Chemical Engineering 150A
Midterm Exam – 2
Tuesday, April 3, 2018
6:05 pm – 6:55 pm

*The exam is 100 points total.*

**Name:** ________________________________ (in Uppercase)

**Student ID:** ___________________________

*You are allowed one 8.5”×11” sheet of paper with your notes on both sides.*

*The exam should have 12 exam pages including the cover page. Additionally, you will find 5 pages with the relevant equations attached to the back of the exam.*

**Instructions:**

1) Only the numbered pages (**front side of the page**) will be graded. Any work done outside of specified area will not be graded.
2) Please sign below saying that you agree to the UC Berkeley honor code.
3) The exam contains two problems.
4) You can use the blank white full pages behind the question pages as scratch sheets, but they will not be graded.
5) If you simplify directly on the handout, we will grade it.

**Honor Code:**

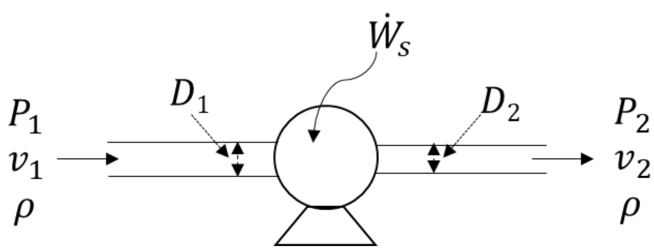
As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.

Signature: _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

**Problem 1: (20 points)**

At the inlet of a pump, the pressure and velocity of the fluid is known, $P_1$ and $v_1$, respectively. The fluid is Newtonian and incompressible of density $\rho$. Shaft work, $\dot{W}_s$, acts on the pump and the pump acts adiabatically. The inlet is of diameter $D_1$, while the outlet is of diameter $D_2$ (where $D_2 < D_1$). Determine the outlet pressure, $P_2$, in terms of $\hat{w}_s$, $v_1$, $\rho$, $P_1$, $D_1$, and $D_2$. You can ignore viscous forces, can assume that the inlet and outlet streams have a plug velocity profile, and can assume that the internal energy change across the pump is zero. The inlet and outlet streams are at the same height. The general energy equation is given below:

$$\frac{\partial}{\partial t}\int_{CV} \rho\left(u + \frac{1}{2}\underline{v}\cdot\underline{v} + gh\right)dV$$
$$= -\int_{CS}\left(u + \frac{1}{2}\underline{v}\cdot\underline{v} + gh + \frac{P}{\rho}\right)(\rho\underline{v}\cdot\underline{n})dA + \dot{Q} + \dot{W}_s$$
$$+ \int_{CS}\left(\underline{\underline{\tau}}^T\underline{n}\right)\cdot\underline{v}\,dS$$




Energy bal:

$$0 = -\int_{CS}\left(u + \frac{1}{2}\underline{v}\cdot\underline{v} + gh + \frac{P}{\rho}\right)(\rho\underline{v}\cdot\underline{n})dS + \dot{W}_s$$

$$0 = -\left(u_1 + \frac{1}{2}v_1^2 + gh_1 + \frac{P_1}{\rho}\right)(-\rho v_1)A_1 - \left(u_2 + \frac{1}{2}v_2^2 + gh_2 + \frac{P_2}{\rho}\right)(\rho v_2)A_2 + \dot{W}_s$$

$$0 = \left(u_1 + \frac{1}{2}v_1^2 + gh_1 + \frac{P_1}{\rho}\right)\dot{m} - \left(u_2 + \frac{1}{2}v_2^2 + gh_2 + \frac{P_2}{\rho}\right)\dot{m} + \dot{W}_s$$

Mass bal:

$$\rho v_1 A_1 = \rho v_2 A_2 = \dot{m}$$

$$v_2 = v_1\left(\frac{D_1}{D_2}\right)^2$$

Back to energy bal:

$$0 = \dot{m}\left((u_1 - \cancel{u_2}) + \frac{1}{2}\left(v_1^2 - \left(v_1\left(\frac{D_1}{D_2}\right)^2\right)^2\right) + g(\cancel{h_1} - h_2) + \frac{1}{\rho}(P_1 - P_2)\right) + \dot{W}_s$$

