PHYSICS 8A, Lecture 2 – Spring 2017
Midterm 2, C. Bordel
Thursday, April 6th, 7pm-9pm

- Student name:
- Student ID #:
- Discussion section #:
- Name of your GSI:
- Day/time of your DS:

Physics Instructions

In the absence of any information about the size of the objects, treat them as point masses. Assume that air resistance is negligible, and consider that the acceleration of gravity has magnitude g at the surface of the Earth.
Remember that you need to show your work in order to get full credit!

Math Information Sheet

- For circle of radius r:

  Circumference: $2\pi r$

  Area: πr^2

- Quadratic Equations
  The solution so the equation $0 = ax^2 + bx + c$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

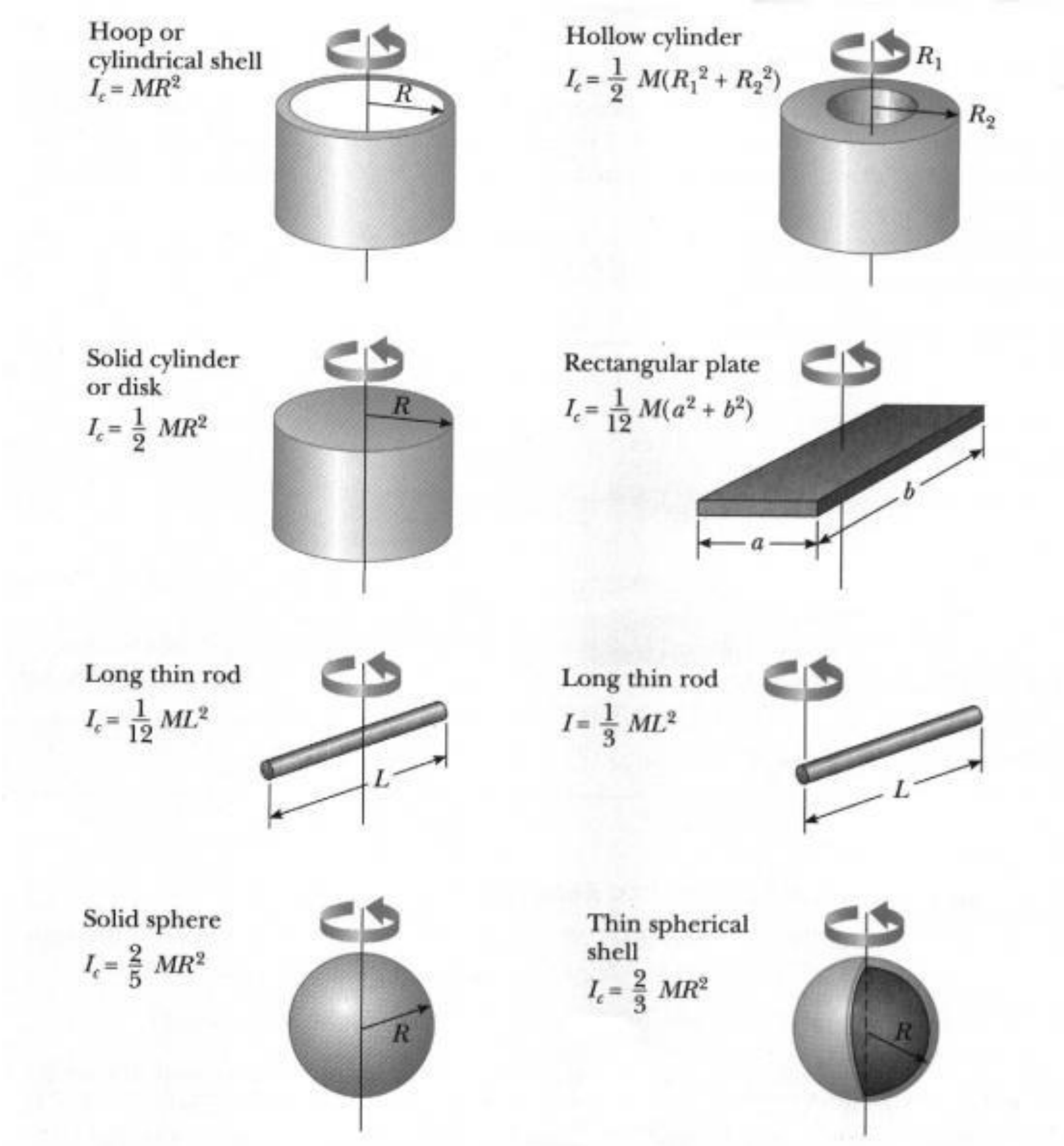
- Derivatives

$$\frac{d(x^\alpha)}{dx} = \alpha x^{\alpha-1}$$

- Integrals

$$\int x^n dx = \frac{1}{n+1} x^{n+1}$$

Rotational inertias



Problem 1 – Ball launcher (25 pts)

A child's toy consists of a block that attaches to a table, a spring connected to that static block, a ball of negligible size, and a launching ramp (Fig.1). The spring has a spring constant k, the ball has a mass m, and the ramp rises a height h above the table, the surface of which is a height H above the floor. The spring rests initially at its equilibrium length and is then compressed a distance L, where the ball is held at rest. The ball is then released and launched up the ramp. When the ball leaves the launching ramp, its velocity vector makes an angle θ with respect to the horizontal. Air resistance and friction are considered negligible.

