University of California, Berkeley, Department of Physics

Physics 7B

Final Exam

Spring 2017

A sheet of notes is provided with this exam. Calculators or other electronic devices are not permitted. Put a box around your final answer and cross out any work you wish the grader to disregard. Try to be neat and organized.

Remember to look over your work. Good Luck!

Problem 1	______/20
Problem 2	______/20
Problem 3	______/20
Problem 4	______/20
Problem 5	______/20
Problem 6	______/15
Problem 7	______/20
Problem 8	______/15
Total	______/150

Physics 7B Spring 2017 - Final Exam Formula Sheet

$$PV = nRT = Nk_BT$$

$$\Delta E_{int} = Q - W$$

$$dE_{int} = dQ - PdV$$

$$\Delta S = \int \frac{dQ}{T}$$

$$\eta = \frac{W}{Q_h}$$

$$PV^\gamma = \text{const.}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \hat{r}}{r^2}$$

$$C = \frac{Q}{V}$$

$$C = \kappa C_0$$

$$U = \frac{Q^2}{2C}$$

$$L = \frac{N\Phi_B}{I}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$U = \frac{1}{2}LI^2$$

$$I = \frac{dQ}{dt}$$

$$U = \int \frac{1}{2}\left(\epsilon_0 |\vec{E}|^2 + \frac{1}{\mu_0}|\vec{B}|^2\right) dV$$

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$
$$d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z}$$
(Cartesian Coordinates)

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial f}{\partial z}\hat{z}$$
$$d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + dz\hat{z}$$
(Cylindrical Coordinates)

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial f}{\partial \phi}\hat{\phi}$$
$$d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin(\theta)d\phi\hat{\phi}$$
(Spherical Coordinates)

$$y(t) = \frac{B}{A}(1 - e^{-At}) + y(0)e^{-At}$$
solves $\frac{dy}{dt} = -Ay + B$

$$y(t) = y_{max}\cos(\sqrt{A}t + \delta)$$
solves $\frac{d^2y}{dt^2} = -Ay$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0\epsilon_0\frac{d\Phi_E}{dt}$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$k_B = 1.4 \times 10^{-23} J/K$$