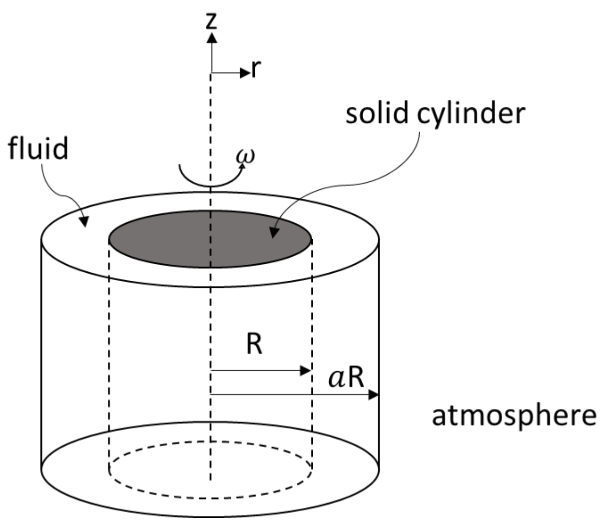
$$0 = \left(\frac{1}{2}\left(v_1^2 - \left(v_1\left(\frac{D_1}{D_2}\right)^2\right)^2\right) + \frac{1}{\rho}(P_1 - P_2)\right) + \widehat{w}_s$$

$$P_2 = \rho\widehat{w}_s + \frac{1}{2}\rho v_1^2\left(1 - \left(\frac{D_1}{D_2}\right)^4\right) + P_1$$

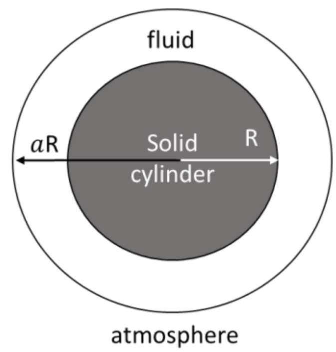
**Problem 2: (80 points)**

An incompressible, Newtonian fluid, of density, $\rho$, and viscosity, $\mu$, flows down the outside of a very long solid cylinder. The fluid has a uniform thickness of $(a-1)R$ and is in contact with air. The solid is rotating at an angular speed of $\omega$ ($\omega$ is small, but not negligible). The cylinder is aligned with gravity in the z-direction and is held in place vertically while it is rotating. You can assume that there are no pressure gradients in the $\theta$ and z-direction.

**Side view**




**Top View**




<u>Note:</u>

1. The flow is fully developed and at steady-state.
2. Make all the assumptions that seem physical to you for velocity and pressure dependences to reduce the complexity. Explicitly state your assumptions motivated from physical arguments in a couple of words.
3. A handout with the Navier-Stokes equations is attached to the exam.
4. If you are running out of time, make sure to write the relevant equations and appropriate boundary conditions to get <u>partial credit</u>.
5. There are <u>**2 parts to this problem**</u>

a) Determine the velocity profile(s) of the fluid. Solve them in terms of the known constants $a, R, \omega, \rho, g$, and $\mu$. You need not obtain the pressure profiles. Box your final velocity profiles.

$\omega$ is small

$$\underline{v} = \cancel{v_r \underline{e}_r} + v_\theta \underline{e}_\theta + v_z \underline{e}_z$$

Symmetry

$$v_\theta = v_\theta(r, \cancel{\theta}, \cancel{z})$$

$v_\theta$ same at all z

Symmetry

$$v_z = v_z(r, \cancel{\theta}, \cancel{z})$$

Fully developed

Continuity Equation as a check:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

Incompressible

$$\cancel{\frac{\partial \rho}{\partial t}} + \rho\left(\frac{1}{r}\frac{\partial(r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}\right) = 0$$

$v_r = 0$ Fully developed

$$\frac{1}{r}\frac{\partial(r \cancel{v_r})}{\partial r} + \frac{1}{r}\cancel{\frac{\partial v_\theta}{\partial \theta}} + \cancel{\frac{\partial v_z}{\partial z}} = 0$$

$$v_\theta \neq v_\theta(\theta)$$

$\theta$ direction:

$$\rho\left(\cancel{\frac{\partial v_\theta}{\partial t}} + \cancel{v_r}\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\cancel{\frac{\partial v_\theta}{\partial \theta}} + \frac{v_\theta \cancel{v_r}}{r} + v_z\cancel{\frac{\partial v_\theta}{\partial z}}\right)$$
$$= -\frac{1}{r}\cancel{\frac{\partial P}{\partial \theta}} + \rho \cancel{g_\theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\cancel{\frac{\partial^2 v_\theta}{\partial \theta^2}} + \frac{2}{r^2}\cancel{\frac{\partial v_r}{\partial \theta}} + \cancel{\frac{\partial^2 v_\theta}{\partial z^2}}\right]$$

$$0 = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right)\right]$$

$$C_1 = \frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)$$

$$C_1 r = \frac{\partial}{\partial r}(r v_\theta)$$

$$C_2 r^2 + C_3 = r v_\theta$$

$$v_\theta(r) = C_2 r + \frac{C_3}{r}$$

$z$ direction:

$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}} + \cancel{v_r}\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\cancel{\frac{\partial v_z}{\partial \theta}} + v_z\cancel{\frac{\partial v_z}{\partial z}}\right) = -\cancel{\frac{\partial P}{\partial z}} + \rho g_z + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\cancel{\frac{\partial^2 v_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right]$$

$$0 = -\rho g + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right]$$

$$\frac{\rho g}{\mu} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)$$

$$\frac{\rho g r}{\mu} = \frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)$$

$$\frac{\rho g r^2}{2\mu} + B_1 = r\frac{\partial v_z}{\partial r}$$

$$\frac{\rho g r}{2\mu} + \frac{B_1}{r} = \frac{\partial v_z}{\partial r}$$

$$v_z(r) = \frac{\rho g r^2}{4\mu} + B_1 \ln(r) + B_2$$

Boundary conditions

1. $@\ r = R, v_\theta(R) = \omega R$
2. $@\ r = aR, \tau_{r\theta}|_{r=aR,Liq} = \tau_{r\theta}|_{r=aR,Air} = 0$
3. $@\ r = R,\ v_z(R) = 0$
4. $@\ r = aR, \tau_{rz}|_{r=aR,Liq} = \tau_{rz}|_{r=aR,Air} = 0$

Apply BC1:

$$v_\theta(R) = \omega R = C_2 R + \frac{C_3}{R}$$

$$\omega = C_2 + \frac{C_3}{R^2}$$

$$C_2 = \omega - \frac{C_3}{R^2}$$

Apply BC2:

$$\tau_{r\theta}(aR) = \ \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] = 0$$

$$\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]_{r=aR} = 0$$

$$\left[\frac{\partial}{\partial r}\left(\frac{C_2 r + \frac{C_3}{r}}{r}\right)\right]_{r=aR} = 0$$

$$\left[\frac{\partial}{\partial r}\left(C_2 + \frac{C_3}{r^2}\right)\right]_{r=aR} = 0$$

$$-\frac{2C_3}{(aR)^3} = 0$$

$$C_3 = 0$$

$$C_2 = \omega$$

$$v_\theta(r) = \omega r$$

Apply BC3:

$$v_z(R) = 0 = \frac{\rho g R^2}{4\mu} + B_1 \ln(R) + B_2$$

Apply BC4:

$$\tau_{rz}|_{r=\kappa R} = \mu\left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right]_{r=aR} = 0$$

$$\left[\frac{\partial v_z}{\partial r}\right]_{r=aR} = 0$$

$$\frac{\rho g a R}{2\mu} + \frac{B_1}{aR} = 0$$

$$B_1 = -\frac{\rho g}{2\mu}(aR)^2$$

Back to BC3:

$$v_z(R) = 0 = \frac{\rho g R^2}{4\mu} - \frac{\rho g}{2\mu}(aR)^2 \ln(R) + B_2$$

$$B_2 = \frac{\rho g}{2\mu}(aR)^2 \ln(R) - \frac{\rho g R^2}{4\mu}$$

$$v_z(r) = \frac{\rho g r^2}{4\mu} - \frac{\rho g}{2\mu}(aR)^2 \ln(r) + \frac{\rho g}{2\mu}(aR)^2 \ln(R) - \frac{\rho g R^2}{4\mu}$$

Velocity profiles:

$$v_z(r) = \frac{\rho g}{4\mu}(r^2 - R^2) + \frac{\rho g (aR)^2}{2\mu}[\ln(R) - \ln(r)]$$

$$v_\theta(r) = \omega r$$

b) Based off of your intuition or velocity profiles derived in part (a), draw the path of two massless colloids or balls, D1 and D2, that were released into the fluid, one at $r = R$ and the other at $r = aR$, when the cylinder rotates $2\pi$.
(The paths need not be drawn exactly).