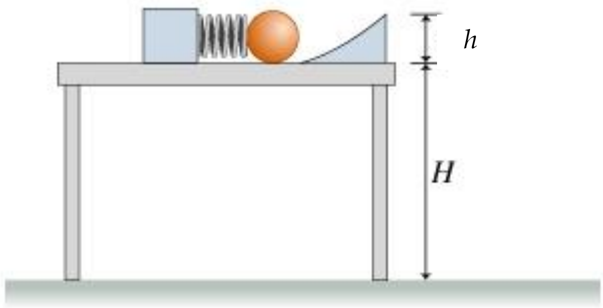


Figure 1

Take the origin of the *x*-*y* coordinate system on the floor along the vertical axis, and where the spring is fully compressed along the horizontal axis.

a. Sketch the variation in potential energy of the ball-spring system as a function of the ball's horizontal displacement. Keep in mind that the ball is *not* attached to the spring, and neglect any recoil of the spring after the ball loses contact with it.

b. Explain how energy transforms between the position of full compression and the equilibrium position of the spring, then from there to the top of the trajectory of the ball, and finally from the top to the floor.

c. Calculate v_r, the speed of the ball when it leaves the launching ramp.

d. Determine the maximum height h_{max} (measured from the floor) reached by the ball.

e. Determine the *velocity* $\vec{v_f}$ of the ball when it hits the floor. Give the two components of the vector.

Problem 2 – Satellite and asteroid (25 pts)

A satellite of mass m is initially put into circular orbit around the Earth (radius R_E, mass M) at radial distance r_1=2R_E from the centre of the Earth. After observing an asteroid that is on a collision course with the Earth, the satellite moves to a higher orbit at radial distance r_2=3R_E.

You may ignore friction with the atmosphere and the motion of the Earth, and assume that both the satellite and asteroid are under the influence of the Earth's gravitational field exclusively.

a. Show that at any radial distance r, the escape speed v_{esc} of an object is larger than its circular speed v by a factor $\sqrt{2}$.

b. Calculate the work done by the gravitational force from r_1=2R_E to r_2=3R_E.

c. Compare the mechanical energy of the satellite on each orbit, assuming uniform circular motion in each case. Explain why it is (not) conserved.

d. Determine the final speed v_f of the asteroid when it hits the Earth's surface, assuming an initial speed v_i at radial distance $2R_E$.

Problem 3 – Fun at the arcade (25 pts)

Two friends go to an arcade to play air hockey. You may neglect friction and assume perfectly elastic collisions between the two pucks of identical mass m.

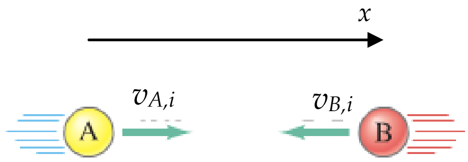


Figure 3.1

a. Explain what physical quantities are conserved during the collisions and why.

b. A head-on collision occurs, where the two pucks have opposite initial velocities (Fig.3.1). Determine the final velocity of each of the two pucks and draw the scenario after the collision.

Then a two-dimensional collision occurs, one of pucks being initially at rest (Fig.3.2).

c. Given that the puck initially in motion goes off with an angle α with respect to its initial direction, determine the two final speeds and the angle β of the second puck with respect to the initial direction (x-axis). Draw the scenario after the collision.

Figure 3.2

d. Explain, without any calculation, what would happen if the 2 pucks were launched at the same speed in opposite directions, as shown in Fig.3.3.

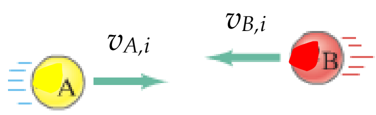


Figure 3.3

Problem 4 – Spinning wheel (25 pts)

A student is standing on a frictionless turntable which is initially at rest. She's holding a wheel of mass m and radius R in a horizontal plane, and the wheel is initially spinning at angular speed ω_1 with respect to its symmetry axis (Fig. 4.1). You may treat the wheel as a cylindrical shell, and consider that the turntable and student have a rotational inertia I. *You may neglect the distance between the axes of the turntable and wheel, or assume that the wheel is held above the person's head.* Assume that the counterclockwise direction (viewed from the top) is positive.

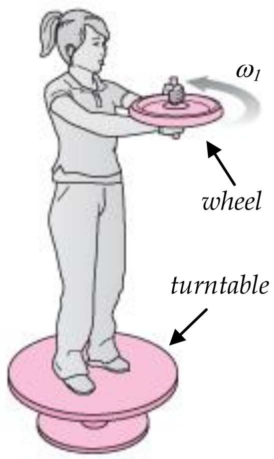


Figure 4.1

Figure 4.2

a. Calculate the net torque acting on the system {turntable+student+wheel} about the vertical central axis.

b. Determine the total angular momentum L.

Then the student turns the wheel upside down, as shown in Fig.4.2.

c. Predict the direction of rotation of the turntable and student after the wheel is reversed. Explain your reasoning.

d. Determine the angular velocity ω_t of the turntable and student after the wheel is reversed.