}{2}kx^2$. [Here x is the extension of the spring, $x = (x_1 - x_2 - l)$, where l is the spring's unstretched length, and I assume that mass 1 remains to the right of mass 2 at all times.] **(b)** Rewrite $\mathcal{L}$ in terms of the new variables $X = \frac{1}{2}(x_1 + x_2)$ (the CM position) and x (the extension), and write down the two Lagrange equations for X and x. **(c)** Solve for $X(t)$ and $x(t)$ and describe the motion.

Problem 2

Two masses m_1 and m_2 move in a plane and interact by a potential energy $U(r) = \frac{1}{2}kr^2$. Write down their Lagrangian in terms of the CM and relative positions $\mathbf{R}$ and $\mathbf{r}$, and find the equations of motion for the coordinates X, Y and x, y. Describe the motion and find the frequency of the relative motion.

Problem 3

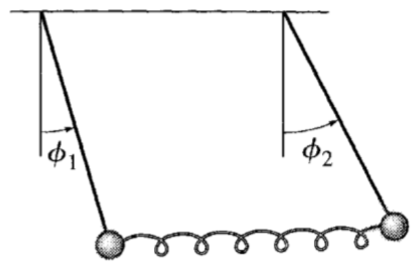
On a certain planet, which is perfectly spherically symmetric, the free-fall acceleration has magnitude $g = g_o$ at the North Pole and $g = \lambda g_o$ at the equator (with $0 \leq \lambda \leq 1$). Find $g(\theta)$, the free-fall acceleration at colatitude θ as a function of θ.

Problem 4

A rigid body consists of three masses fastened as follows: m at $(a, 0, 0)$, $2m$ at $(0, a, a)$, and $3m$ at $(0, a, -a)$. **(a)** Find the inertia tensor $\mathbf{I}$. **(b)** Find the principal moments and a set of orthogonal principal axes.

Problem 5

Consider two identical plane pendulums (each of length L and mass m) that are joined by a massless spring (force constant k) as shown below . The pendulums' positions are specified by the angles ϕ_1 and ϕ_2 shown. The natural length of the spring is equal to the distance between the two supports, so the equilibrium position is at $\phi_1 = \phi_2 = 0$ with the two pendulums vertical. **(a)** Write down the total kinetic energy and the gravitational and spring potential energies. [Assume that both angles remain small at all times. This means that the extension of the spring is well approximated by



$L(\phi_2 - \phi_1)$.] Write down the Lagrange equations of motion. **(b)** Find and describe the normal modes for these two coupled pendulums.

Problem 6

A bead of mass m is threaded on a frictionless wire that is bent into a helix with cylindrical polar coordinates (ρ, ϕ, z) satisfying $z = c\phi$ and $\rho = R$, with c and R constants. The z axis points vertically up and gravity vertically down. Using ϕ as your generalized coordinate, write down the kinetic and potential energies, and hence the Hamiltonian $\mathcal{H}$ as a function of ϕ and its conjugate momentum p. Write down Hamilton's equations and solve for $\ddot{\phi}$ and hence $\ddot{z}$. Explain your result in terms of Newtonian mechanics and discuss the special case that $R = 0$.