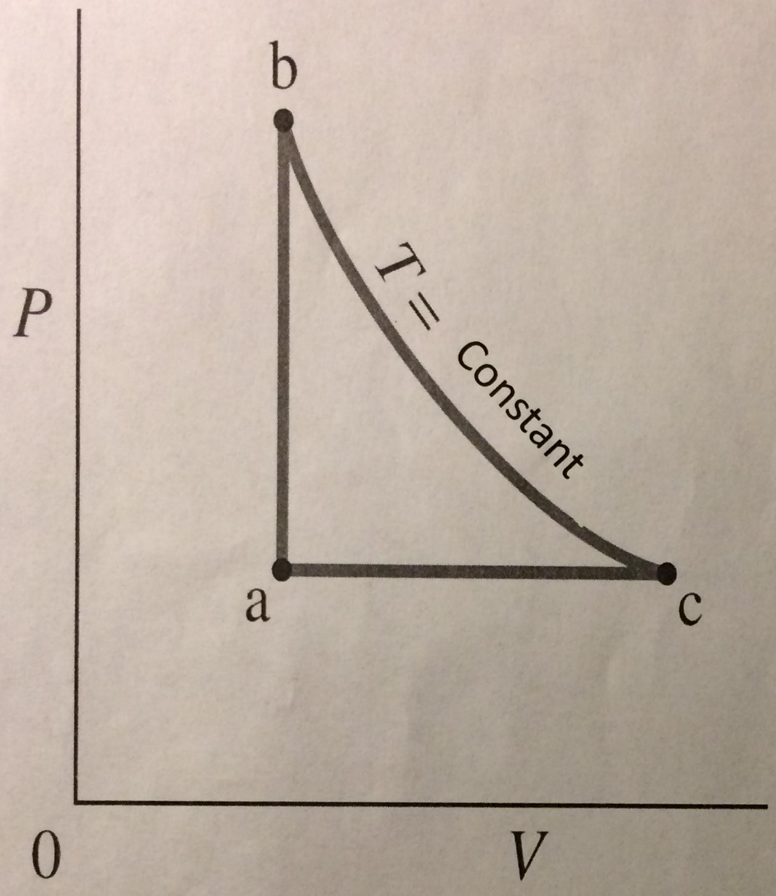
$$R = 8.3 J/(\text{mol} \cdot K)$$

$$\epsilon_0 = 8.9 \times 10^{-12} C^2/(N \cdot m^2)$$

$$\mu_0 = 4\pi \times 10^{-7} T \cdot m/A$$

$$c = 3 \times 10^8 m/s$$

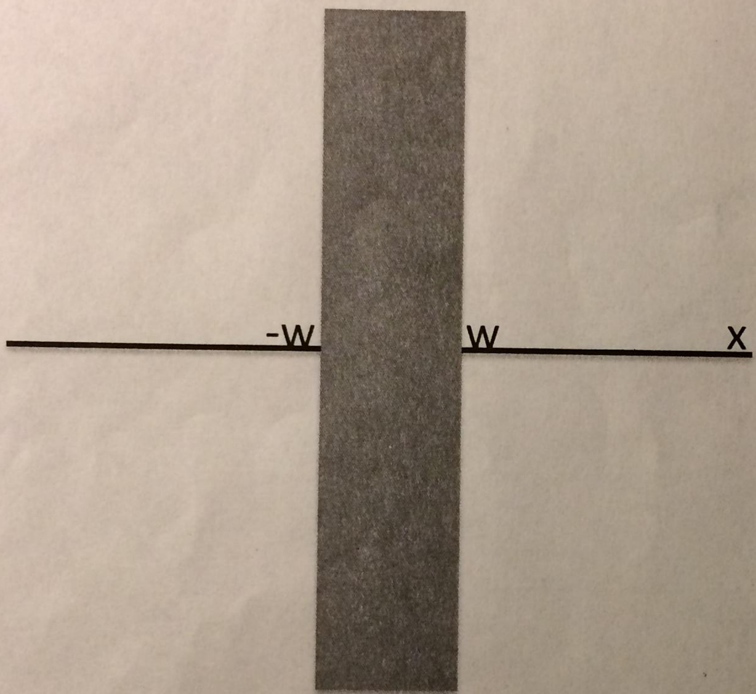
Problem 1. [20 points.]



The diagram above shows a reversible heat engine operating with n moles of a monatomic ideal gas. Path a→b is at constant volume. Path b→c is at constant temperature. Path c→a is at constant pressure. The system variables you may use in your answers to parts a) –g) below are T_a, P_a, V_a, P_b, T_b, and V_c

a) **[2 points]** Which direction (clockwise or counter-clockwise) should the heat engine operate if the working gas is to perform positive work?
b) **[2 points]** What is the work done by the gas going from a to b?
c) **[2 points]** What is the work done by the gas going from b to c?
d) **[2 points]** What is the work done by the gas going from c to a?
e) **[3 points]** What is the heat exchange going from a to b?
f) **[3 points]** What is the heat exchange going from b to c?
g) **[3 points]** What is the heat exchange going from c to a?
h) **[3 points]** **(Messy**, don't waste time here) Find an algebraic expression for the efficiency in terms of T_a and T_b

Problem 2. [20 points]



An infinite slab of width 2W, centered at x=0 and bounded by the planes x= -W and x= W, as shown above, has a non-uniform volume charge density $\rho = b\,|x|$, where b is a positive constant. Give your answer to the questions below in terms of b,W, and appropriate physical constants.

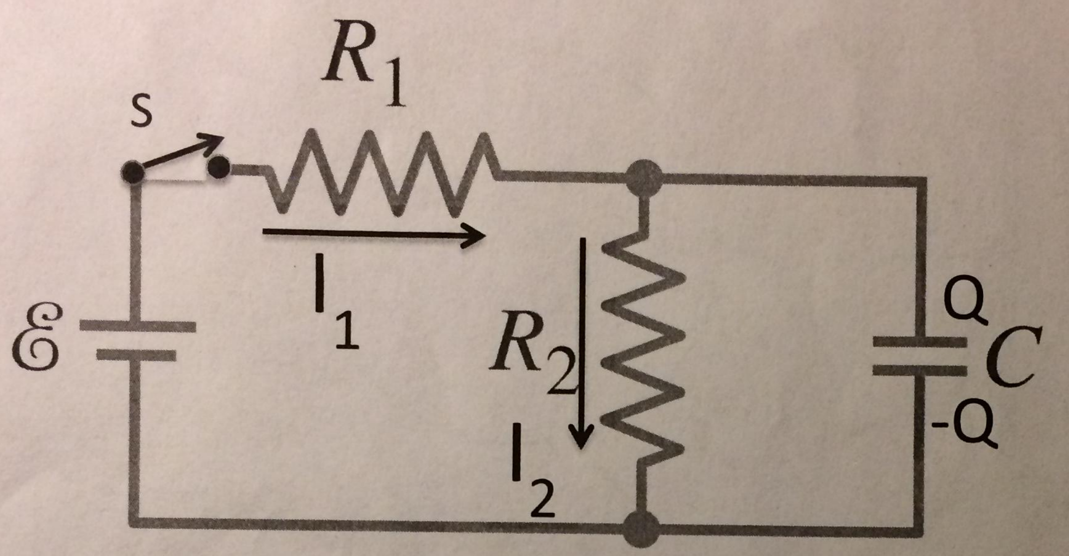
Use Gauss' Law to find the magnitude of the electric field for:

a) **[8 points]** $|x| > W$
b) **[8 points]** $|x| < W$
c) **[4 points]** Sketch the direction of E at points x= -3w/2 and x= 3w/2.

$$E\,2A = \frac{\int \rho\, dV}{\varepsilon_0} = \frac{2A\int_0^x bx\,dx}{\varepsilon_0}$$

$$E = \frac{2}{\varepsilon_0}\, b\, \frac{x^2}{2}$$

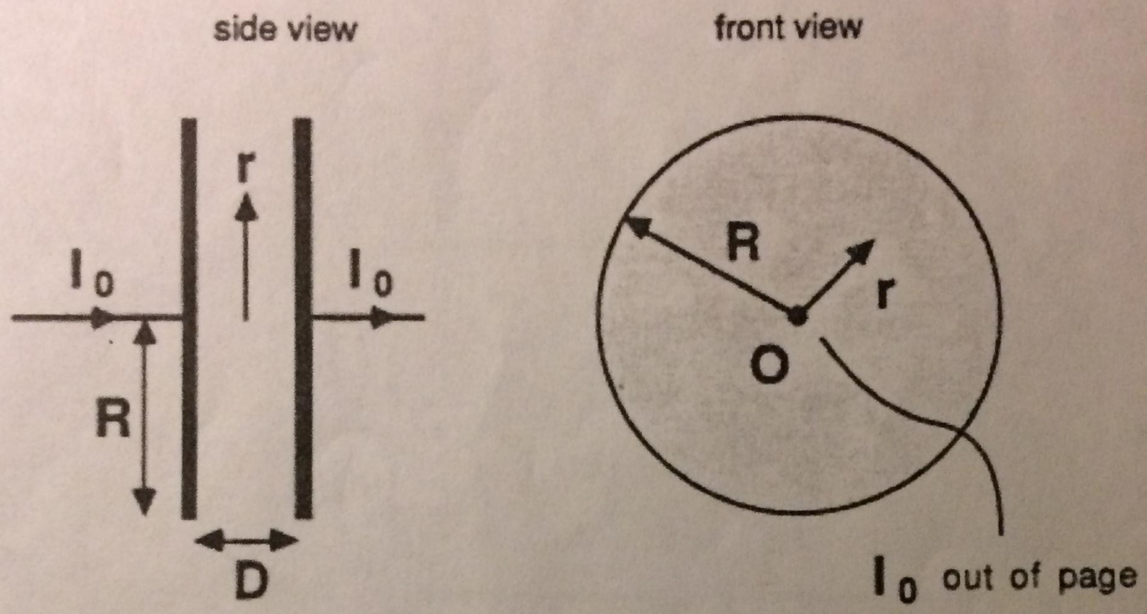
Problem 3. [20 points]



In the RC circuit above, after the switch is closed at time t=O, currents I_1 and I_2 flow through the resistors R_1, R_2, and the capacitor C has a charge Q(t).
The capacitor C is initially uncharged: Q(t=0)=0.

a) **[4 points]** What are the values of I_1 and I_2, and Q immediately after the switch is closed?
b) **[4 points]** What are the values of I_1 and I_2, and Q a very long time after the switch is closed?
c) **[4 points]** Write down a set of three independent equations for the unknowns of I_1 and I_2, and Q.
d) **[4 points]** Combine your equations from part c) to get a single first order differential equation for Q.
e) **[4 points]** [This may be *harder*. Don't *waste* time here]. What is the time constant for charging the circuit?

