UNIVERSITY OF CALIFORNIA AT BERKELEY

Physics 105 – Lecture Section 2

Spring 2014

MIDTERM EXAMINATION

Please keep this exam booklet closed until the beginning of the exam is announced.

Please try to do all your work on the front and back pages of this exam. If that is not enough space, you may attach additional paper. Please write your name on all pages.

No calculators or other electronic devices are allowed.

Please circle your final answer.

NAME:

STUDENT ID NUMBER:

SIGNATURE:

	SCORE
1. 35 points	
2. 35 points	
3. 30 points	
Total: 100 points	

Problem 1 [35 points]

A large clock on the wall has an hour hand, a minutes hand, and a seconds hand. A fourth hand is added which makes one clockwise revolution per second (This number is mentioned here because it helps to visualize the dynamics. For this problem, use the symbol ω for the constant angular velocity of the fourth hand). The style of the fourth hand is shown in the diagram (the fourth hand extends on both sides of the pivot in the center of the clock). The fourth hand is made of a hollow glass tube, and a small particle of mass m can slide inside the tube without friction. We will study the dynamics of the particle under the assumption that it does not collide with an end of the tube. We define a generalized coordinate q which specifies the location of the particle within the tube relative to the pivot, with positive values indicating the longer end and negative values indicating the shorter. We define x and y axes, with the x axis pointing to the right and the y axis pointing up. The acceleration due to the Earth's gravitational field is to be taken as the familiar constant vector $\mathbf{g}$ pointing in the negative y direction.

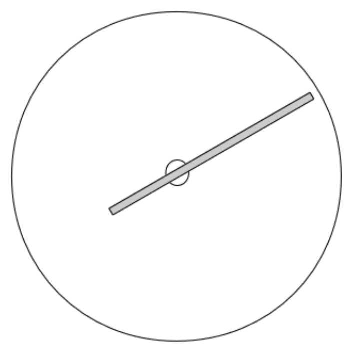
(a) [5 points] Find expressions for the x and y coordinates of the particle in terms of q and the time t, with the definition that the fourth hand point straight up at $t = 0$.

(b) [5 points] Find the Lagrangian.

(c) [5 points] Find the equation of motion. There are similarities to an equation we have studied extensively, but there is an important sign difference.

(d) [10 points] Find a particular solution by trying the form $q = A\cos(\omega t)$

(e) [10 points] Find the general solution.



<u>Problem 2</u> [35 points]

A droid named R2D2 has fallen out of a spaceship, and now he is orbiting a distant planet by himself. Fortunately R2D2 is in an elliptical orbit with $r_{min} = r_R$ and $r_{max} = (25/7)\ r_R$, where r_R is the International Rescue Orbit Radius. If R2D2 can change his orbit to be a circle of radius r_R he will certainly be rescued. R2D2 has some lecture notes with him which are useful because he can throw them to change his orbit. Use M = mass of the planet, m = mass of R2D2, and mass of the notes = $m/3$.

(a) [10 points] Before R2D2 throws the notes, that is, while he and the notes are still traveling together, let v_m denote their speed at perigee. Find v_m. Your answer should be expressed in terms of G, M and r_R.

(b) [5 points] Find v_R, the speed of an object in a circular orbit of radius r_R around this planet.

(c) [10 points] At perigee, R2D2 throws the notes in the same direction he is traveling in order to slow himself down. Find v_{nd}, the necessary speed of the notes relative to R2D2 immediately after the throw.

(d) [10 points] Now that R2D2 is safe, commence the celebrations by calculating the eccentricity of the orbit of the notes.

<u>Problem 3</u> [30 points]
A locomotive moves a railroad car on tracks that are horizontal and straight. Inside the railroad car is a laboratory with x and y coordinate axes defined so that the x axis is parallel to the tracks and the y axis points up. These axes are fixed relative to the car. A piece of wire is bent into the shape of a parabola and mounted in the x-y plane. The equation for the parabola is $y = b\,x^2$. A bead of mass m slides without friction on the wire. We will use the x coordinate of the bead as its generalized coordinate for this problem. The locomotive moves the car back and forth along the tracks according to $x_{car} = A \sin(\omega\, t)$. The coordinates of the bead relative to the tracks and ground (an inertial frame) are x_g and y_g. The acceleration due to the Earth's gravitational field is to be taken as the familiar constant vector **g** pointing in the negative y direction.

(a) [5 points] Find expressions for x_g and y_g in terms of the generalized coordinate x and the time t.
(b) [5 points] Write down the Lagrangian.
(c) [10 points] Write down the equation of motion.
(d) [10 points] At $t = 0$, the generalized coordinate x and its time derivative are zero. Find the leading order term in the Taylor series expansion of $x(t)$ about t = 0.