UNIVERSITY OF CALIFORNIA AT BERKELEY

Physics 105 – Lecture Section 1

Spring 2014

MIDTERM EXAMINATION

Please keep this exam booklet closed until the beginning of the exam is announced.

Please try to do all your work on the front and back pages of this exam. If that is not enough space, you may attach additional paper. Please write your name on all pages.

No calculators or other electronic devices are allowed.

Please circle your final answer.

NAME:

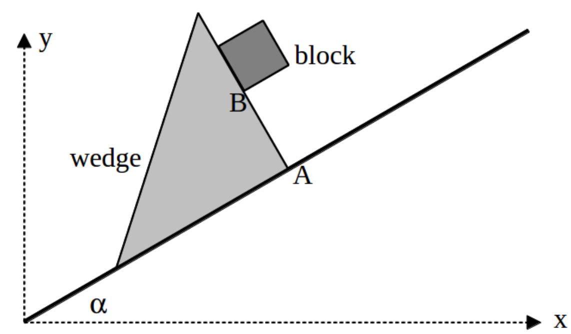
STUDENT ID NUMBER:

SIGNATURE:

	SCORE
1. 30 points	
2. 35 points	
3. 35 points	
Total: 100 points	

<u>Problem 1</u> [30 points]
A triangular wedge of mass *M* has a right angle at one corner. The wedge slides without friction on a slope (angle α relative to the horizontal). A block of mass *m* slides without friction along one edge of the wedge, as shown in the diagram. Let A be the vertex of the wedge that has the right angle. Let B be the vertex of the block that is in contact with the wedge and closer to A. We define generalized coordinates q_1 and q_2 as follows. q_1 is the distance from the origin to A. q_2 is the distance from A to B. The acceleration due to the Earth's gravitational field is to be taken as the familiar constant vector **g** pointing in the negative *y* direction.

(a) [7 points] What are the x and y coordinates of the points A and B as functions of q_1 and q_2?
(b) [7 points] What is the kinetic energy T of the system?
(c) [7 points] What is the potential energy U of the system? Of course, U is only defined up to an overall additive constant. Choose this constant so that U = 0 when q_1 and q_2 are 0.
(d) [7 points] Write out the Lagrangian, and determine the equations of motion (the two Lagrange equations).
(e) [deleted]
(f) [2 points] What do d^2q_1/dt^2 and d^2q_2/dt^2 become when α is set to 0?



<u>Problem 2</u> [35 points]
The Starship Enterprise (mass = m) is in a circular orbit of radius R around Venus (mass = M), and the engines have been shut down for routine maintenance. Unfortunately the Klingons have launched a giant glob of clay (mass = $m/4$), and it collides completely inelastically with the Enterprise. We must do some calculations to see if the Enterprise's new orbit will cause it to crash into Venus. The glob of clay was in an elliptical orbit in the same plane as the Enterprise's orbit, with r_{min} equal to R, and r_{max} equal to 8R. The angular momentum vectors of the Enterprise and the clay about the center of Venus were oppositely directed before the collision. Assume $m << M$.

(a) [5 points] Find v_{E0}, the speed of the Enterprise immediately before the collision. Your answer should be expressed in terms of G, M and R.
(b) [10 points] Find v_{C0}, the speed of the clay immediately before the collision.
(c) [5 points] Find v_{EC}, the speed of the Enterprise and clay stuck together immediately after the collision.
(d) [10 points] Find the eccentricity of the new orbit.
(e) [5 points] Find r_{min} of the new orbit. This result will be used by the crew of the Enterprise to determine if they will board the escape pods.

<u>Problem 3</u> [35 points]
Two particles with masses m_1 and m_2 are on a frictionless horizontal table. An *x*-*y* coordinate system is used on the surface of the table. There are no springs or any other forces acting on the particles in the horizontal plane (Thus we may take U = 0).

(a) [5 points] Write down the Lagrangian in terms of the CM position ***R*** and the relative position ***r***, using polar coordinates (r, ϕ). You can quote the result; you do not need to derive it.

(b) [10 points] Find the Lagrange equation for r. The Lagrange equation for ϕ tells you that there is a conserved quantity. Use this to remove the explicit reference to $d\phi/dt$ in the Lagrange equation for r (This is "the equivalent one-dimensional problem").

(c) [5 points] At time $t = 0$, m_1 is at rest at the origin, and m_2 has position (x_0, 0) and velocity (0, v_0). Of course m_1 remains at the origin, and m_2 moves with constant velocity. Using this information, write down an expression for the distance between the particles as a function of time, r(t). [A one-line answer is all that is required here.]

(d) [10 points] Using the same initial conditions, find the *z*-component of the total angular momentum about the center of mass.

(e) [5 points] Verify that your answer to part (c) satisfies the differential equation you found in part (b).