Problem 4. [20 points] Displacement Current

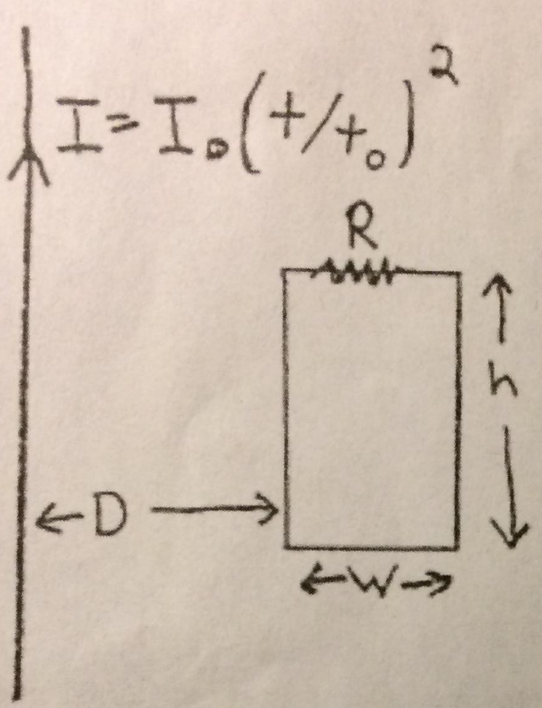


A long conducting wire (as shown) is interrupted by a parallel-plate capacitor consisting of two circular plates each of radius R and separation D (assume D << R). The current is switched on at time t = 0 and for time t>0 the current is held at constant value $I_0>0$. Before being switched on the capacitor has zero charge.

Neglect fringe field effects in answers to the questions below.

a) **[6 points]** Find the magnitude of the electric field E between the plates as a function of time for t > 0.
b) **[3 points]** Sketch the electric field lines on the left diagram above. Indicate the direction clearly.
c) **[2 points]** Find the electric flux between the plates.
d) **[6 points]** Find the magnitude of the magnetic field B between the plates as a function of r and t. Your result should include the cases of both r < R and r > R.
e) **[3 points]** Sketch the magnetic field lines on the right side diagram above. Indicate the direction clearly.

Problem 5. [20 points]



a) A time dependent current $I=I_0\,(t/t_0)^2$ flows along an infinite wire (t is time; I_0 and t_0 are constants). A rectangular wire loop with height h and width w is placed a distance D from the wire. It is oriented with its sides of length h parallel to the wire, as shown. The loop has a resistor R and the rest of the loop has negligible resistance. Use variable r as the distance from the wire. Express your answer to the question in terms of R, w, h, D, t, I_0, t_0, r and any fundamental constants you need.

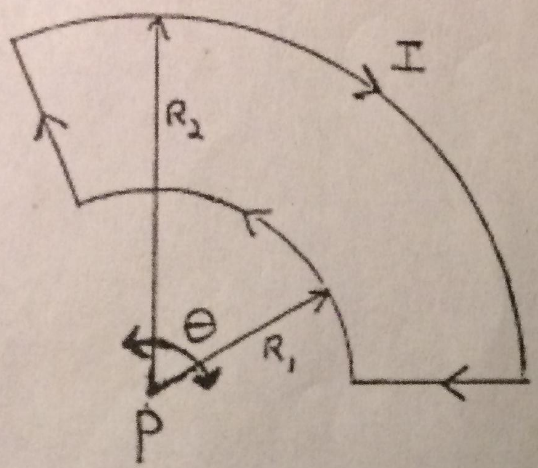
b) **[5 points]** Use Ampere's law to find the time dependent magnetic field (magnitude and direction) due to the current in the wire.

c) **[5 points]** Find the magnetic flux Φ_B through the loop.

d) **[5 points]** Find the induced EMF and current in the loop (magnitude and direction).

e) **[5 points]** Find the net force (magnitude and direction) on the loop.

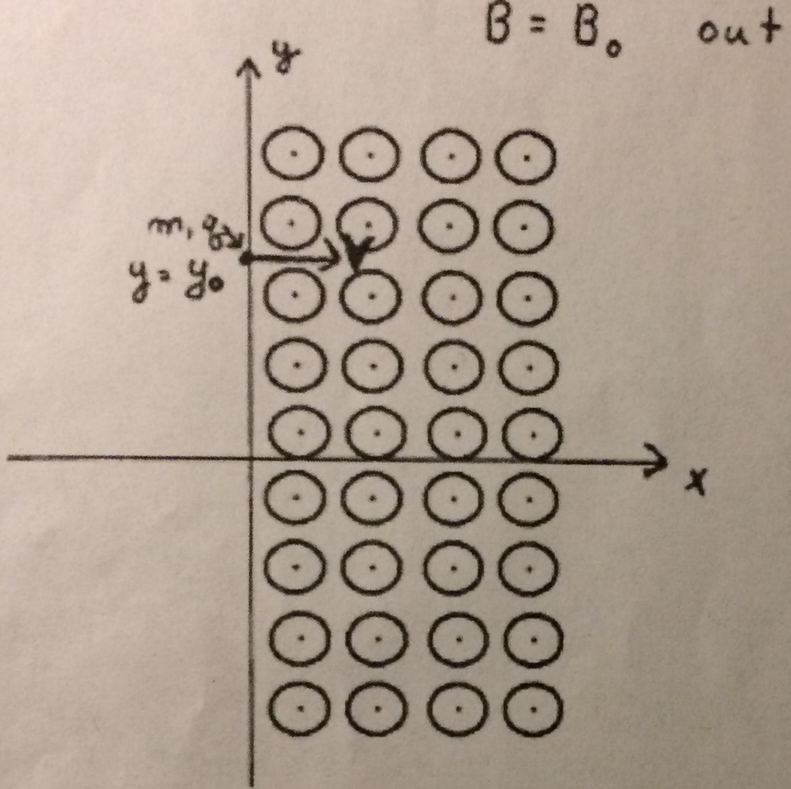
Problem 6. [15 points] Biot-Savart law



A closed loop with current I consists of two arcs of radii R_1 and R_2, and angle θ are connected by straight sections, as shown.

[**Magnitude 11 points. Direction 4 points**] Find the magnetic field B (magnitude and direction) at the point P (P is at the center of the arcs).

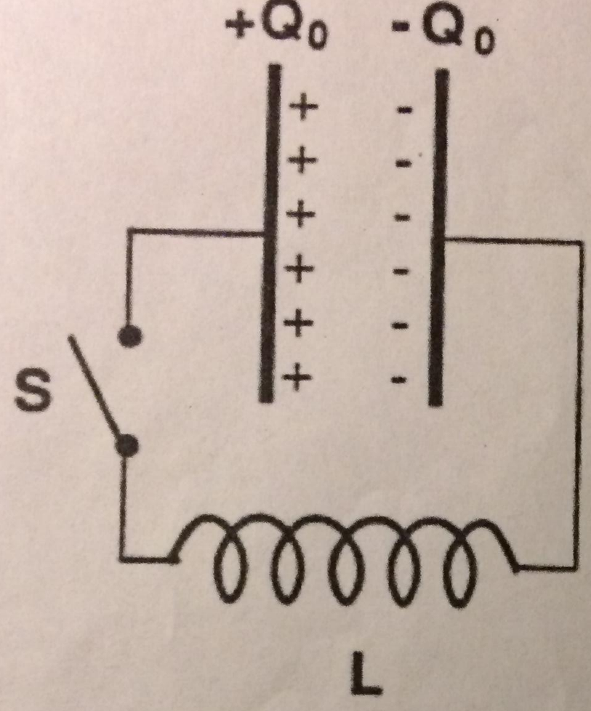
Problem 7. [20 points] Particle in cross fields



A particle of mass m and charge $q>0$ and velocity v in the x-direction moves into the region $x>0$ at $y=y_0$ which has a magnetic field B_{out} of the page (+z-direction).

a) **[5 points]** Sketch the subsequent motion of the particle.
b) **[5 points]** At what distance from the entry point does the particle exit the field, and with what velocity (magnitude and direction)
c) **[5 points]** How much time does the particle spend in the magnetic field.
d) **[5 points]** Suppose that the region $x> 0$ also had, in addition to the magnetic field, an electric field $E=B_0v$ in the positive y-direction. Describe the motion of the particle after entering the region $x>0$ with the combined fields. (The precise value of E is important.)

Problem 8. [15 points]



An L-C circuit consists of a coil of an inductor with self-inductance L and a capacitor, of capacitance C. The capacitor plates initially have charges $\pm Q_0$. The switch S is initially open and is then closed at time $t = 0$. Express your answers in terms of Q_0, L, C, and t.

a) **[8 points]** Find the oscillatory voltage across the capacitor as a function of time.
b) **[3 points]** Find the current in the circuit as a function of time.
c) **[4 points]** Find the maximum stored energy in the inductor.